# Mechanical Engineering

First – year

2025-2026

| المنهاج الاسبوعي النظري | |
|---|---|
| | **Material Covered** |
| Weeks 1&2 | • Demonstrates knowledge about the Introduction to mechanics, Force systems, Scalar & vector quantities,<br>• Able to identify and apply the Parallelogram law, Triangle law, Forces & components. |
| Weeks 3&4 | • Able to identify and apply the Moment of a force , Varignon's theorem, and their Applications |
| Weeks 5&6 | • Demonstrates knowledge of the Couples,<br>• Able to identify Resolution of a force into a force & a couple. |
| Weeks 7&8 | • Demonstrates knowledge and correctly compute the Resultant of force systems, Resultant of concurrent force system, Resultant of parallel force system, Resultant of non-concurrent force system. |
| Weeks 9&10 &11 | • Demonstrates knowledge, identify and correctly compute Equilibrium of force system, Free body diagram, Equilibrium of concurrent force system, Equilibrium of parallel force system, Equilibrium of non-concurrent force system. |
| Week 12 | • Demonstrates knowledge of the Types of beams, Supports, and loads, Equilibrium of beams. |
| Weeks 13&14 ,15 | • Demonstrates knowledge of the Trusses<br>• Able to analyses the trusses, method of Joint, method of section. |

**References:**

1. Engineering Mechanics / F.L. Singer
2. Engineering Mechanics / Mclean & Nelson
3. Engineering Mechanics / J.F. Shelley
4. Engineering Mechanics / A. Higdon & W.B. Stiles
4. Mechanics for Engineers / Statics / F.P. Beer , E.R. Johnston,Jr

**Mechanics:** Is the branch of physics that considers the action of forces on bodies or fluids that are at rest or in motion. The primary topics of mechanics are **Statics** and **Dynamics**.

**Engineering mechanics:** Is the branch of engineering that applies the principles of mechanics to mechanical design (i.e., any design that must take into account the effect of forces).

**Statics:** Deals with the study of forces acting on physical bodies in static equilibrium. (Not motion)

**Dynamics**: Is that branch of mechanics which deals with the motion of bodies under the action of forces.

## Quantities

**الكميات الاتجاهية و غير الاتجاهية (العددية)**

**Scalars:** A scalar is a physical quantity that has magnitude but no direction such as: Time, length, mass, speed.

**Vector:** Is a physical quantity that has both magnitude and direction. For example: force, velocity.

# The study of elementary mechanics rests on six fundamental principles, based on experimental evidence.

**The Parallelogram Law for the Addition of Forces.**

- Two forces acting on a particle may be replaced by a single force, called their *resultant*
- **The Principle of Transmissibility.** The conditions of equilibrium or of motion of a rigid body remain unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action

**Newton's Three Laws of Motion.** Formulated by Sir Isaac Newton in the late seventeenth century, these laws can be stated as follows:

**FIRST LAW.** If the resultant force acting on a particle is zero, the particle remains at rest (if originally at rest) or moves with constant speed in a straight line (if originally in motion)

**SECOND LAW.** If the resultant force acting on a particle is not zero, the particle has an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

$\mathbf{F} = m\mathbf{a}$

where **F**, $m$, and **a** represent, respectively, the resultant force acting on the particle, the mass of the particle, and the acceleration of the particle expressed in a consistent system of units.

**THIRD LAW.** The forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite sense .

- W of the weight of a particle of mass m as
- W = m x g
- g equals 9.81 $m/s^2$ or 32.2 $ft/s^2$.

# Systems of Units

- Four fundamental concepts just discussed are the so-called *kinetic units*, i.e., the units of ***length***, ***time***, ***mass***, and ***force***.
- **1-International System of Units (SI Units).** In this system, SI units, the base units are the units of **length**, **mass**, and **time**, and they are called, respectively, the **meter (m)**, the **kilogram (kg)**, and the **second (s)**.
- It is called the newton (N) and is defined as the force that gives an acceleration of 1 $m/s^2$ to a body of mass 1 kg .

- $W = mg$
  = (1 kg) (9.81 m/s2)
  = 9.81 N

$$1\ \text{km} = 1000\ \text{m} \quad 1\ \text{mm} = 0.001\ \text{m}$$
$$1\ \text{Mg} = 1000\ \text{kg} \quad 1\ \text{g} = 0.001\ \text{kg}$$
$$1\ \text{kN} = 1000\ \text{N}$$

## Table 1.2 Principal SI Units Used in Mechanics

| Quantity | Unit | Symbol | Formula |
|---|---|---|---|
| Acceleration | Meter per second squared | . . . | $m/s^2$ |
| Angle | Radian | rad | † |
| Angular acceleration | Radian per second squared | . . . | $rad/s^2$ |
| Angular velocity | Radian per second | . . . | rad/s |
| Area | Square meter | . . . | $m^2$ |
| Density | Kilogram per cubic meter | . . . | $kg/m^3$ |
| Energy | Joule | J | N·m |
| Force | Newton | N | $kg·m/s^2$ |
| Frequency | Hertz | Hz | $s^{-1}$ |
| Impulse | Newton-second | . . . | kg·m/s |
| Length | Meter | m | ‡ |
| Mass | Kilogram | kg | ‡ |
| Moment of a force | Newton-meter | . . . | N·m |
| Power | Watt | W | J/s |
| Pressure | Pascal | Pa | $N/m^2$ |
| Stress | Pascal | Pa | $N/m^2$ |
| Time | Second | s | ‡ |
| Velocity | Meter per second | . . . | m/s |
| Volume | | | |
| Solids | Cubic meter | . . . | $m^3$ |
| Liquids | Liter | L | $10^{-3}\ m^3$ |
| Work | Joule | J | N·m |

†Supplementary unit (1 revolution = $2\pi$ rad = 360°).

‡Base unit.

# 2-U.S. Customary Units.

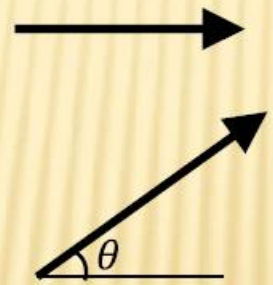
Most practicing American engineers still commonly use a system in which the base units are those of **length, force**, and **time**. These units are, respectively, the ***foot*** **(ft),** the ***pound*** **(lb),** and the ***second*** **(s)**.

- ***foot is defined as 0.3048 m.***
- ***pound*** **(lb)*= 0.453 592 43 kg.***
- **g =32.2ft/s$^2$**
- Other U.S. customary units frequently encountered in engineering problems are the ***mile*** **(mi), equal to 5280 ft**; the ***inch*** **(in.) equal to (1/12) ft**; and the *kilopound* (kip), equal to 1000 lb.
- **1 mile=1609m , 1ft=0.3048m , 1in=2.54 cm**

# Forces

**Force:** Is the action of one body on another. A force tends to move a body in the direction of action. Force is a vector quantity (Magnitude and direction).

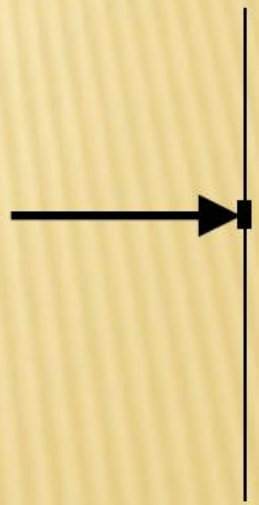
| Magnitude | Direction | Point of action. |
|---|---|---|
| 10 kN | East | |
| 1000 N | North | |




**10 kN East**

**10 kN**





## Engineering Mechanics

**Force**: Is the action of one body on another. A force tends to move a body in the direction of action. Force is a vector quantity (Magnitude and direction).

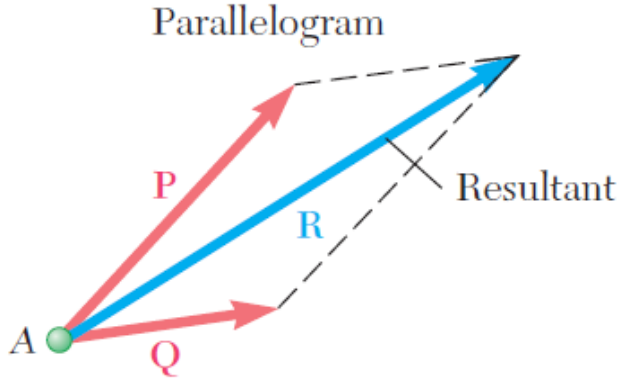
1. Magnitude.
2. Direction.
3. Point of action.

### Units

| Quantity | Dimensional Symbol | SI Units | |
|---|---|---|---|
| | | Unit | Symbol |
| Mass | m | Kilogram | kg |
| Length | L | Meter | m |
| Time | T | Second | s |
| Force | F | Newton | N |

# Force on a Particle: Resultant of Two Forces

- A force represents the action of one body on another. It is generally characterized by its **point of application**, its **magnitude**, and its **direction**.
- Experimental evidence shows that two forces **P** and **Q** acting on a particle ***A*** as Fig. Shown can be replaced by a single force **R** that has the same effect on the particle .This force is called the **resultant**
- . The diagonal that passes through *A* represents the resultant. This method for finding the resultant is known as the **parallelogram** law for the addition of two forces. This law is based on experimental evidence; it cannot be proved or derived mathematically.



- **Combination of forces.**
- **Resolution of forces.**

## Combination of forces (Adding Forces)

**Parallelogram law. (Graphical Solution).**



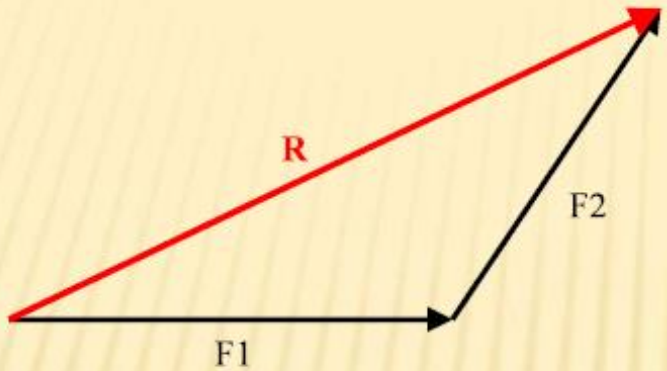
**Triangle law.**



# Triangle law

When two vectors (Forces) are jointed head to tail

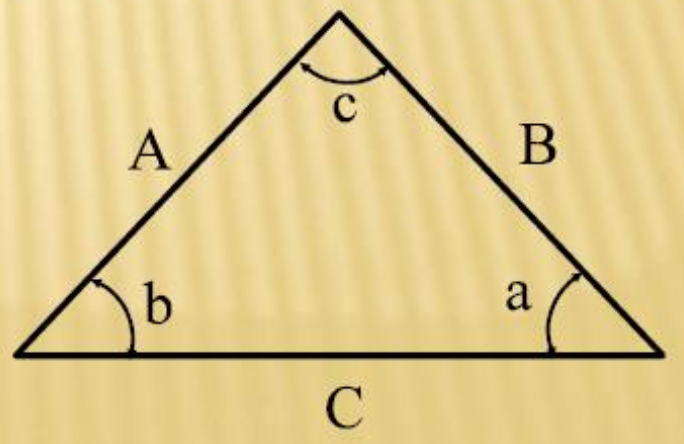
Draw the resultant vectors (forces) by completing the triangle.



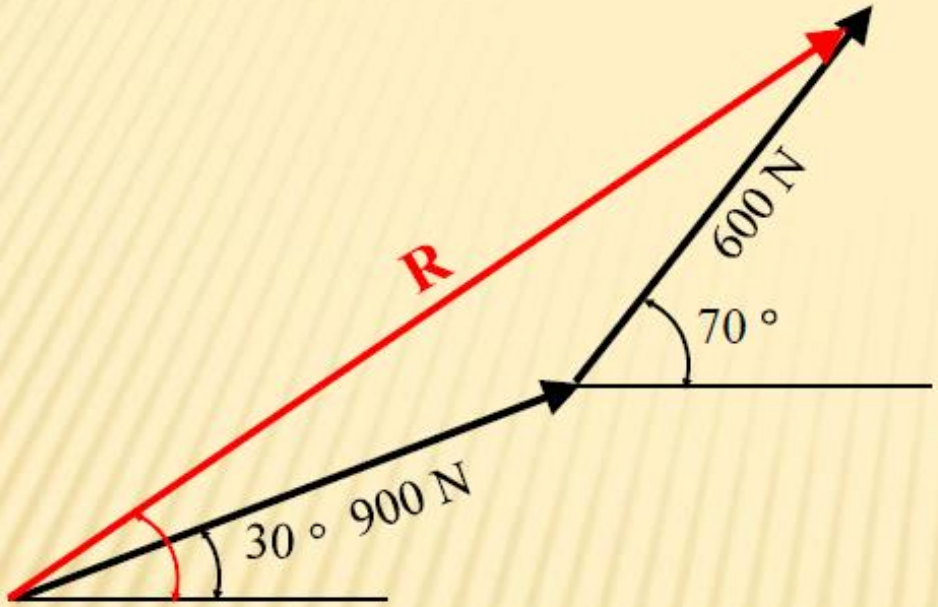

**Trigonometry:** Resultant Force and Components from Law of Cosines and Law of Sines

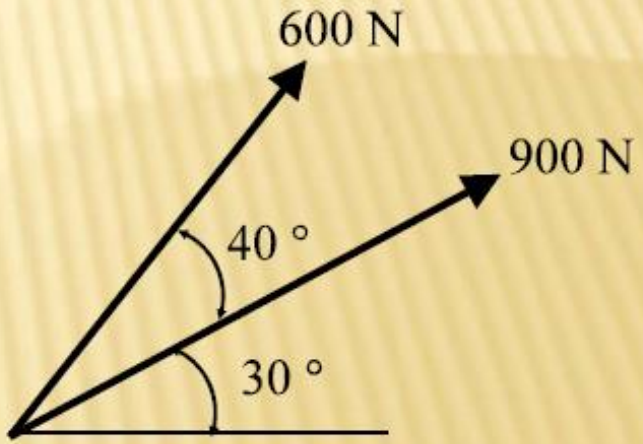
**Cosine Law** $$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

**Sine Law** $$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$



**Example**: Compute the magnitude and direction the resultant of the two forces shown in figure.



$R = ?$ $\theta = ?$

## Cosine Law

$$R = \sqrt{900^2 + 600^2 - 2(900)(600)\cos 140} = 1413.269\ \text{N}$$

$$\theta = 30 + \beta$$

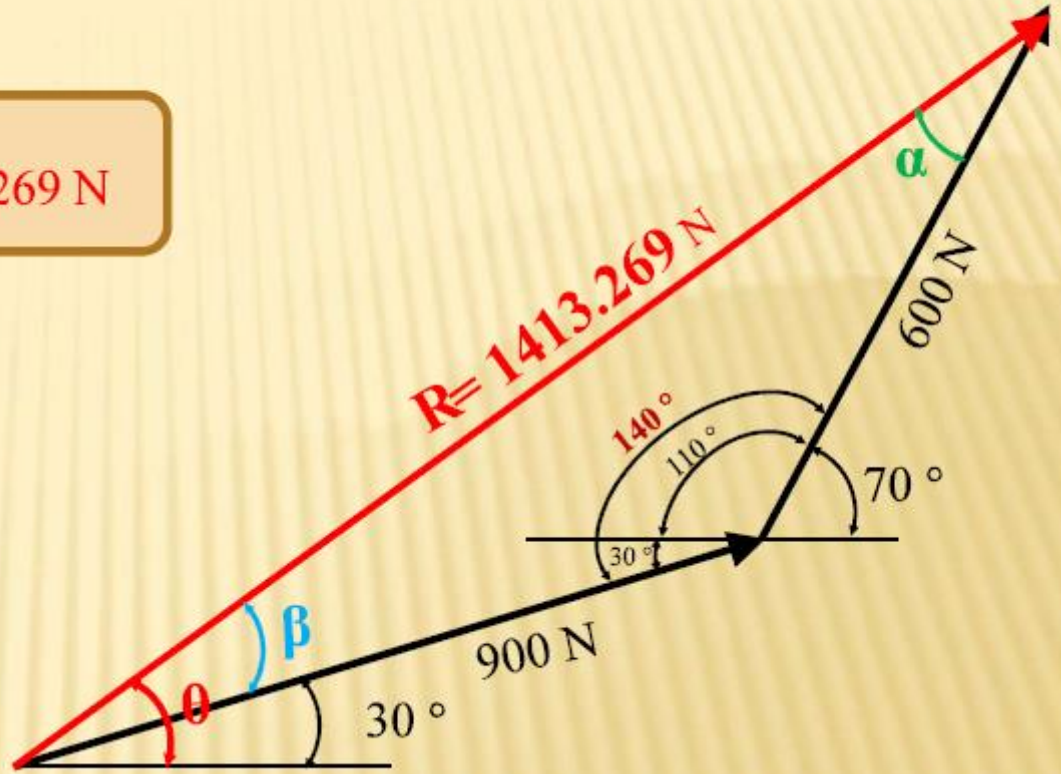
## Sine Law

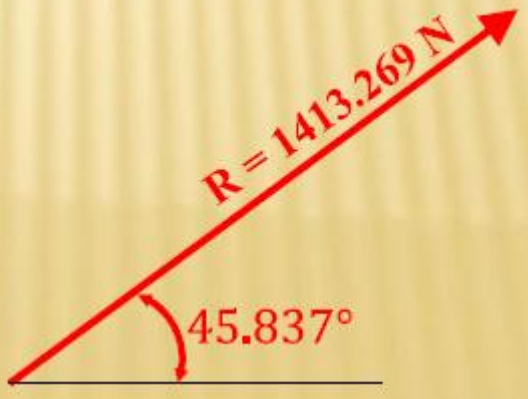
$$\frac{600}{\sin\beta} = \frac{1413.269}{\sin 140} = \frac{900}{\sin\alpha}$$

$$\frac{600}{\sin\beta} = \frac{1413.269}{\sin 140}$$

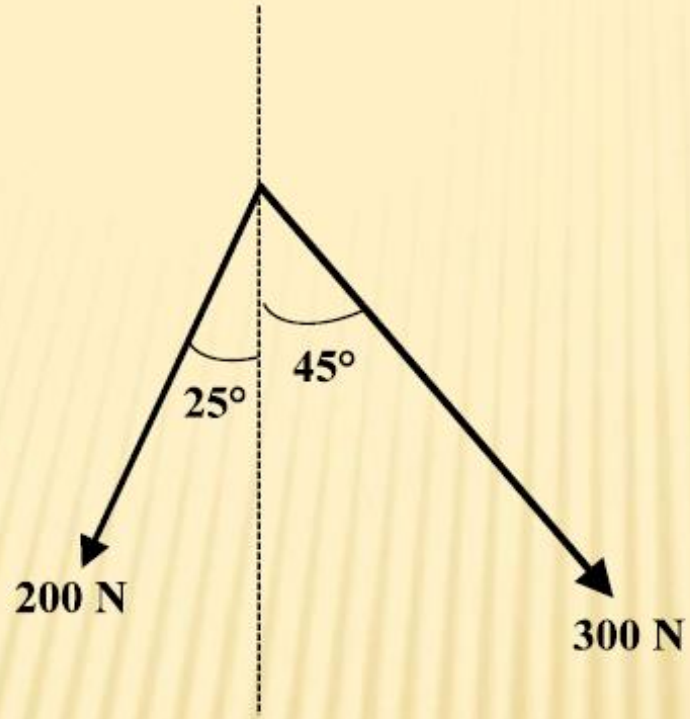
$$\beta = 15.837°$$

$$\therefore \theta = 30 + 15.837 = 45.837°$$



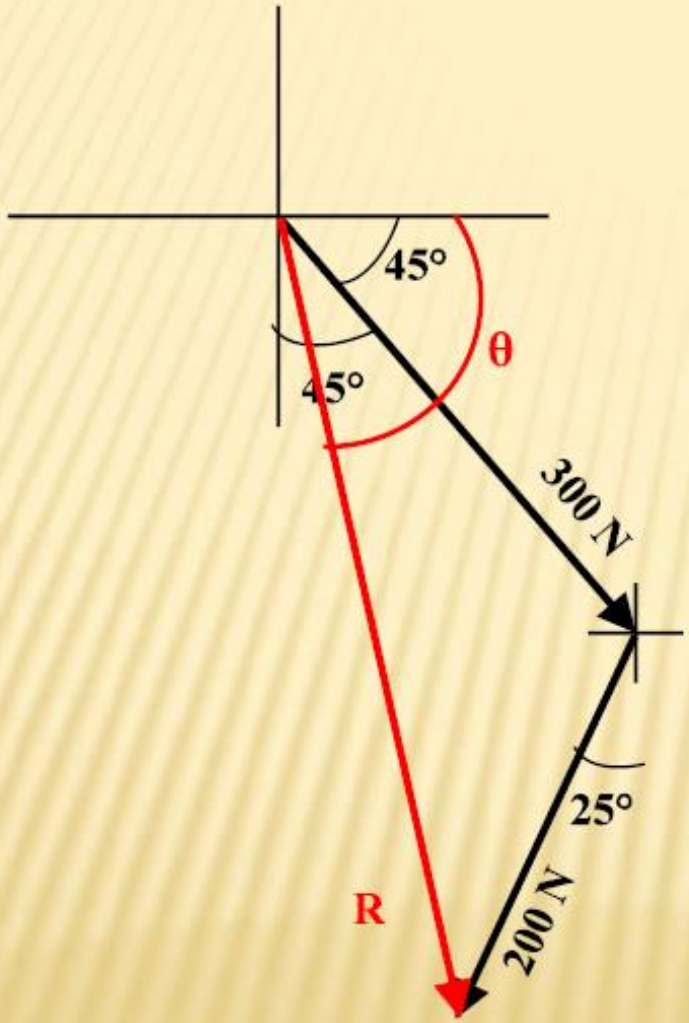



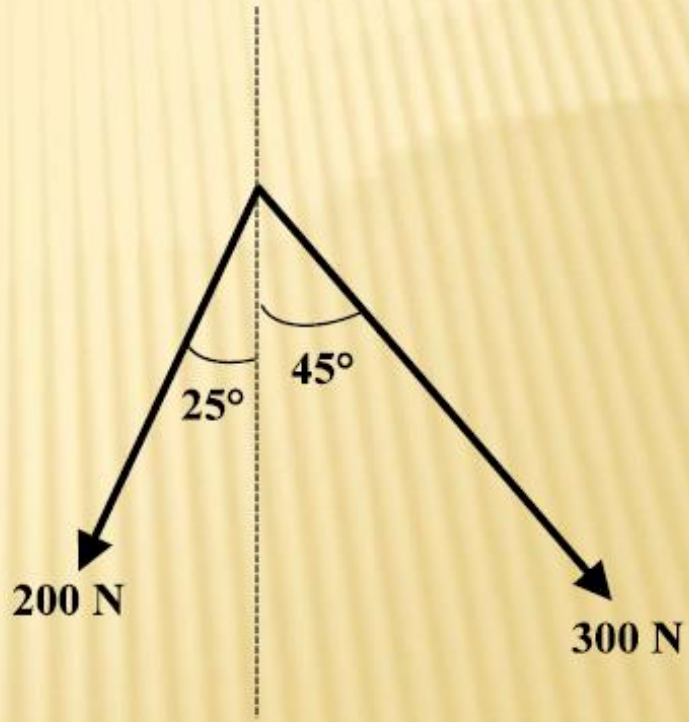
**H.W.** : Compute the magnitude and direction the resultant of the two forces shown in figure.



# Combination of forces

**Example**: Compute the magnitude and direction the resultant of the two forces shown in figure.



$R = ?$ $\theta = ?$

$$\theta = 45° + \beta$$

## Cosine Law

$$R = \sqrt{300^2 + 200^2 - 2(300)(200)\cos 110} = 413.573 \text{ N}$$

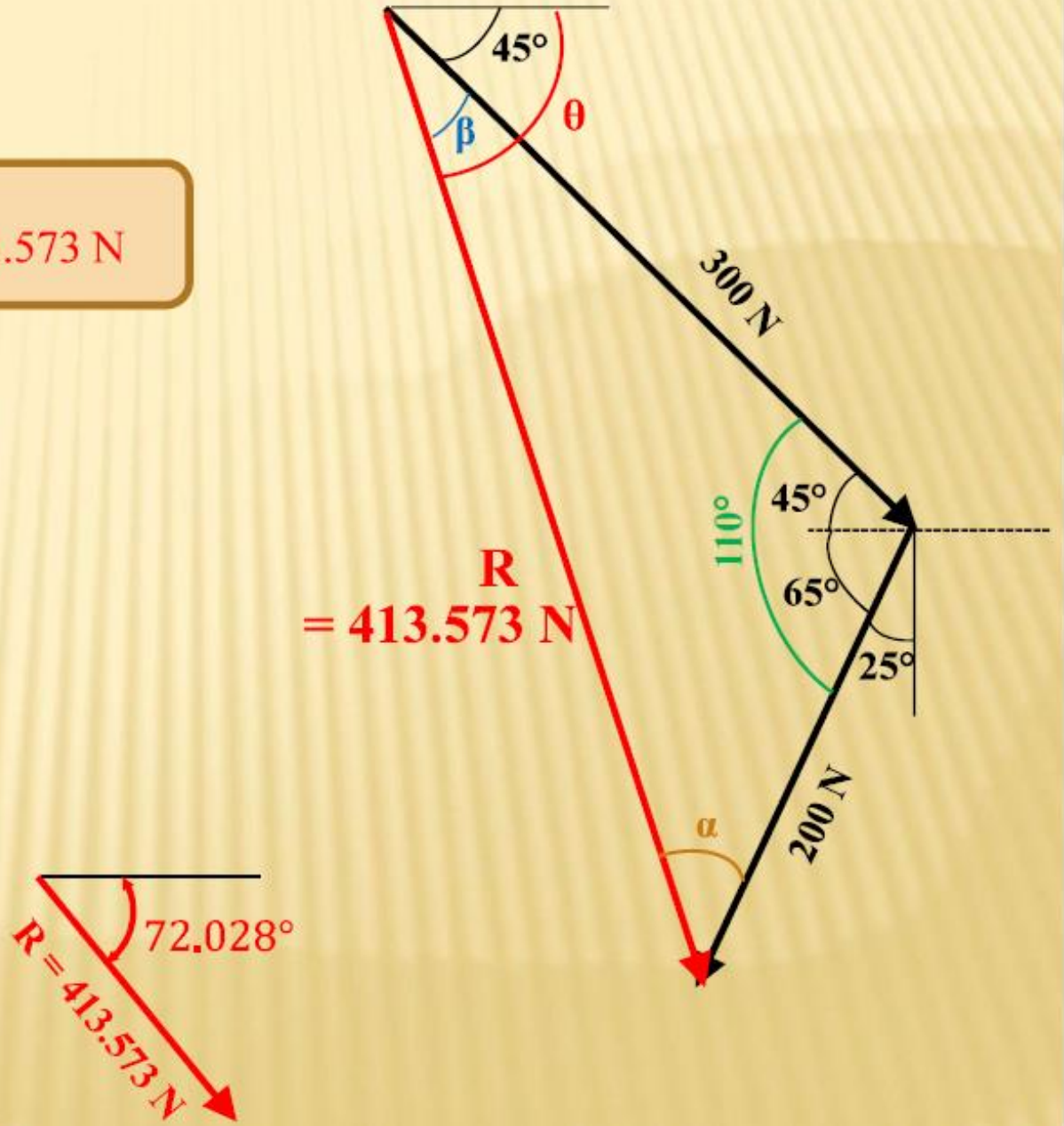
## Sine Law

$$\frac{200}{\sin\beta} = \frac{413.573}{\sin 110} = \frac{300}{\sin\alpha}$$

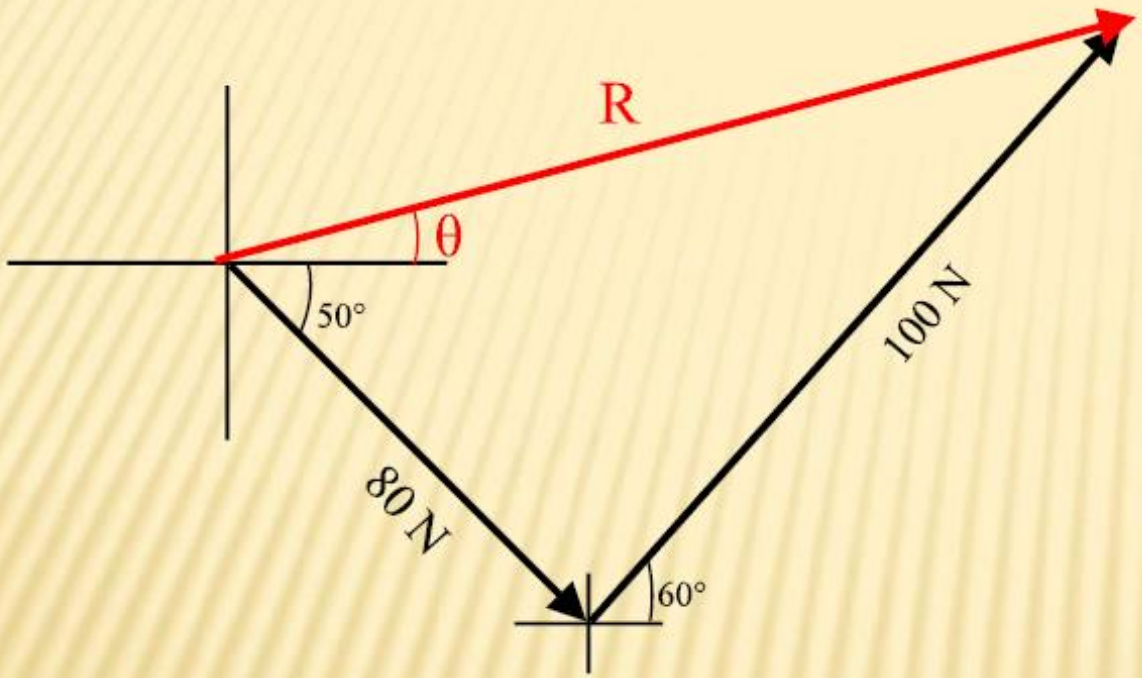
$$\frac{200}{\sin\beta} = \frac{413.573}{\sin 110}$$

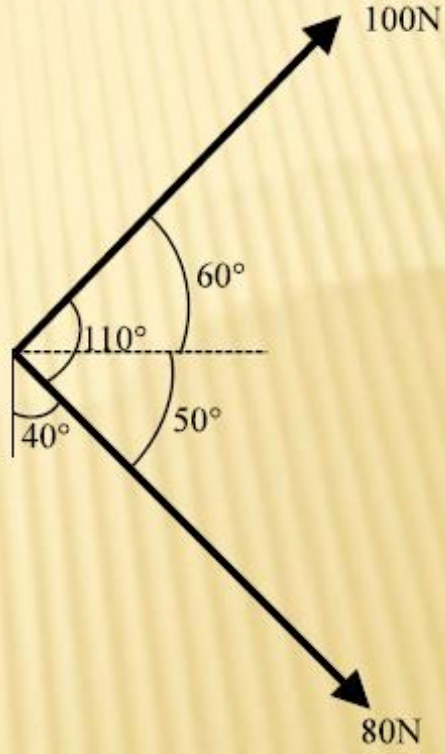
$$\beta = 27.028°$$

$$\therefore \theta = 45 + 27.028 = 72.028°$$



**Example**: Compute the magnitude and direction the resultant of the two forces shown in figure.



$R = ?$ $\theta = ?$

$$\theta = \beta - 50°$$

## Cosine Law

$$R = \sqrt{100^2 + 80^2 - 2(100)(80)\cos 70} = 104.536 \text{ N}$$

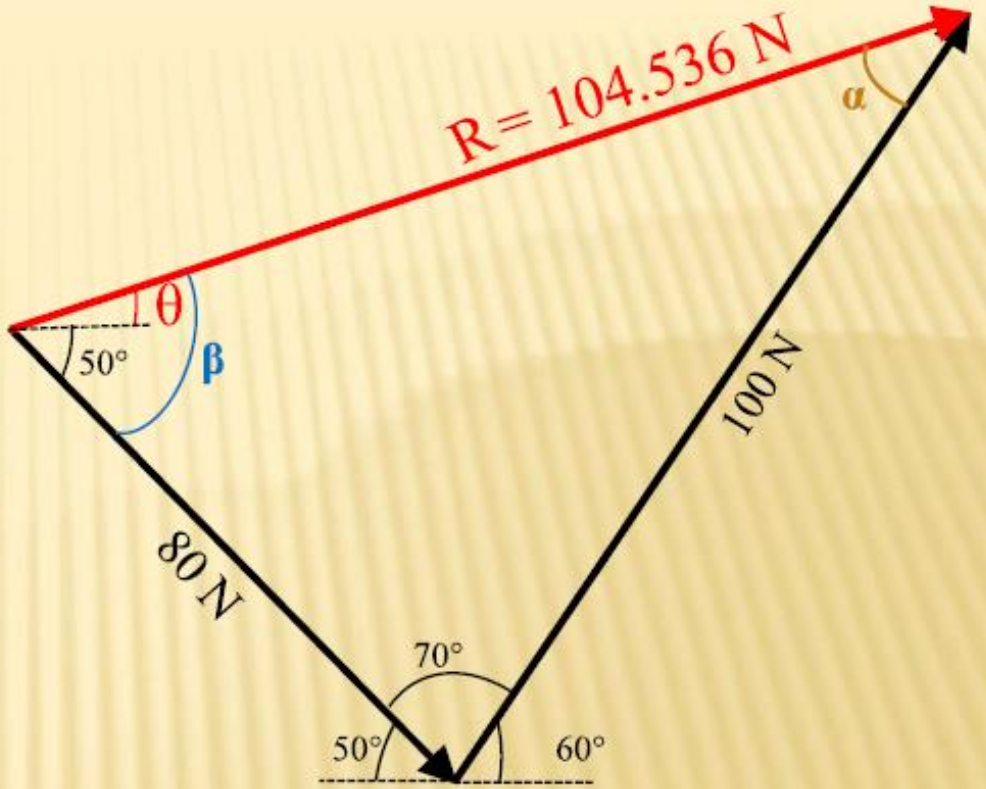
## Sine Law

$$\frac{100}{\sin\beta} = \frac{104.536}{\sin 70} = \frac{80}{\sin\alpha}$$

$$\frac{100}{\sin\beta} = \frac{104.536}{\sin 70}$$

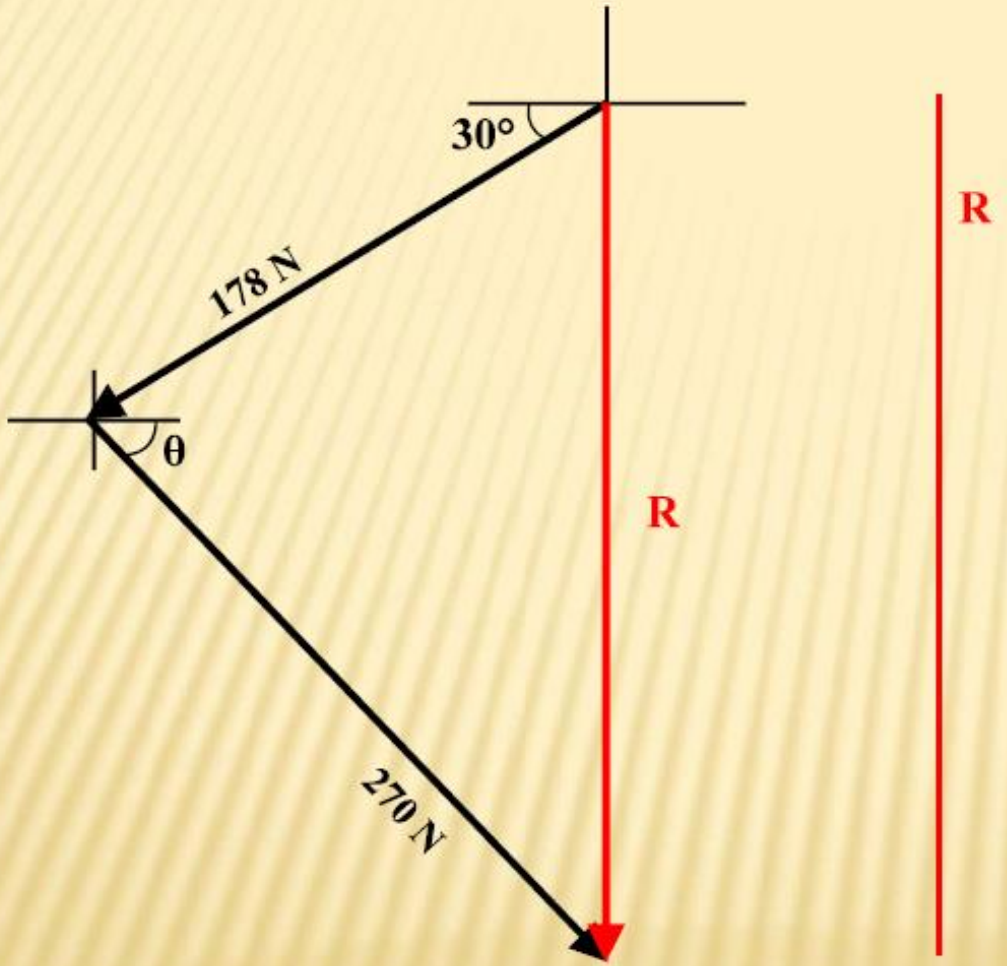
$$\beta = 64.016°$$

$$\therefore \theta = 64.016° - 50 = 14.016°$$

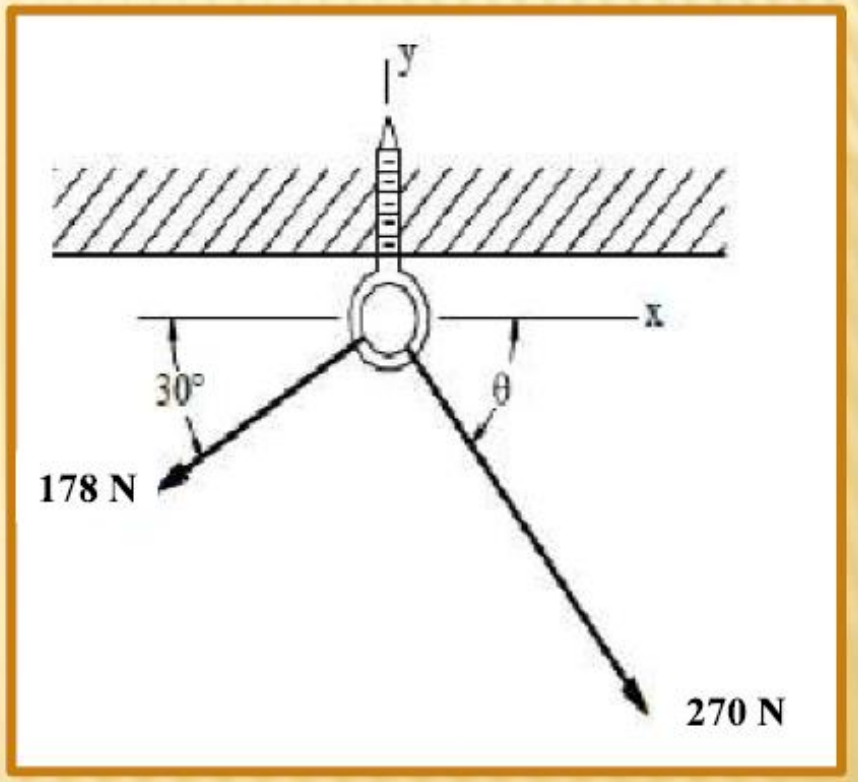





Example: The resultant of the two forces acting on the screw eye is known to be vertical Determine the angle **θ** and the magnitude of the **resultant.**





$\beta = 30° + \theta$

$\theta = 90° - \alpha$

## Sine Law

$$\frac{178}{sin\alpha} = \frac{R}{sin\,\beta} = \frac{270}{sin\,60}$$

$$\frac{178}{sin\alpha} = \frac{270}{sin\,60}$$

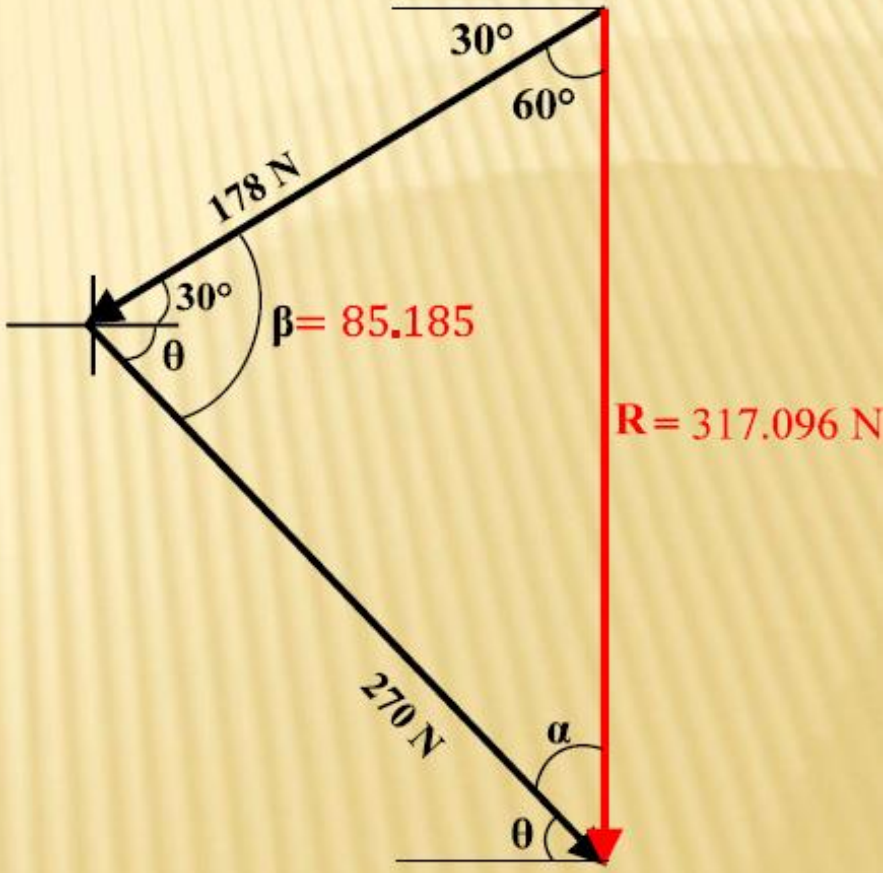
$$\alpha = 34.815°$$

$\theta = 90° - 34.815 = \mathbf{55.185°}$

$\beta = 30° + 55.185 = 85.185\ °$

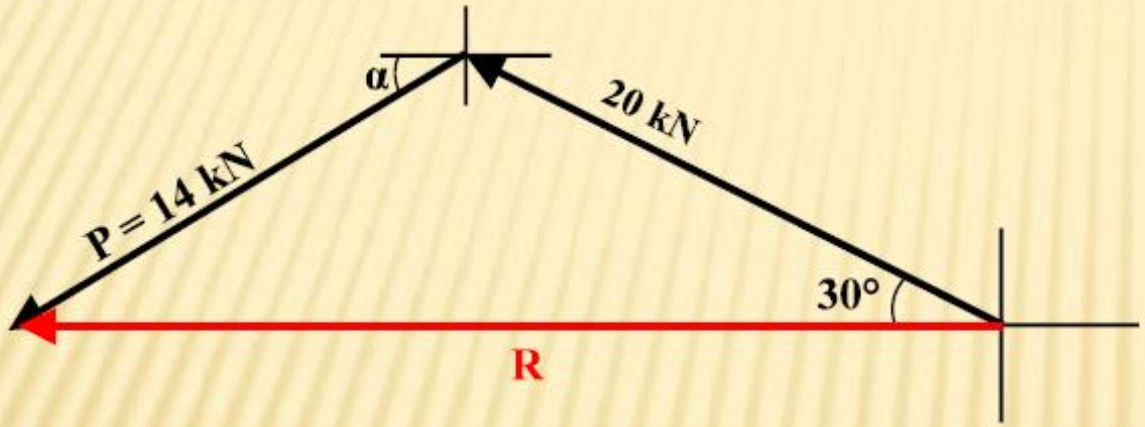
## Cosine Law

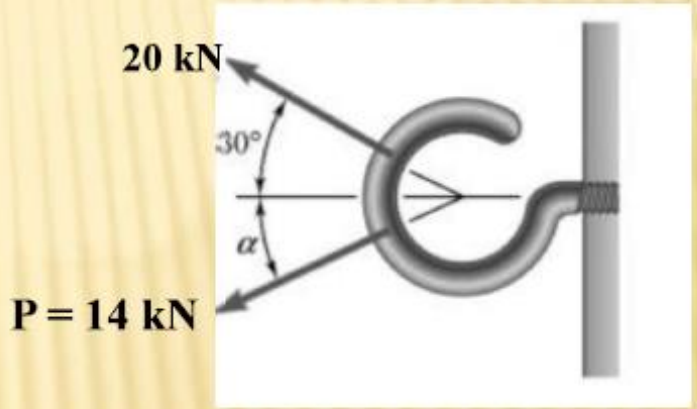
$$R = \sqrt{178^2 + 270^2 - 2\,(178)(270)cos\,85.185} = 317.096\ N$$



**Example:** Two forces are applied as shown in figure to a hook support. Using trigonometry (analytical solution) and knowing the magnitude of **P** is 14 kN, determine:

1. The required angle **α** if the resultant (R) of the two forces applied to the support is to be horizontal.
2. The corresponding magnitude of the resultant (**R**).






$R = ?$ $\alpha = ?$

## Sine Law

$$\frac{20}{\sin\alpha} = \frac{R}{\sin\beta} = \frac{14}{\sin 30}$$

$$\frac{20}{\sin\alpha} = \frac{14}{\sin 30}$$

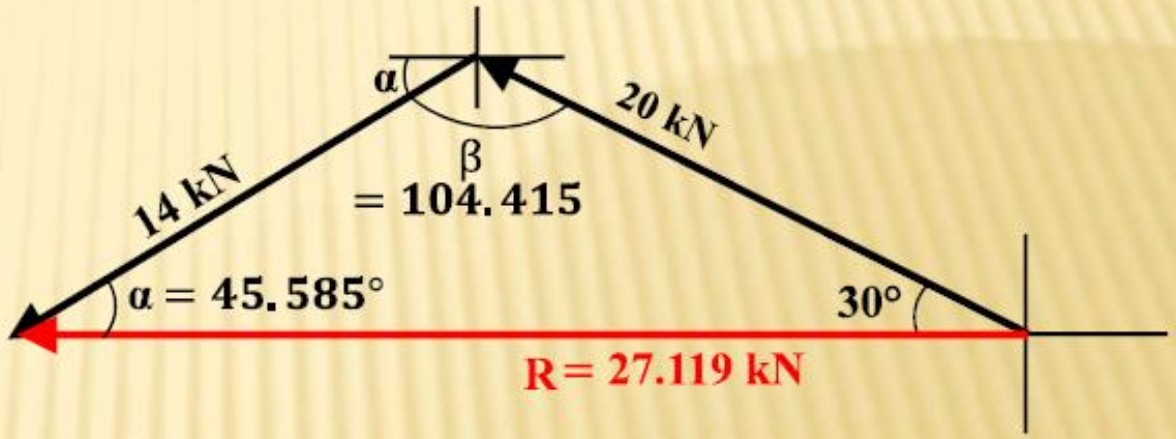
$$\alpha = 45.585°$$

$$\beta = 180° - 30 - 45.585 = 104.415°$$

$$\frac{R}{\sin 104.415} = \frac{14}{\sin 30}$$

$$R = 27.119 \text{ kN}$$

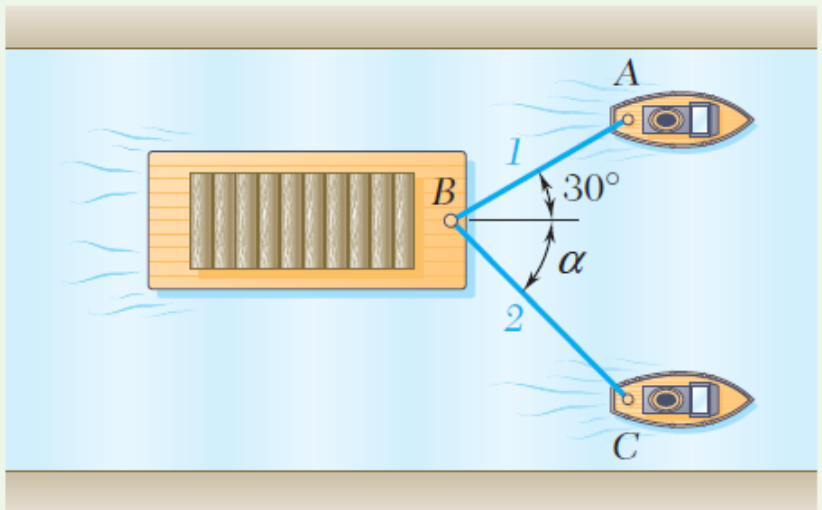
OR




## Cosine Law

$$R = \sqrt{14^2 + 20^2 - 2(14)(20)\cos 104.415} = 27.119 \text{ KN}$$

## Sample Problem 2.2




Two tugboats are pulling a barge. If the resultant of the forces exerted by the tugboats is a 5000-lb force directed along the axis of the barge, determine (*a*) the tension in each of the ropes, given that $\alpha = 45°$, (*b*) the value of $\alpha$ for which the tension in rope *2* is minimum.

**Trigonometric Solution.** Use the triangle rule. Note that the triangle in Fig. 2 represents half of the parallelogram shown in Fig. 1. Using the law of sines,

$$\frac{T_1}{\sin 45^\circ} = \frac{T_2}{\sin 30^\circ} = \frac{5000 \text{ lb}}{\sin 105^\circ}$$

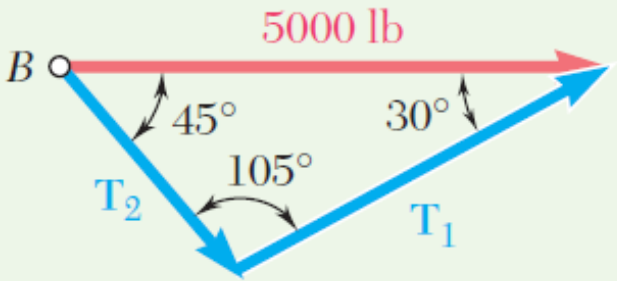



**Fig. 2** Triangle rule applied to add forces $\mathbf{T}_1$ and $\mathbf{T}_2$.

With a calculator, compute and store the value of the last quotient. Multiply this value successively by sin 45° and sin 30°, obtaining

$$T_1 = 3660 \text{ lb} \qquad T_2 = 2590 \text{ lb}$$

**b. Value of $\alpha$ for Minimum $T_2$.** To determine the value of $\alpha$ for which the tension in rope *2* is minimum, use the triangle rule again. In Fig. 3, line *1-1′* is the known direction of $\mathbf{T}_1$. Several possible directions of $\mathbf{T}_2$ are shown by the lines *2-2′*. The minimum value of $T_2$ occurs when $\mathbf{T}_1$ and $\mathbf{T}_2$ are perpendicular (Fig. 4). Thus, the minimum value of $T_2$ is

$$T_2 = (5000 \text{ lb}) \sin 30^\circ = 2500 \text{ lb}$$

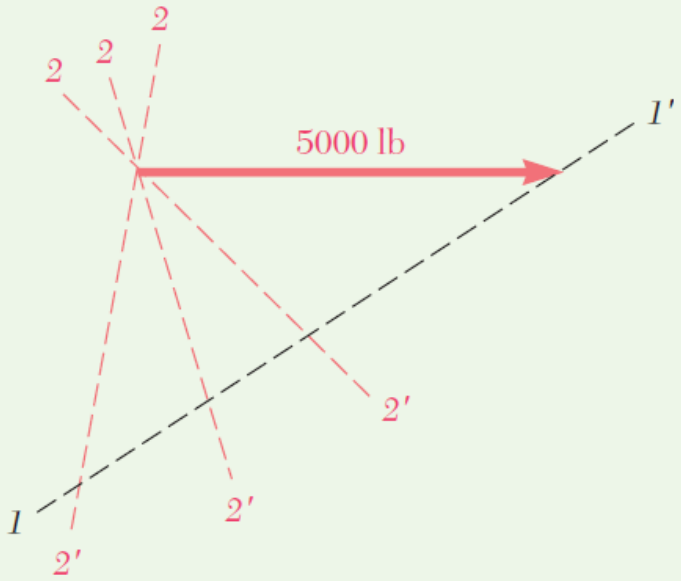



**Fig. 3** Determination of direction of minimum $\mathbf{T}_2$.

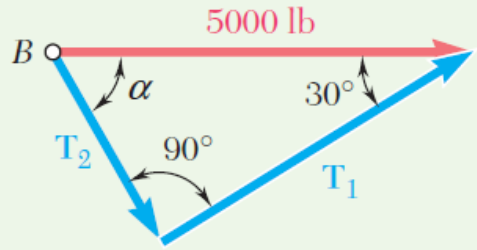



**Fig. 4** Triangle rule applied for minimum $\mathbf{T}_2$.

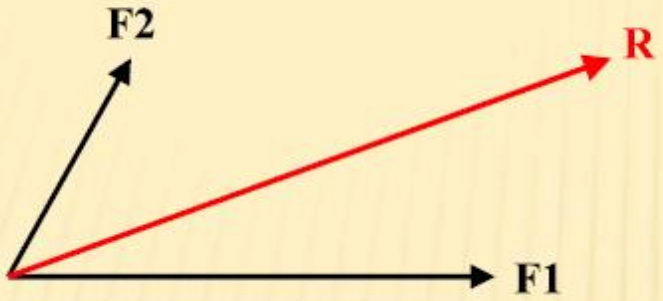
Corresponding values of $T_1$ and $\alpha$ are

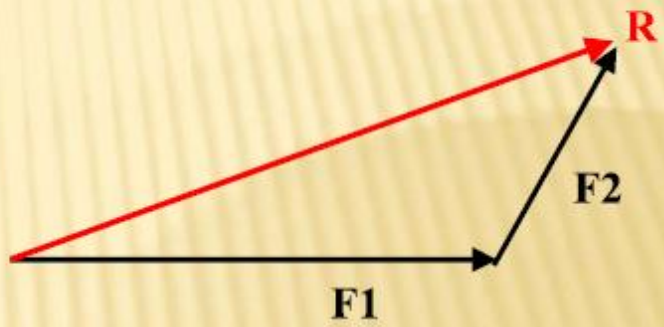
$$T_1 = (5000 \text{ lb}) \cos 30^\circ = 4330 \text{ lb}$$

$$\alpha = 90^\circ - 30^\circ \qquad \alpha = 60^\circ$$

# Resolution of Forces
## Forces and Components

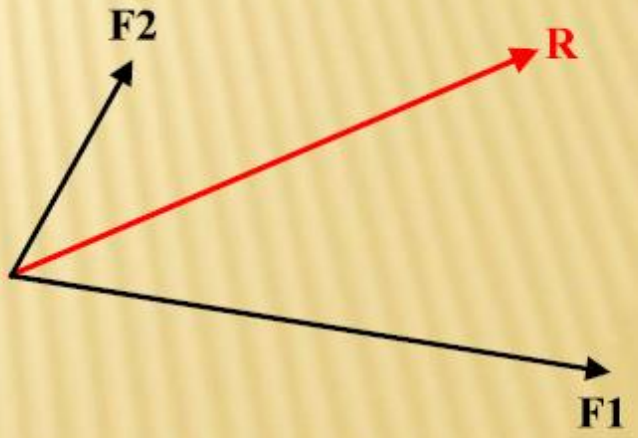
- Combination of forces.




- Resolution of Forces



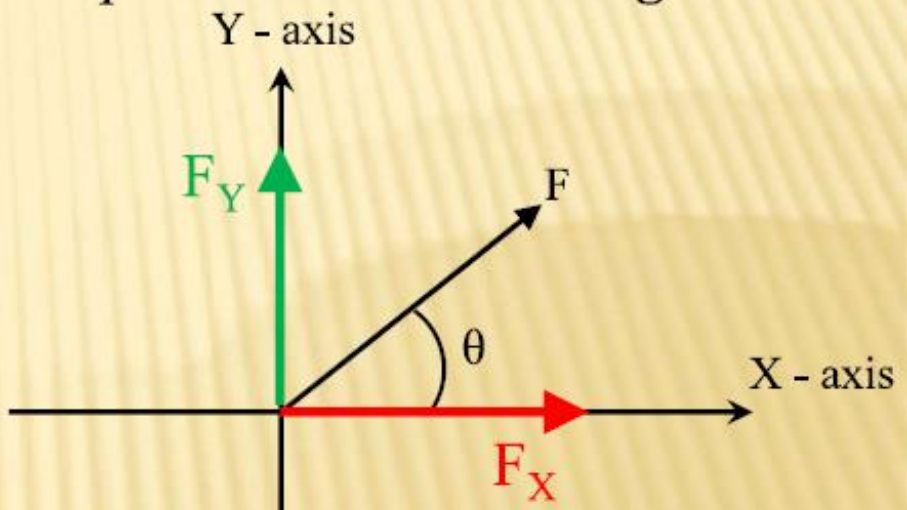



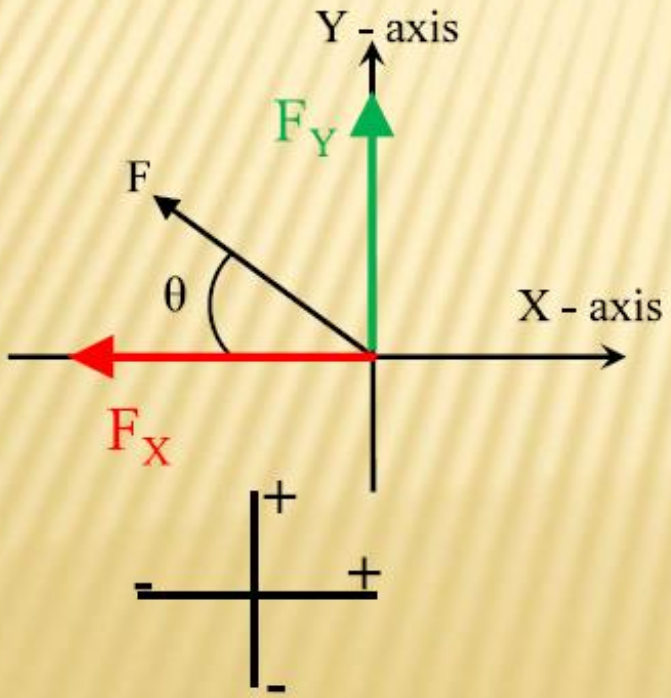


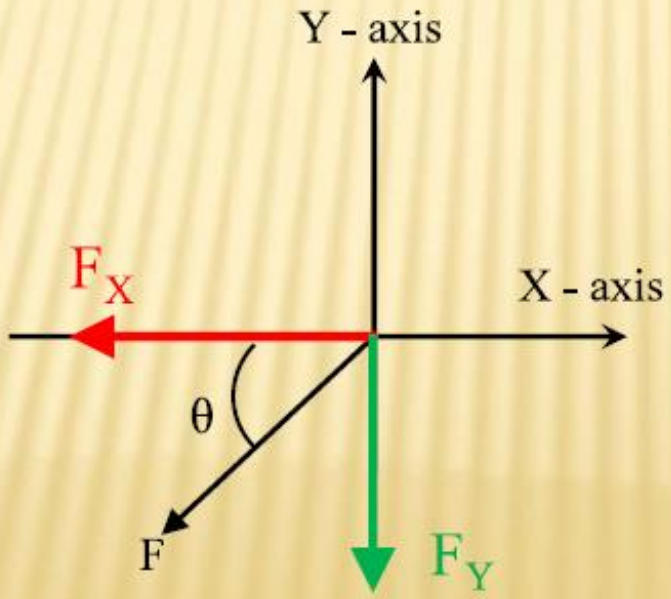

## Resolving a Force into Rectangular Components

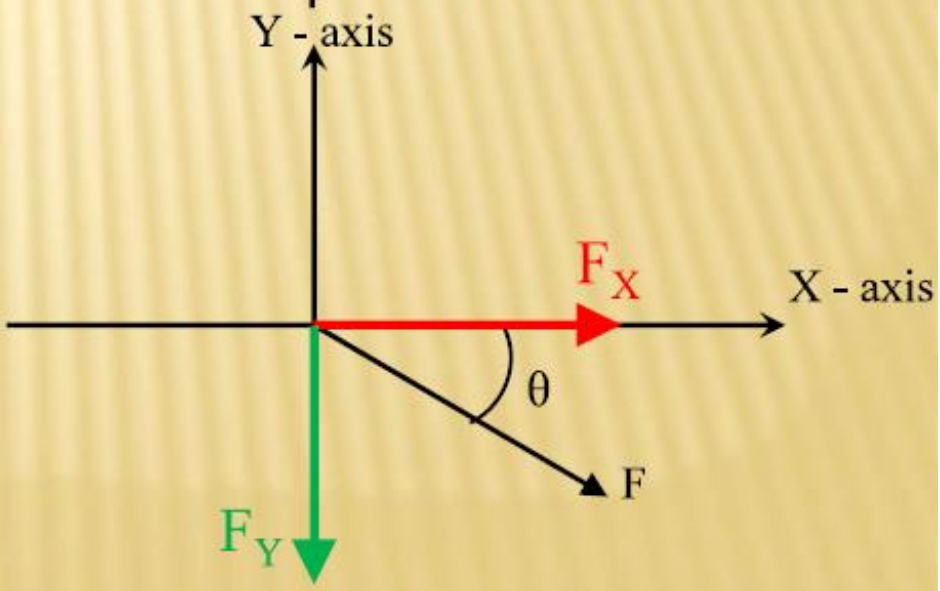
When a force resolved into **perpendicular** axes, the resulting components called **rectangular components.**

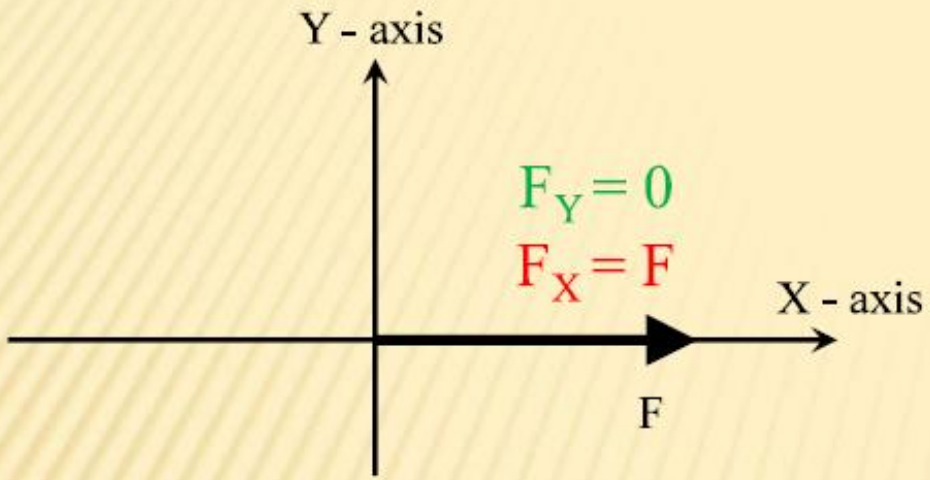
One of the most important and useful concept is the resolution of a force into two perpendicular components, in the X and Y axes as denoted by $F_X$ and $F_Y$ components.

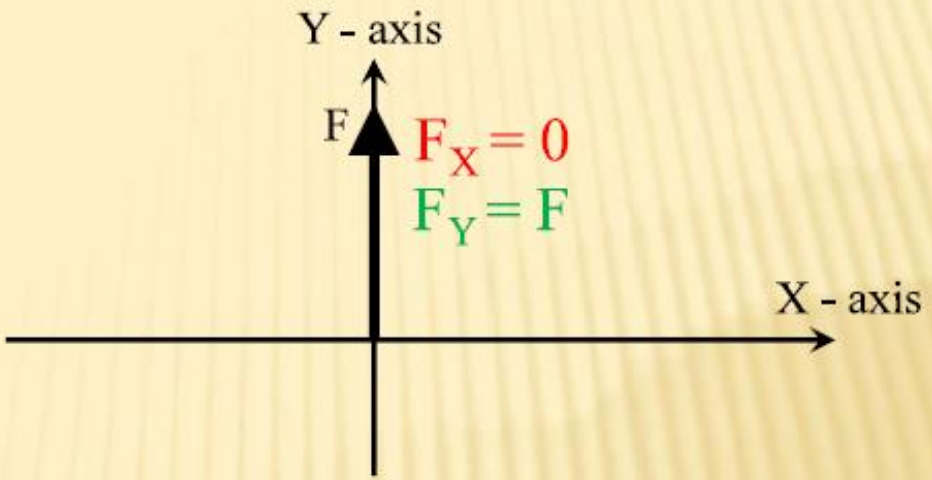




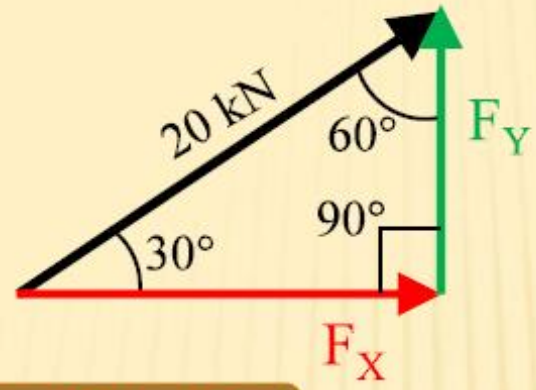



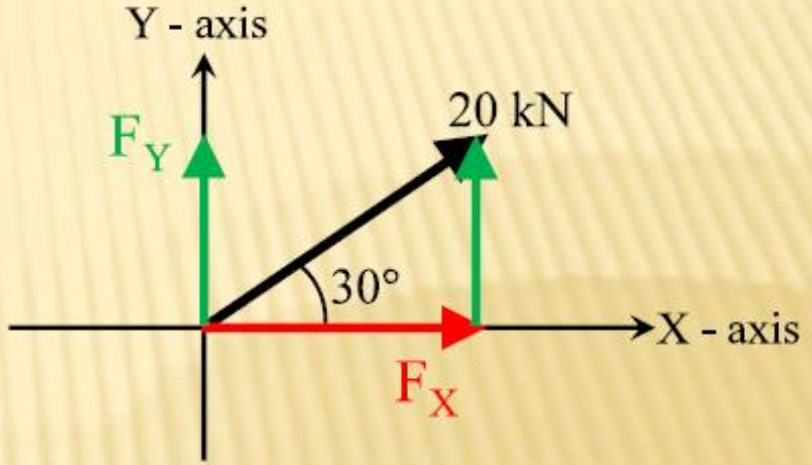

**ملاحظـــة: العلاقة بين القوى و مركباتها**

**١. قيمة القوة دائما اكبر من أي من مركباتها.**

**٢. دائما اتجاه القوة الاصلية يقع بين اتجاه مركباتها.**

**Example**: Resolve the 20 kN force shown in figure into X and Y components.




**Sine law**

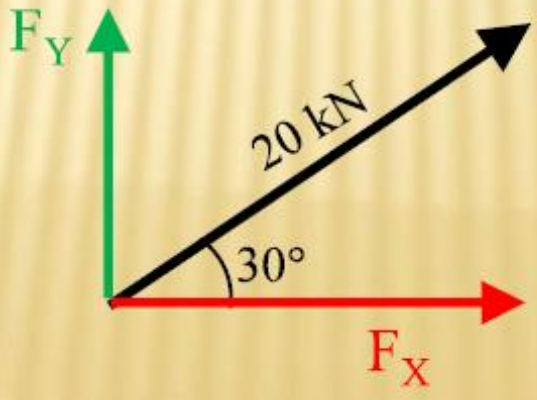
$$\frac{F_x}{sin60} = \frac{20}{sin\ 90} = \frac{F_Y}{sin\ 30}$$

$$\frac{F_x}{sin60} = \frac{20}{sin\ 90} \Longrightarrow F_X = 17.321\ kN$$

$$\frac{20}{sin\ 90} = \frac{F_Y}{sin\ 30} \Longrightarrow F_Y = 10\ kN$$

$$F_X = 20\ cos\ 30 = 17.321\ kN$$

$$F_Y = 20\ sin\ 30 = 10kN$$

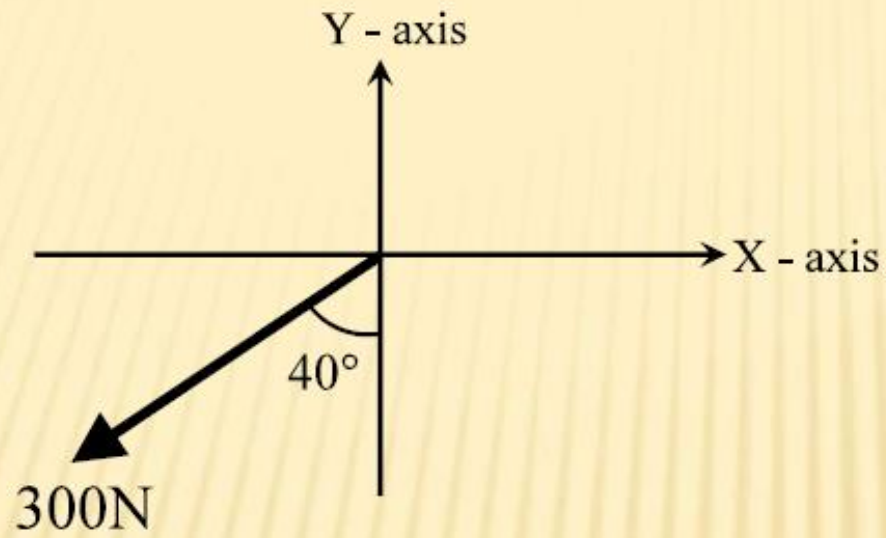



**في حالة المركبات المتعامدة:**

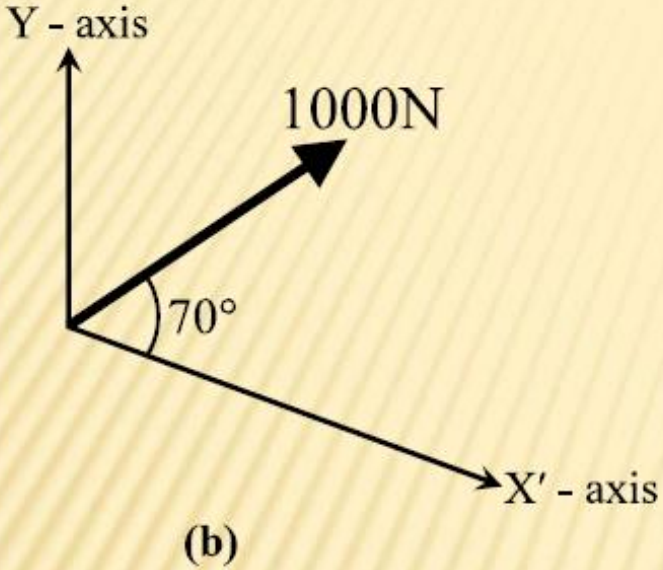
**المركبة القريبة من الزاوية تأخذ cosθ**

**المركبة البعيدة من الزاوية تأخذ sinθ**

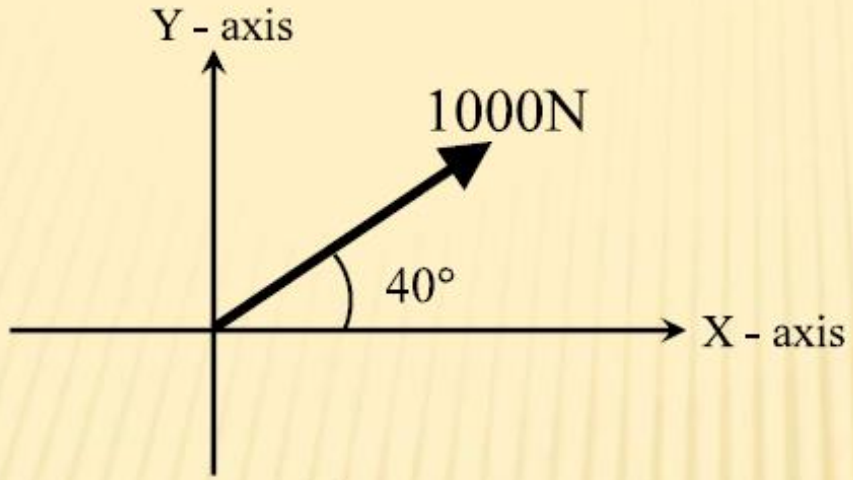
**H.W.**: Resolve the 300 N force shown in figure into X and Y components.



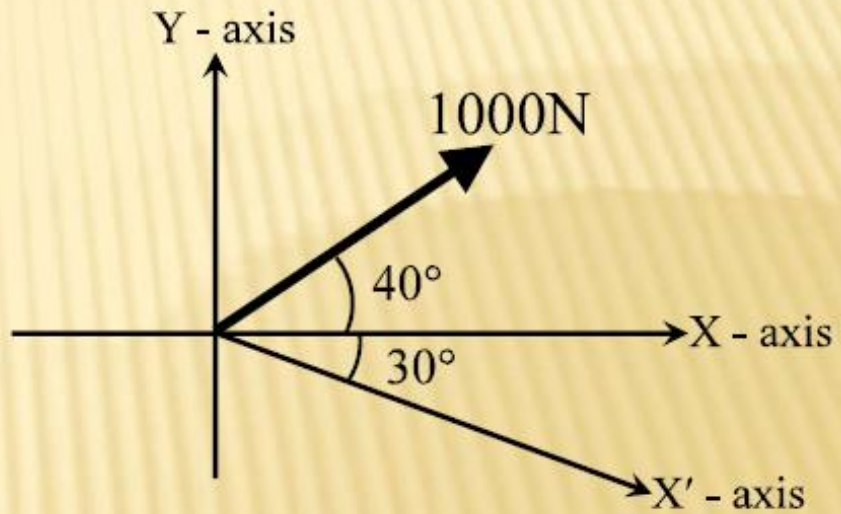
**Example**: Resolve the 1000 N force shown in figure into components in the **(a)** X and Y direction and **(b)** X′ and Y direction.




**(b)**



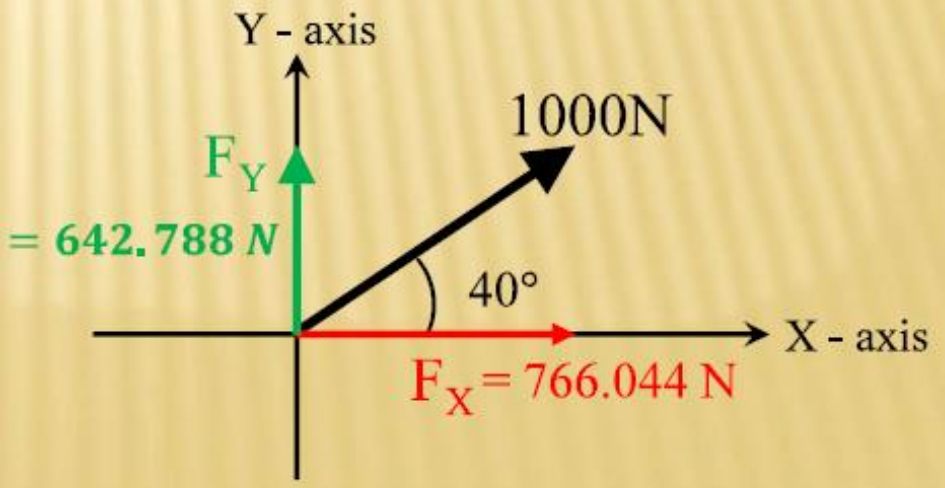

**(a)**



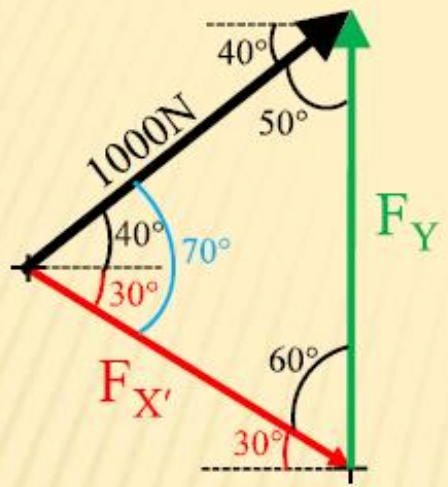

## (a) X and Y direction

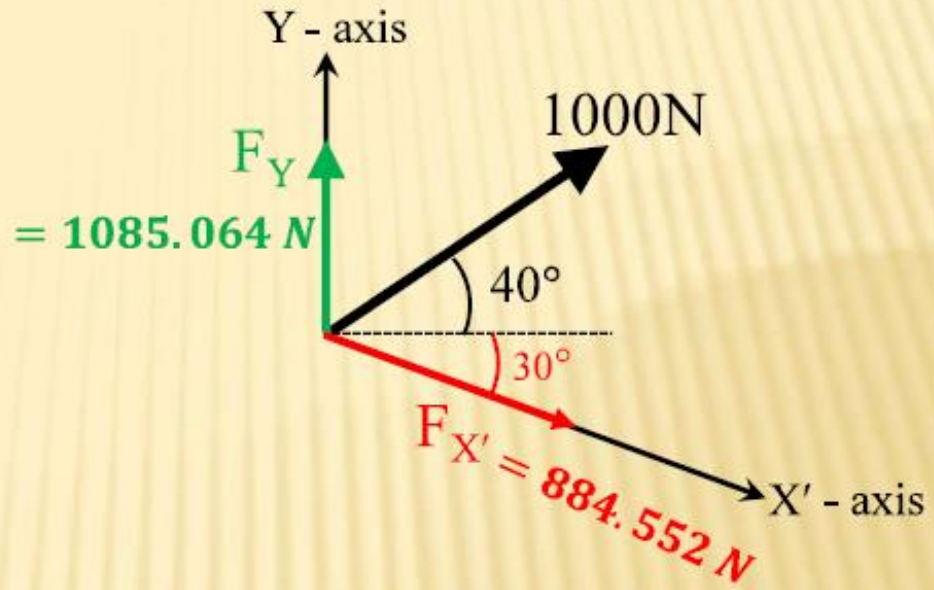
$$F_X = 1000\ cos\ 40 = 766.044\ N$$

$$F_Y = 1000\ sin\ 40 = 642.788\ N$$



**(b)** X′ and Y direction



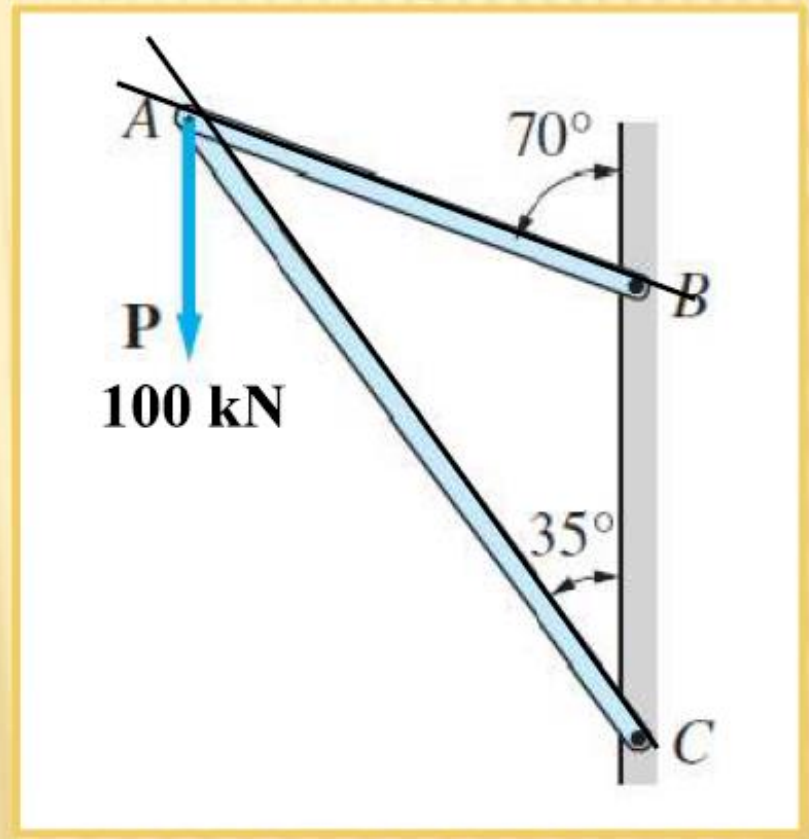

**Sine law**

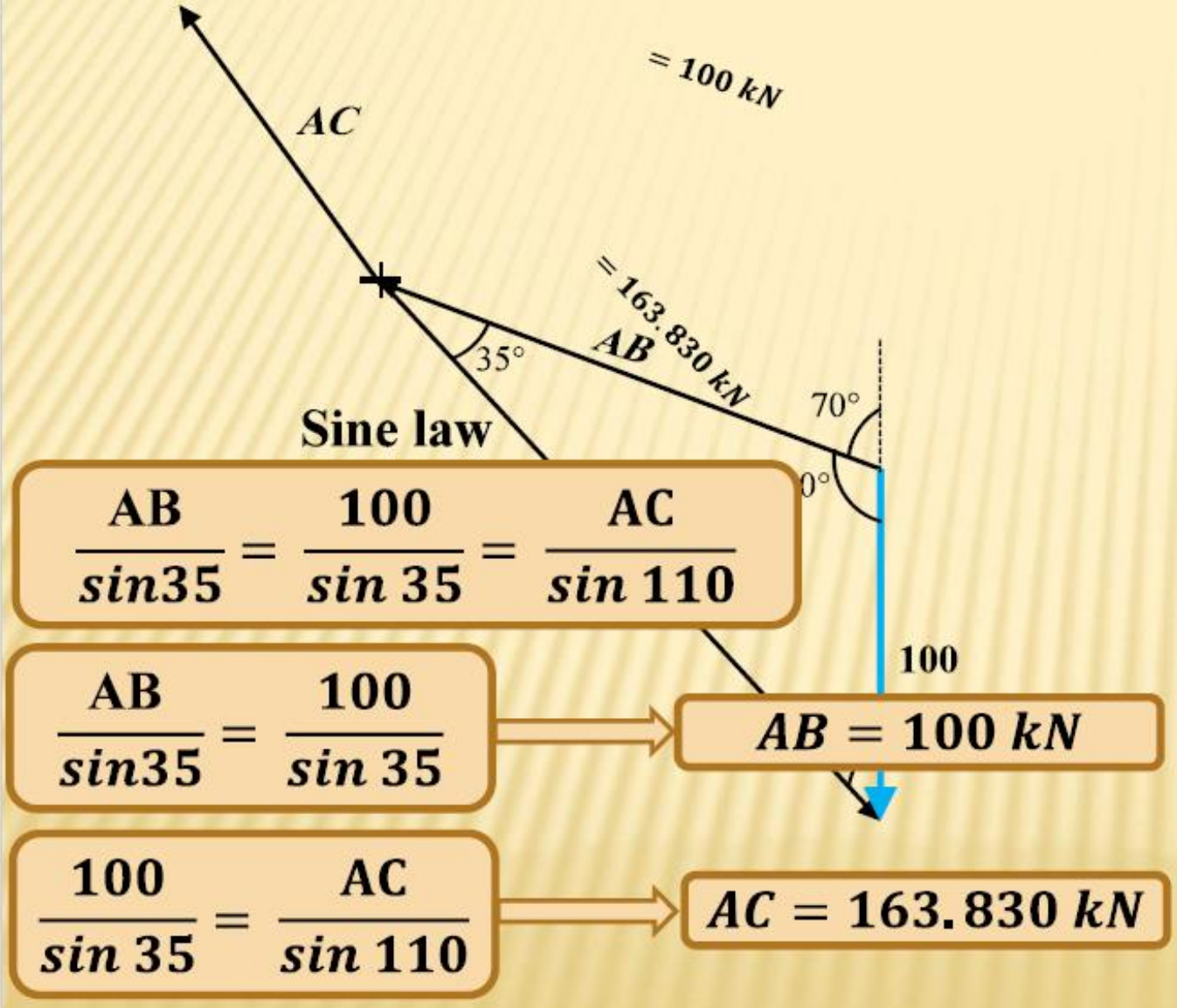
$$\frac{F_{X'}}{sin50} = \frac{1000}{sin\ 60} = \frac{F_Y}{sin\ 70}$$

$$\frac{F_{X'}}{sin50} = \frac{1000}{sin\ 60} \Longrightarrow F_x{}' = 884.552\ N$$

$$\frac{1000}{sin\ 60} = \frac{F_Y}{sin\ 70} \Longrightarrow F_Y = 1085.064\ N$$

**Example:** The vertical force **P** of magnitude 100 kN is applied to the frame shown in figure below. Resolve **P** into components that are parallel to the members ***AB*** and ***AC***.



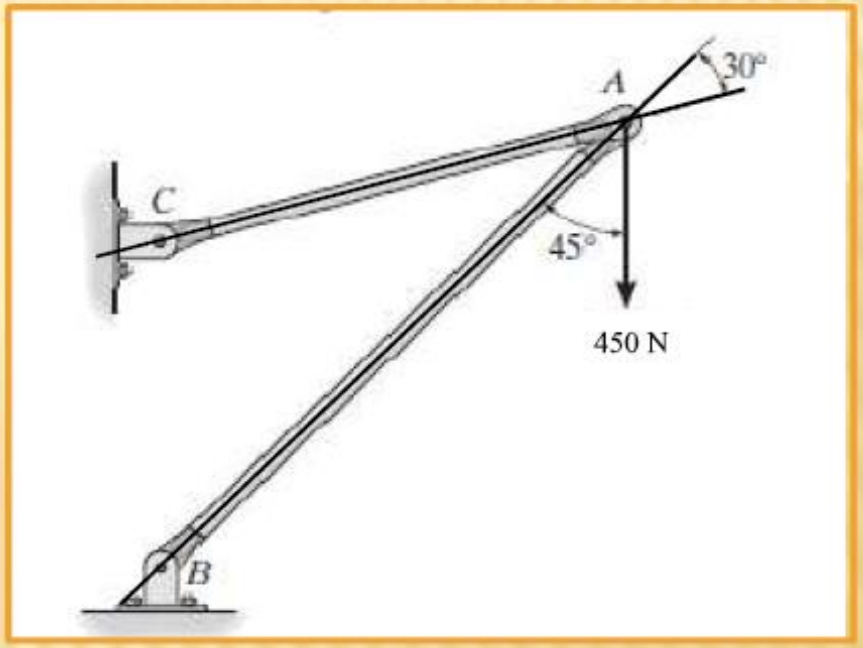

**Sine law**

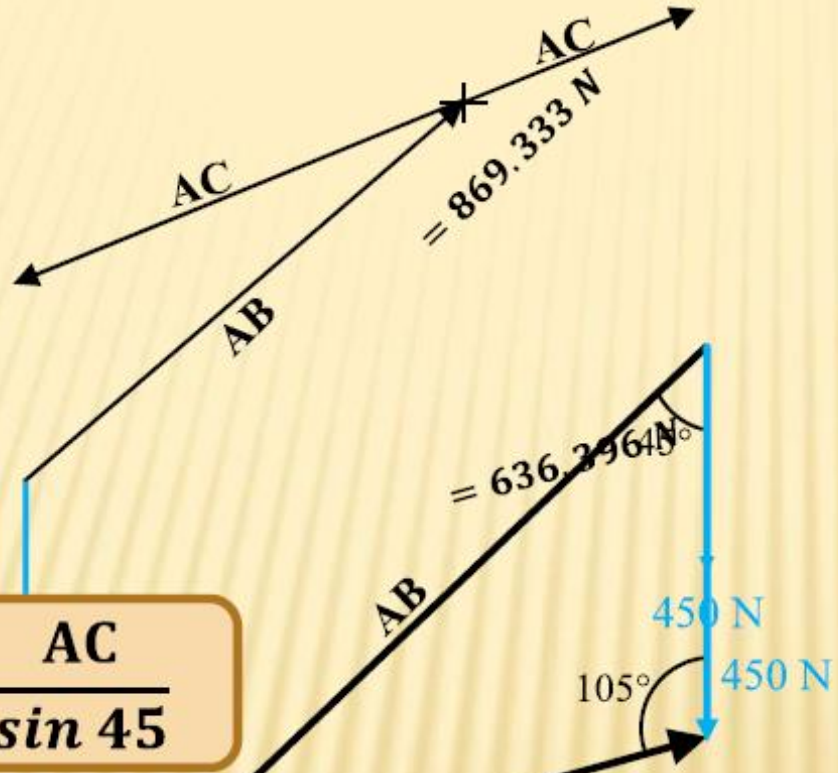
$$\frac{AB}{sin35} = \frac{100}{sin\ 35} = \frac{AC}{sin\ 110}$$

$$\frac{AB}{sin35} = \frac{100}{sin\ 35} \Rightarrow AB = 100\ kN$$

$$\frac{100}{sin\ 35} = \frac{AC}{sin\ 110} \Rightarrow AC = 163.830\ kN$$

**Example**: Resolve the force (450 N) shown in figure below into two components acting along the members (**AB**) and (**AC**).




**Sine law**

$$\frac{AB}{sin105} = \frac{450}{sin\ 30} = \frac{AC}{sin\ 45}$$

$$\frac{AB}{sin105} = \frac{450}{sin\ 30} \Rightarrow AB = 869.333\ N$$

$$\frac{450}{sin\ 30} = \frac{AC}{sin\ 45} \Rightarrow AC = 636.396\ N$$

**2.3** Two structural members $B$ and $C$ are bolted to bracket $A$. Knowing that both members are in tension and that $P = 10$ kN and $Q = 15$ kN, determine graphically the magnitude and direction of the resultant force exerted on the bracket using ($a$) the parallelogram law, ($b$) the triangle rule.

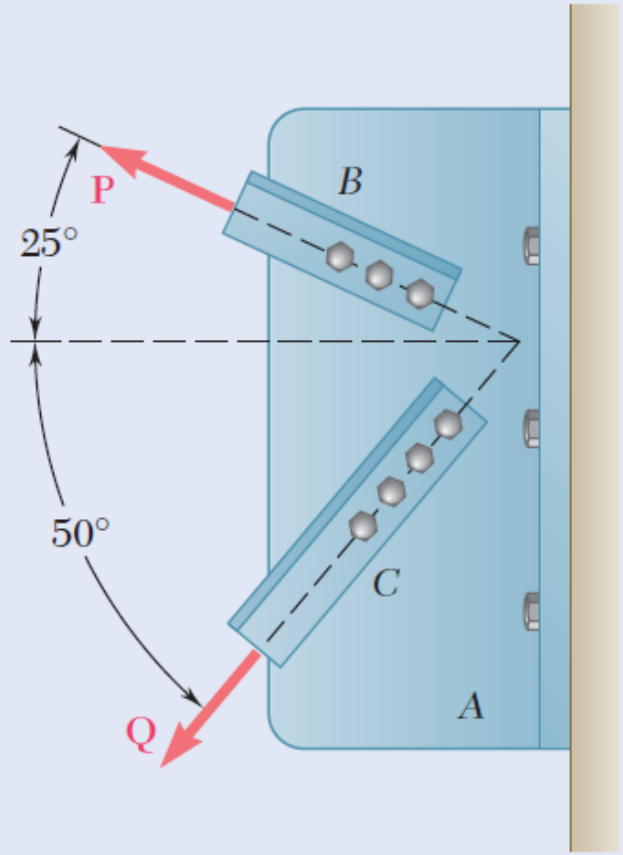
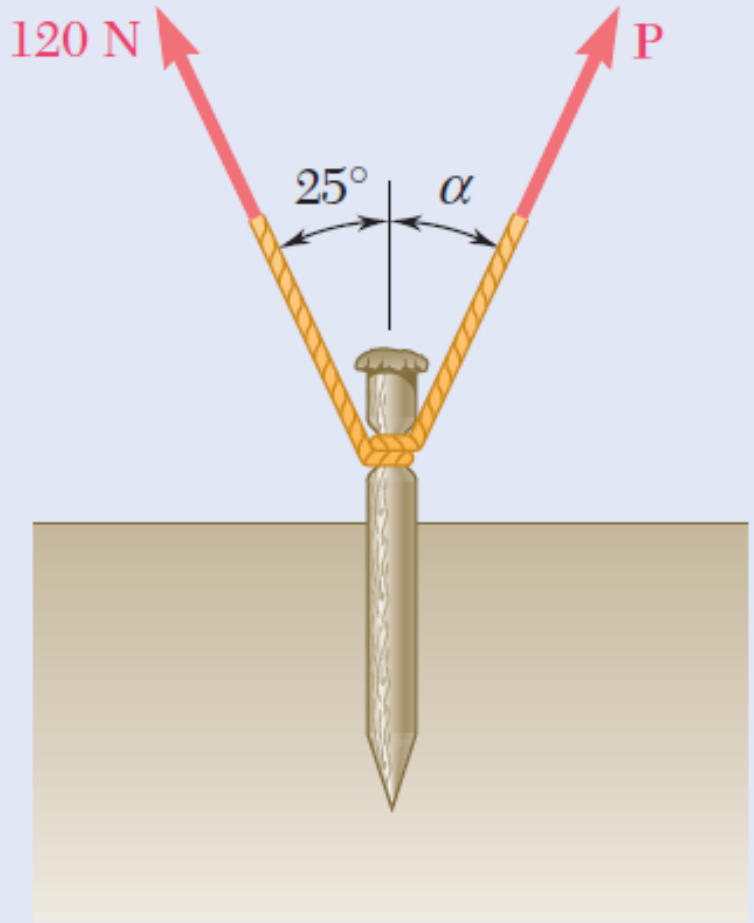



Fig. *P2.3* and P2.4

**2.5** A stake is being pulled out of the ground by means of two ropes as shown. Knowing that $\alpha = 30°$, determine by trigonometry (*a*) the magnitude of the force **P** so that the resultant force exerted on the stake is vertical, (*b*) the corresponding magnitude of the resultant.



**2.6** A telephone cable is clamped at $A$ to the pole $AB$. Knowing that the tension in the left-hand portion of the cable is $T_1 = 800$ lb, determine by trigonometry ($a$) the required tension $T_2$ in the right-hand portion if the resultant **R** of the forces exerted by the cable at $A$ is to be vertical, ($b$) the corresponding magnitude of **R**.

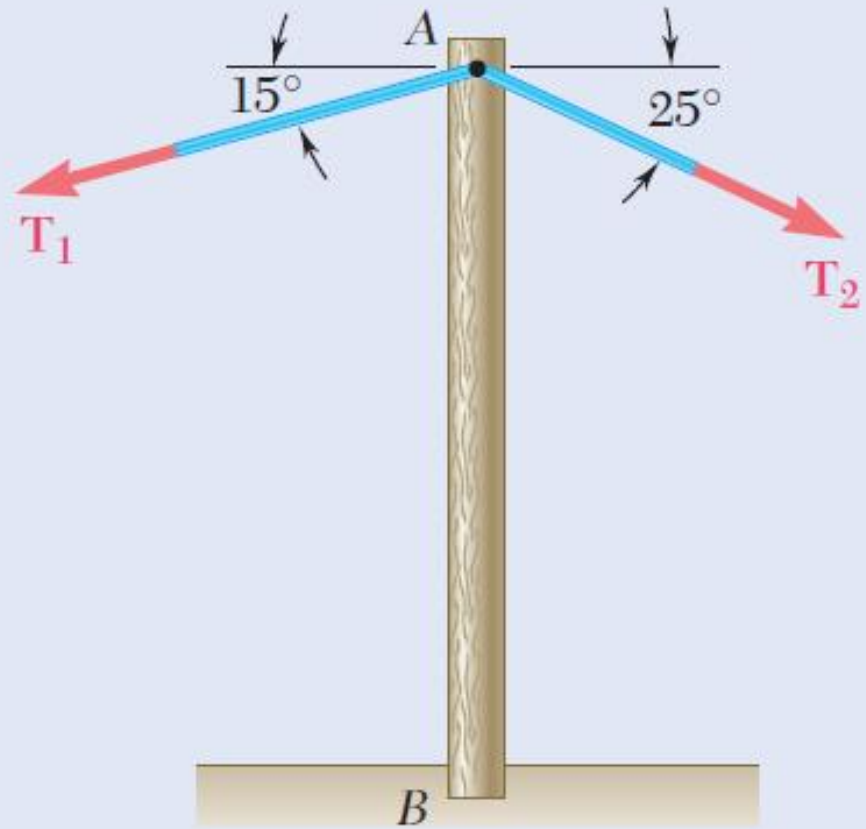



**Fig. P2.6 and *P2.7***

**2.9** A disabled automobile is pulled by means of two ropes as shown. Knowing that the tension in rope $AB$ is 3 kN, determine by trigonometry the tension in rope $AC$ and the value of $\alpha$ so that the resultant force exerted at $A$ is a 4.8-kN force directed along the axis of the automobile.

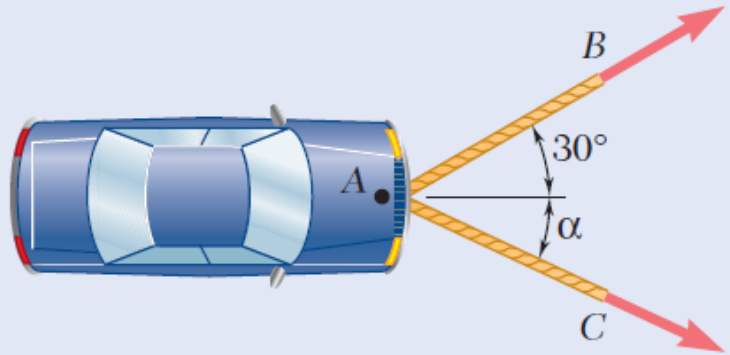



Fig. P2.9

**2.10** Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (*a*) the required angle $\alpha$ if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

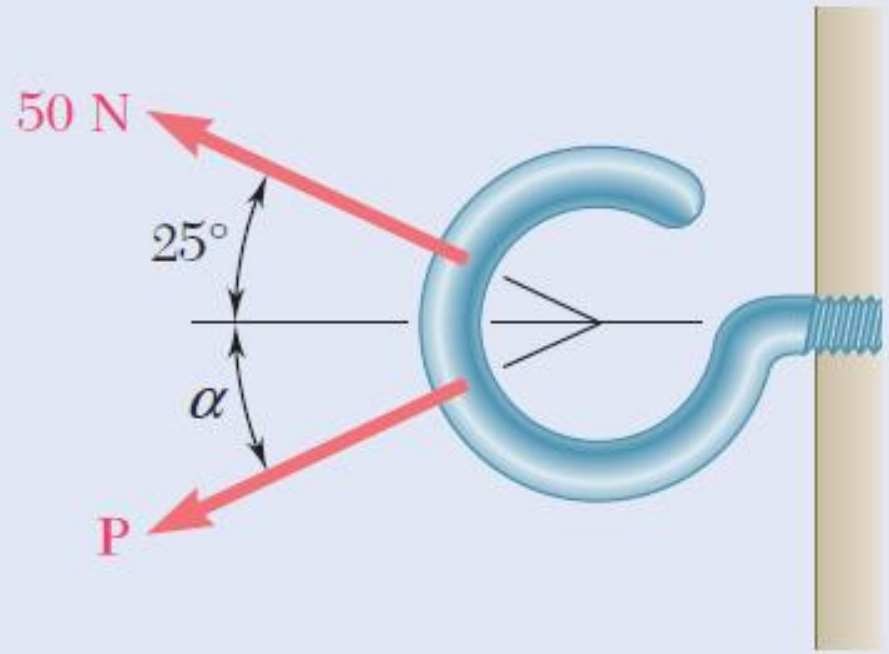
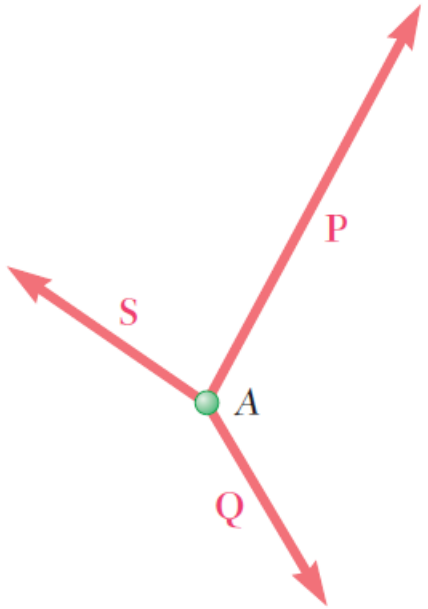



**Fig. P2.10**

# Addition of Forces by Summing **X** and **Y** Components

- When we need to add **three** or more forces, we **cannot** obtain any practical **trigonometric** solution from the force polygon that defines the resultant of the forces. **In this case**, the best approach is to obtain an analytic solution of the problem by **resolving each force into two rectangular components.**

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y$$




(*a*)

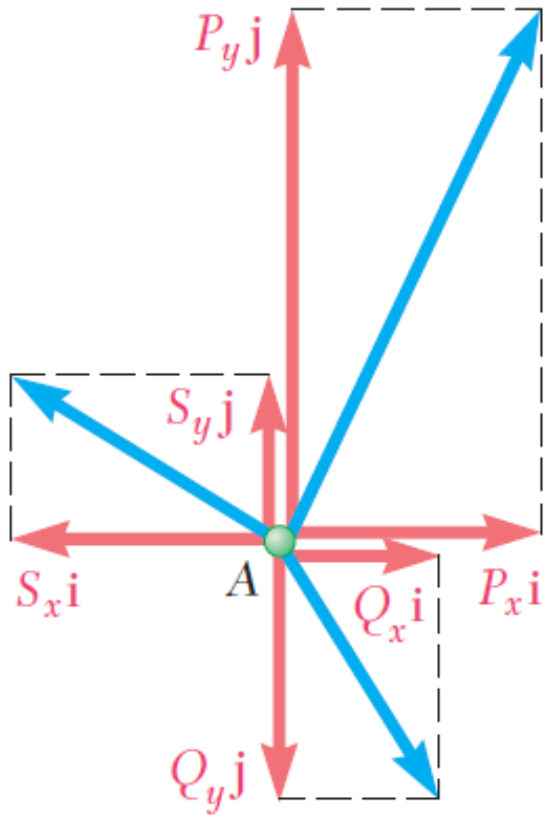
**Fig. 2.21** (*a*) Three forces acting on a particle.

We thus conclude that **when several forces are acting on a particle, we obtain the scalar components $R_x$ and $R_y$ of the resultant R by adding algebraically the corresponding scalar components of the given forces**.*(Clearly, this result also applies to the addition of other vector quantities,such as velocities, accelerations, or momenta.)*

In practice, determining the resultant **R** is carried out in three steps,

as illustrated

**1.** Resolve the given forces (Fig. 2.21*a*) into their $x$ and $y$ components (Fig. 2.21*b*).




(*b*)

**Fig. 2.21** (*b*) Rectangular components of each force.

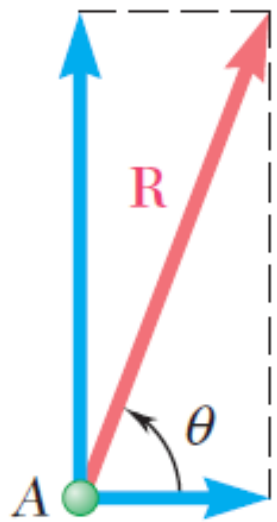
**2.** Add these components to obtain the $x$ and $y$ components of **R** (Fig. 2.21$c$).




($c$)

**Fig. 2.21** ($c$) Summation of the components.

**3.** Apply the parallelogram law to determine the resultant $\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$ (Fig. 2.21*d*).

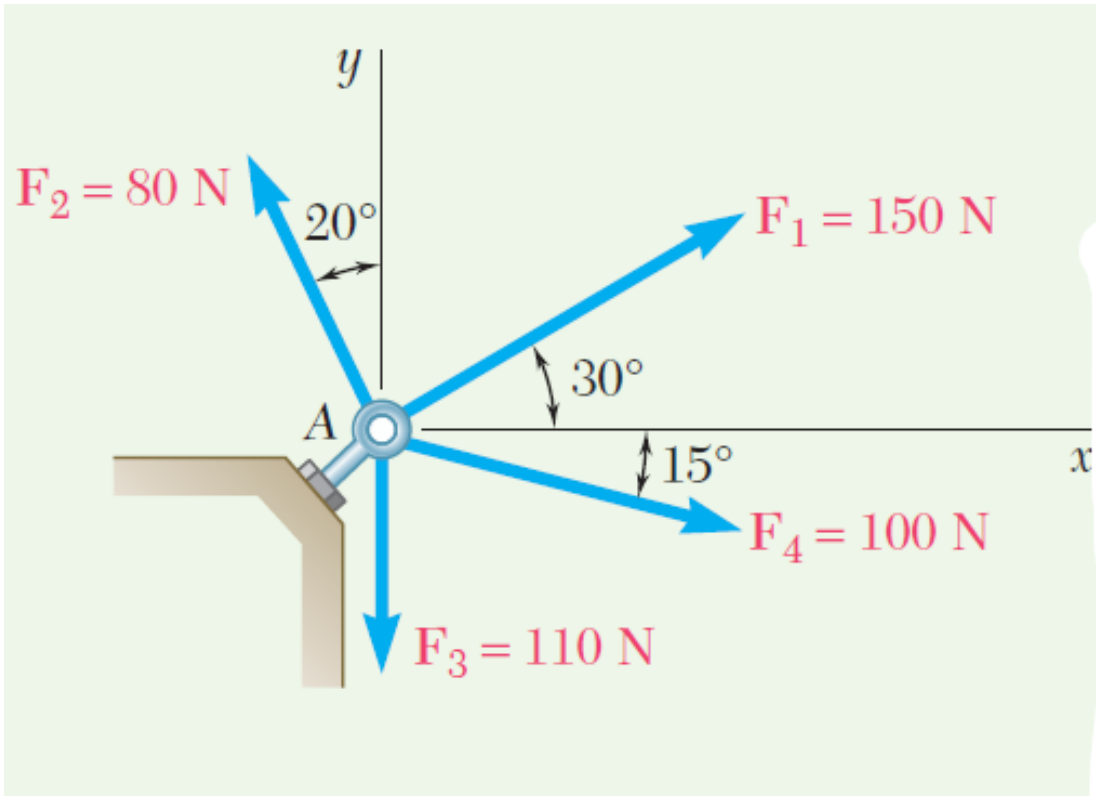



(*d*)

**Fig. 2.21** (*d*) Determining the resultant from its components.

# Sample Problem 2.3

Four forces act on bolt $A$ as shown. Determine the resultant of the forces on the bolt.



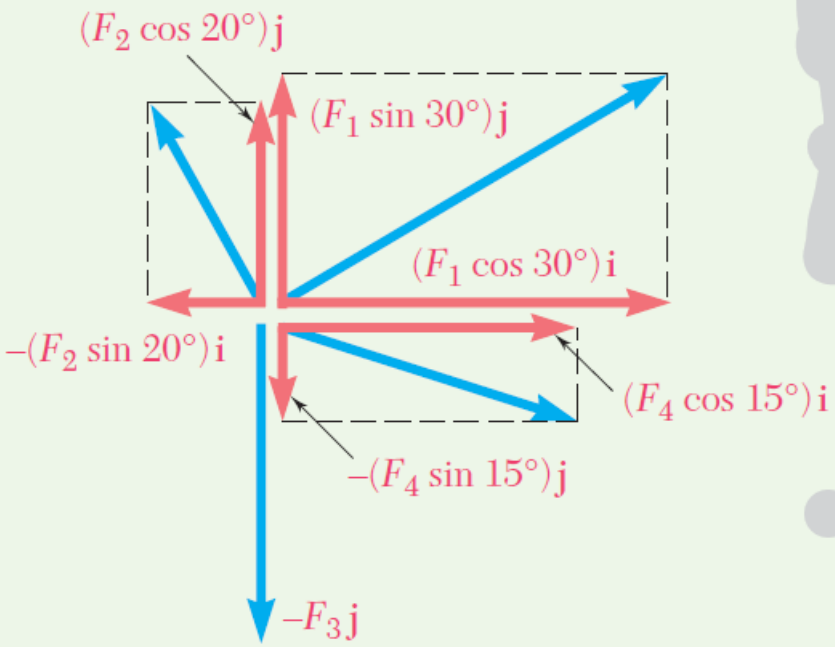



**Fig. 1** Rectangular components of each force.




**Fig. 2** Resultant of the given force system.

## ANALYSIS:

| Force | Magnitude, N | $x$ Component, N | $y$ Component, N |
|---|---|---|---|
| $\mathbf{F}_1$ | 150 | +129.9 | +75.0 |
| $\mathbf{F}_2$ | 80 | −27.4 | +75.2 |
| $\mathbf{F}_3$ | 110 | 0 | −110.0 |
| $\mathbf{F}_4$ | 100 | +96.6 | −25.9 |
| | | $R_x = +199.1$ | $R_y = +14.3$ |

Thus, the resultant **R** of the four forces is

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} \qquad \mathbf{R} = (199.1\ \text{N})\mathbf{i} + (14.3\ \text{N})\mathbf{j}$$

You can now determine the magnitude and direction of the resultant. From the triangle shown in Fig. 2, you have

$$\tan\alpha = \frac{R_y}{R_x} = \frac{14.3\ \text{N}}{199.1\ \text{N}} \qquad \alpha = 4.1°$$

$$R = \frac{14.3\ \text{N}}{\sin\alpha} = 199.6\ \text{N} \qquad \mathbf{R} = 199.6\ \text{N} \measuredangle 4.1°$$

**2.22** Determine the $x$ and $y$ components of each of the forces shown.

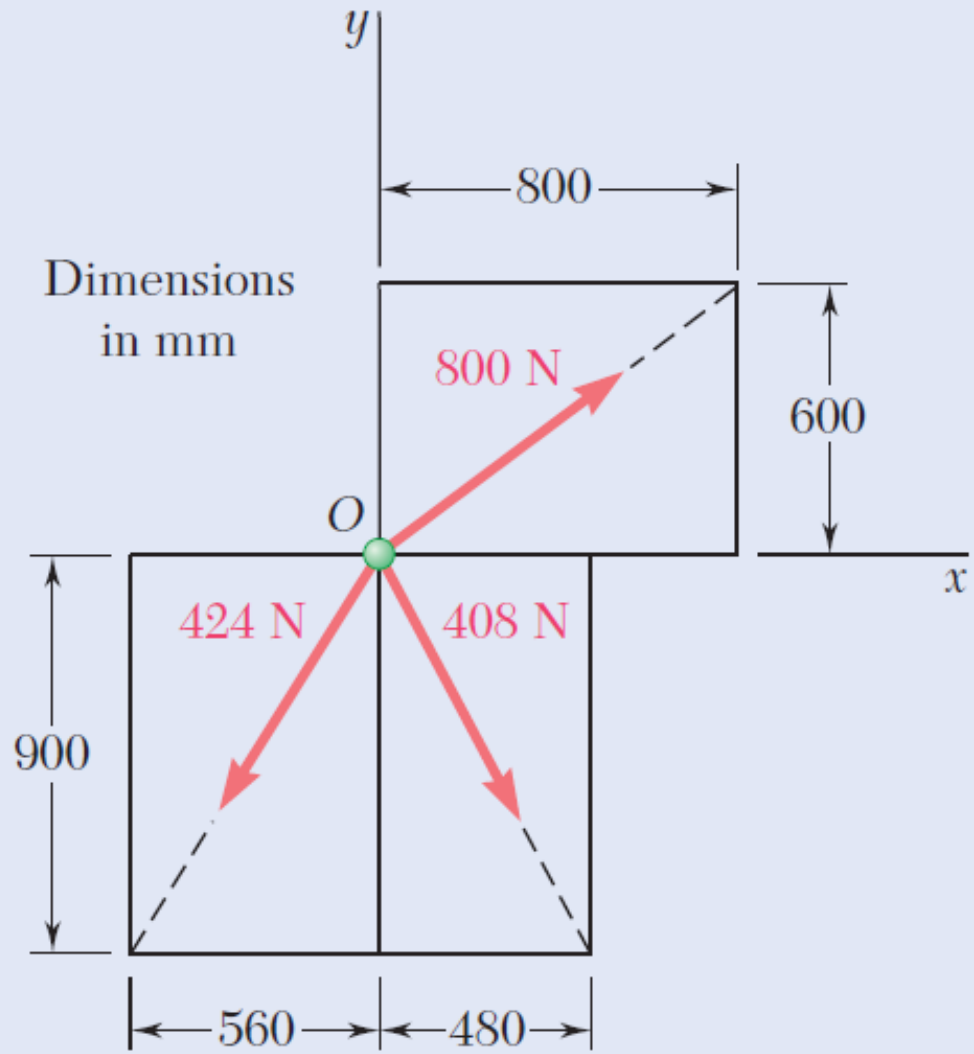
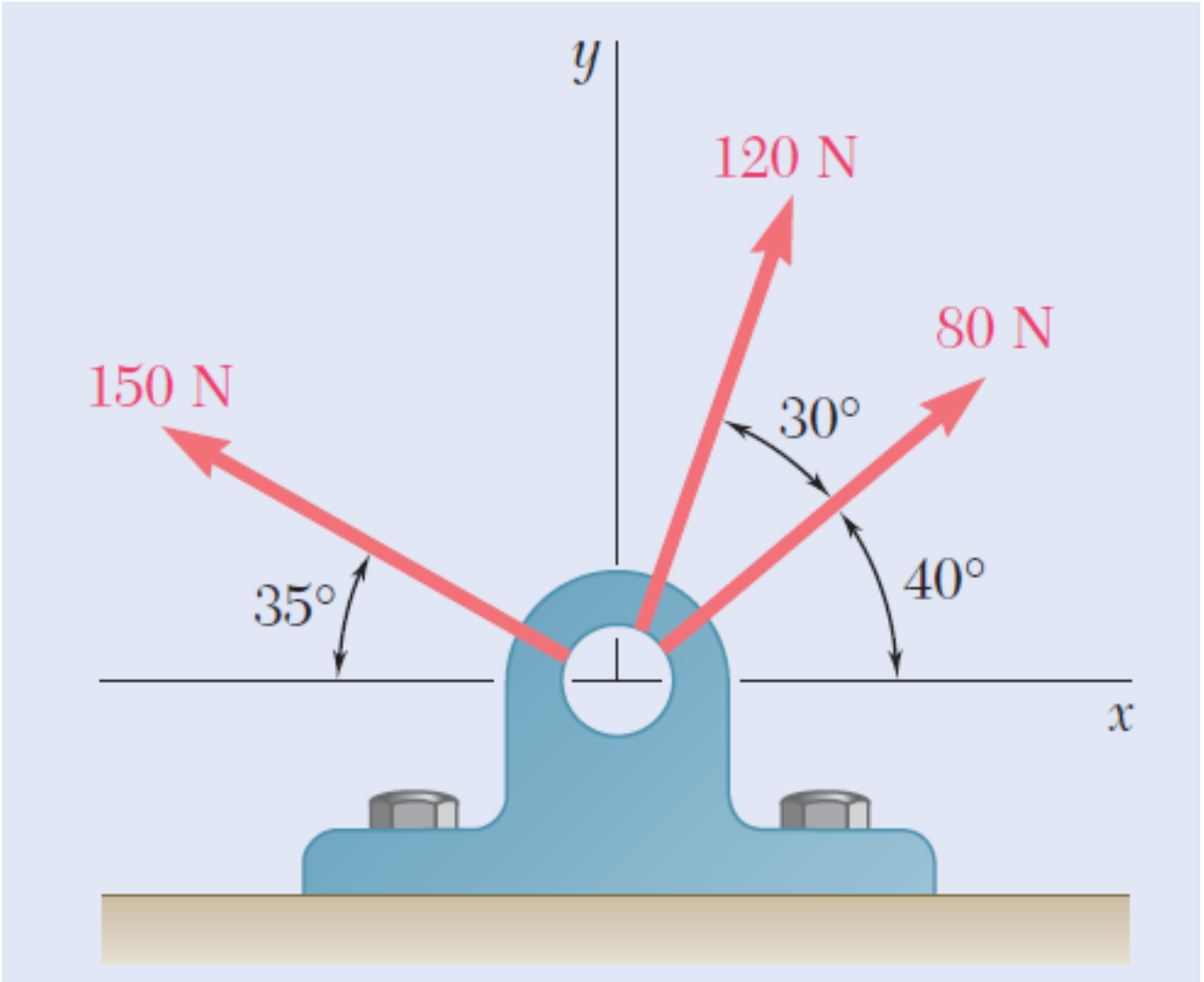



Fig. P2.22

**d 2.24** Determine the $x$ and $y$ components of each of the forces shown.



**2.25** Member $BC$ exerts on member $AC$ a force **P** directed along line $BC$. Knowing that **P** must have a 325-N horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

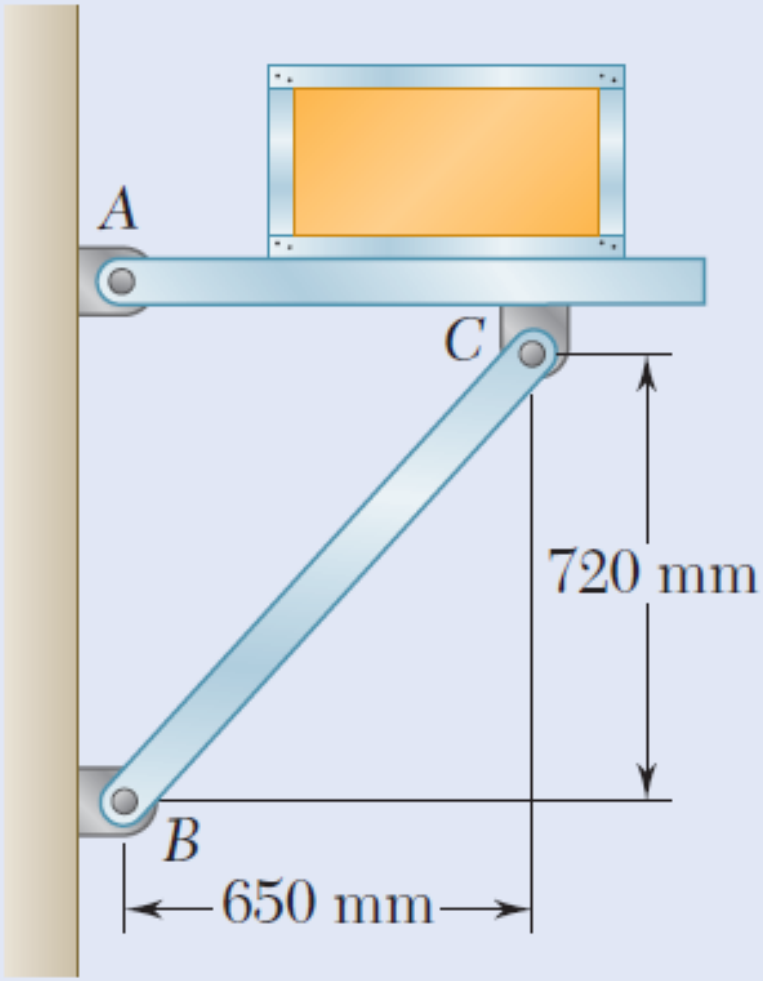



Fig. P2.25

**2.26** Member *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 300-lb horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

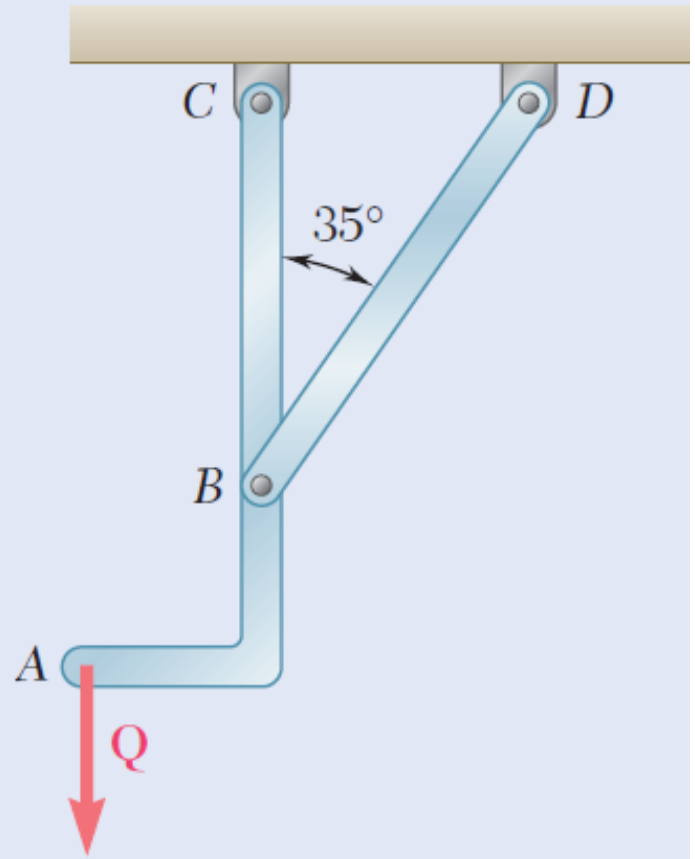



Fig. P2.26

**2.27** The hydraulic cylinder $BC$ exerts on member $AB$ a force **P** directed along line $BC$. Knowing that **P** must have a 600-N component perpendicular to member $AB$, determine $(a)$ the magnitude of the force **P**, $(b)$ its component along line $AB$.

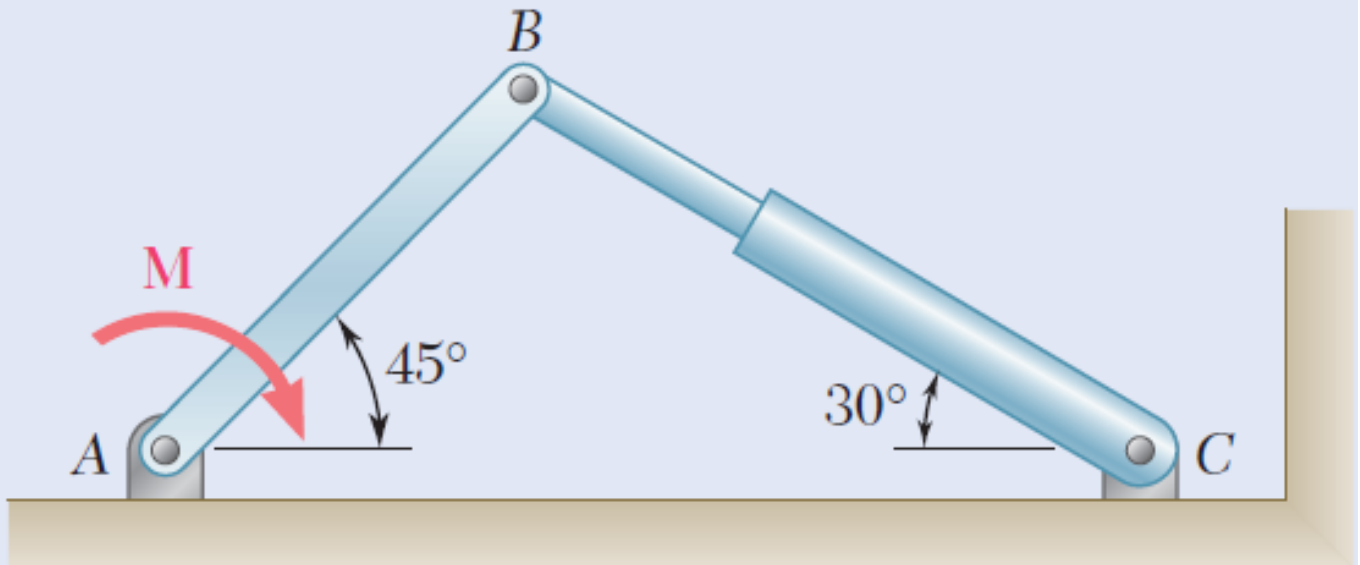



Fig. P2.27

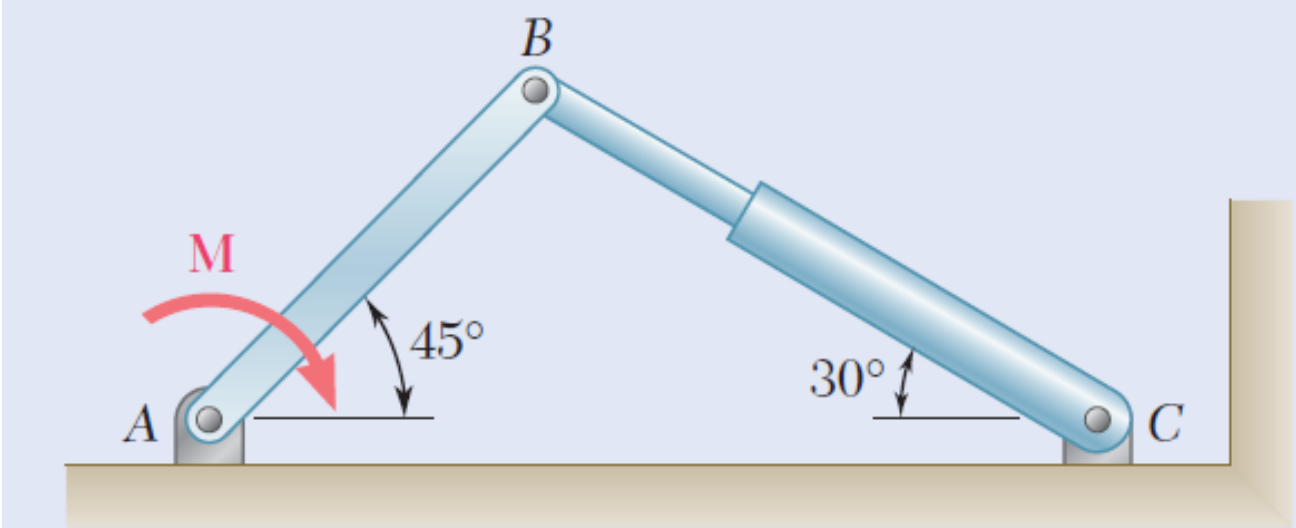



Fig. P2.27

90

Px

45

75

Py=600 N

P

30

**2.29** The hydraulic cylinder *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 750-N component perpendicular to member *ABC*, determine (*a*) the magnitude of the force **P**, (*b*) its component parallel to *ABC*.

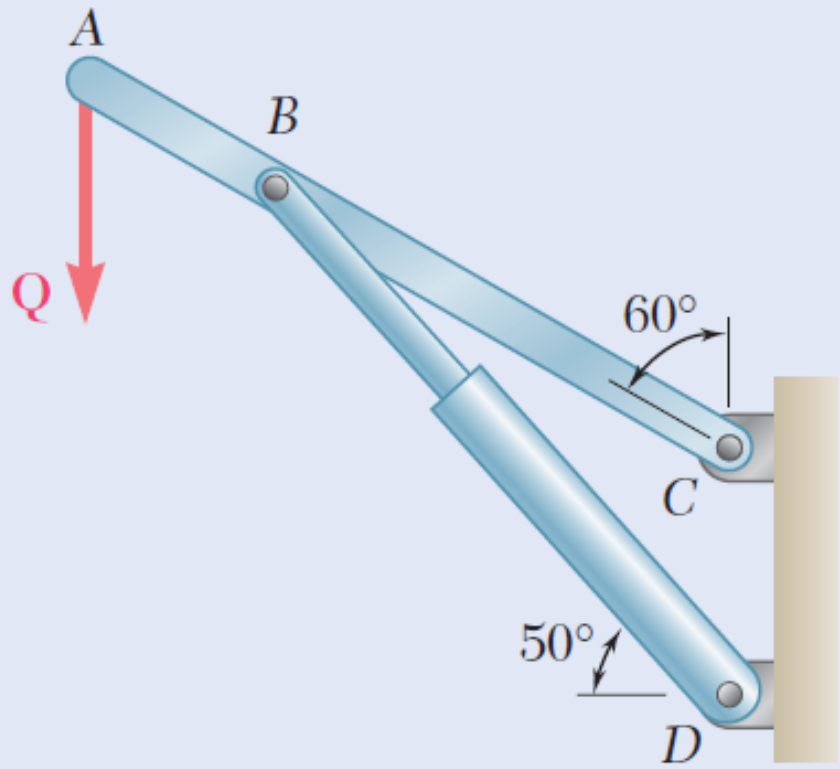



**Fig. P2.29**

**2.36** Knowing that the tension in rope $AC$ is 365 N, determine the resultant of the three forces exerted at point $C$ of post $BC$.

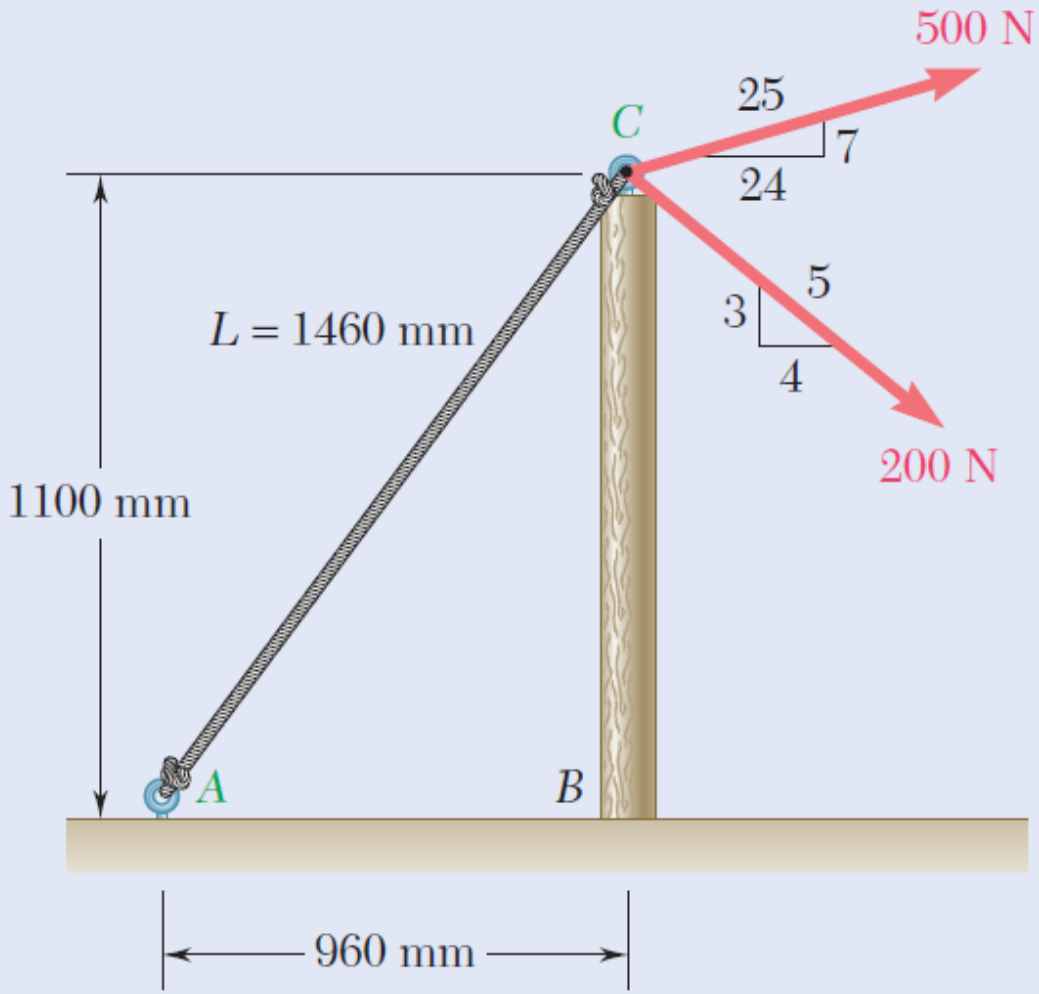
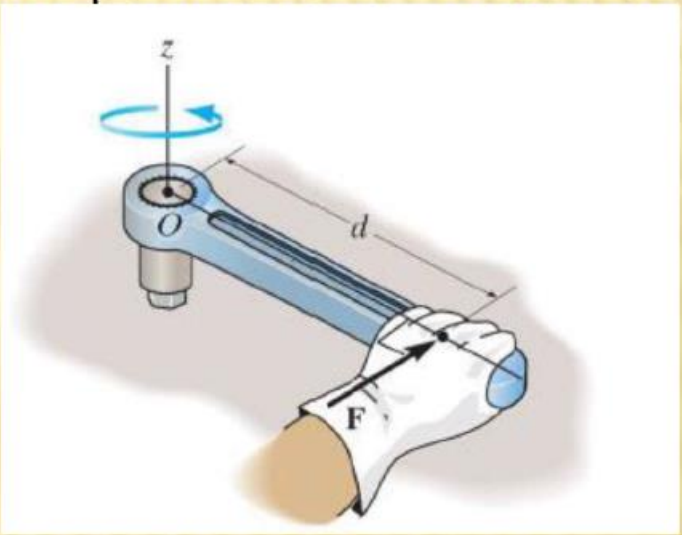



Fig. P2.36

# Moment of a force

**Moment (M):** Is the tendency of a force to rotate a body about a certain point or axis in addition to move a body in the direction of its application.



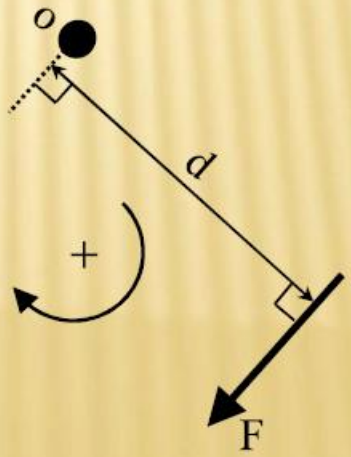

**This rotational effect depends on**

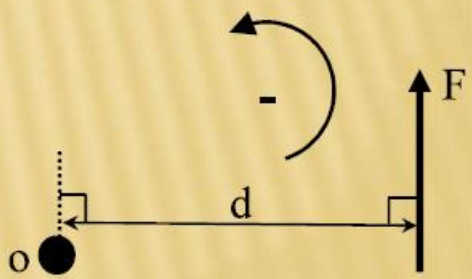
- The magnitude of the force **(F)**.
- The perpendicular distance between the point and the line of action of the force **(d).**

$$M_o = F\ x\ d$$

**Sense of Moment**

- Plus sign (+) for clockwise moments.
- Minus sign(-) for counterclockwise moments.




**Units Of moment**

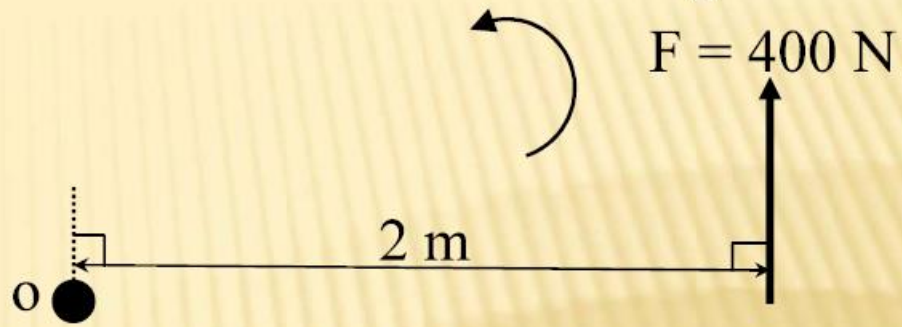
Unit force x Unit length

| | | |
|---|---|---|
| N | mm | $= N\ .mm$ |
| kN | m | $= kN\ .m$ |

**Example:** Calculate the moment about point **O** from the force **F = 400 N** as shown in figure:

$$M_o = F\ x\ d$$

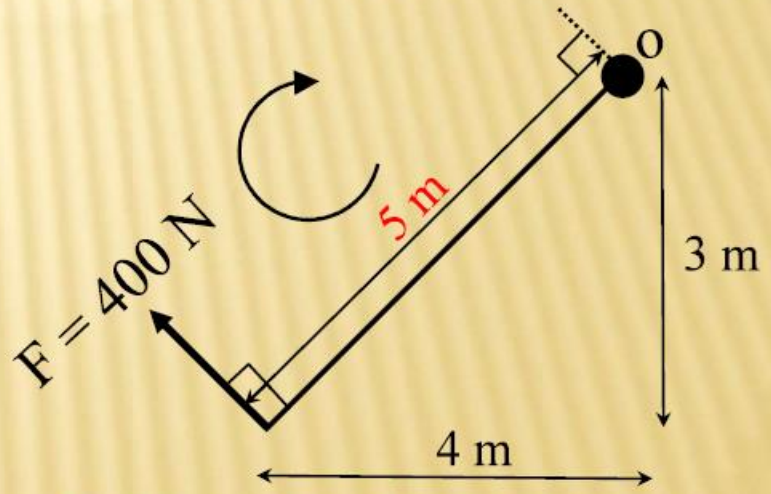
$$M_o = -400\ x\ 2 = -800\ N.m$$ **Counter colckwise**




**Example:** Calculate the moment about point **O** from the force **F = 400 N** as shown in figure:

$$M_o = F\ x\ d$$

$$M_o = 400\ x\ 5 = 2000\ N.m$$ **colckwise**




**Example:** Calculate the moment about point **O** from the force **F = 400 N** as shown in figure:

$$M_o = F\ x\ d \Longrightarrow M_o = 400\ x\ 0 = 0$$

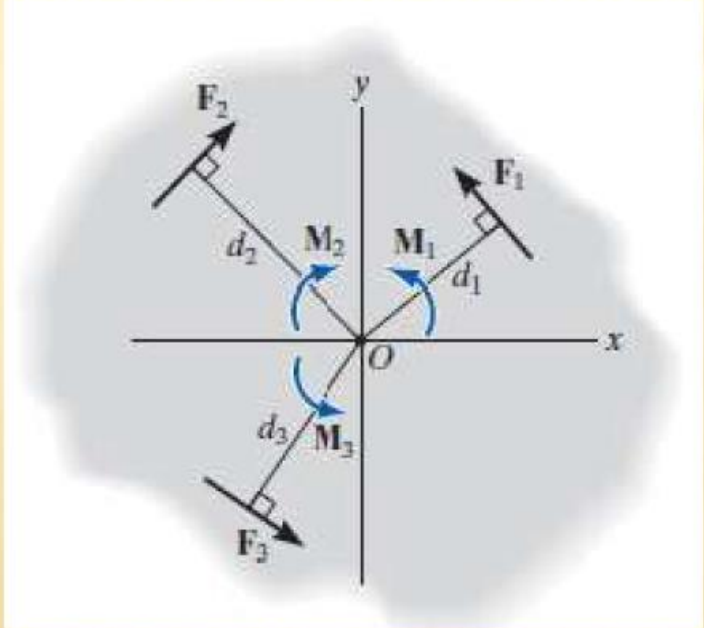



**NOTE** **Moment of force is zero when the perpendicular distance of line of action of force from the point of pivot is zero**

**Moment of the resultant of several forces:** the moment about a given point O of the resultant of several forces is equal to the sum of the moments of the various forces about the same point O

$$(MR)_o = \sum M_o = \sum (F \,.d)$$

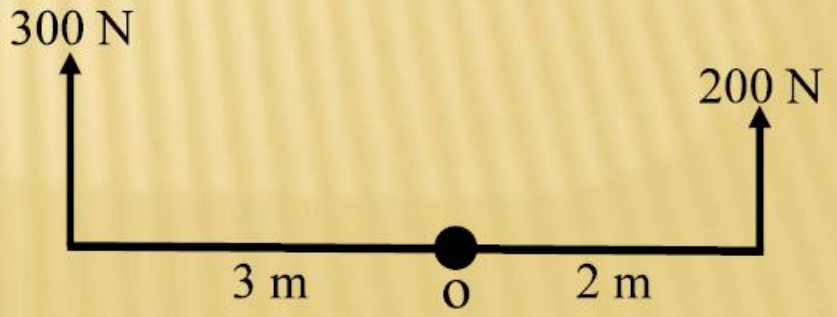
$$(MR)_o = -F1.d1 + F2\,.d2 \; - F3\,.d3$$



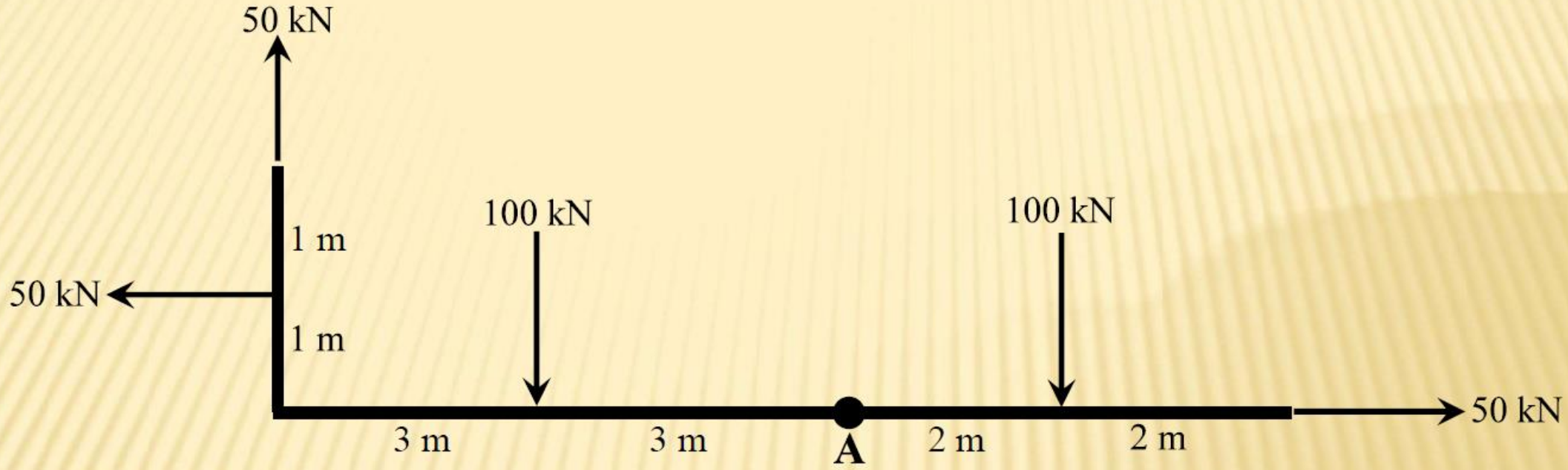

**Example:** Calculate the moment about point **O** for the forces as shown in figure:

$$M_o = \sum M_o = \sum (F \,.d)$$

$$M_o = (300\ x\ 3) - (200\ x\ 2) = 500\ N.m$$ **colckwise**



**Example:** Calculate the moment about point **A** for the forces as shown in figure:



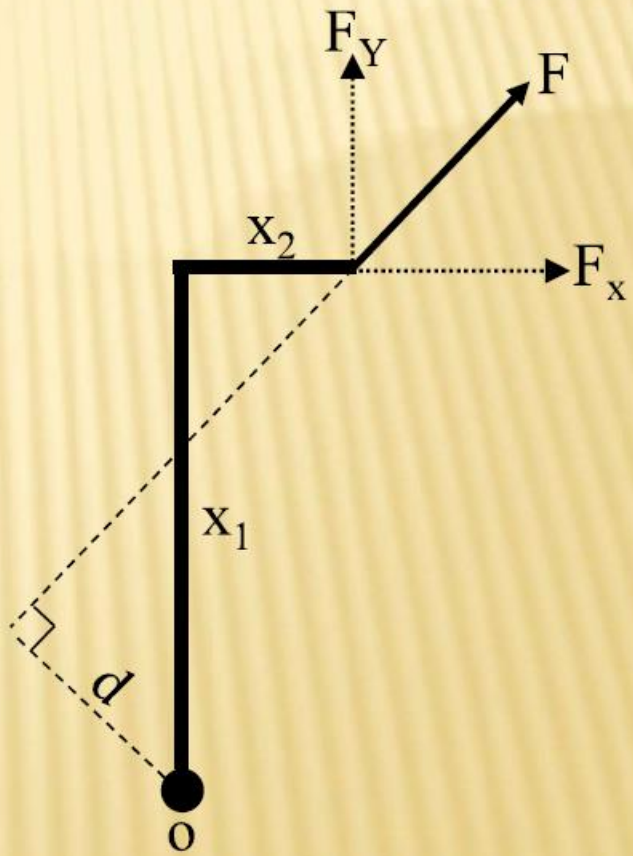

$$M_A = \sum M_A = \sum (F \,.\, d)$$

$$M_A = (100\ x\ 2) + (50\ x\ 6) - (100\ x\ 3) - (50x1) + (50\ x\ 0) = 150\ kN.m$$ **colckwise**

## Varignon's Theorem

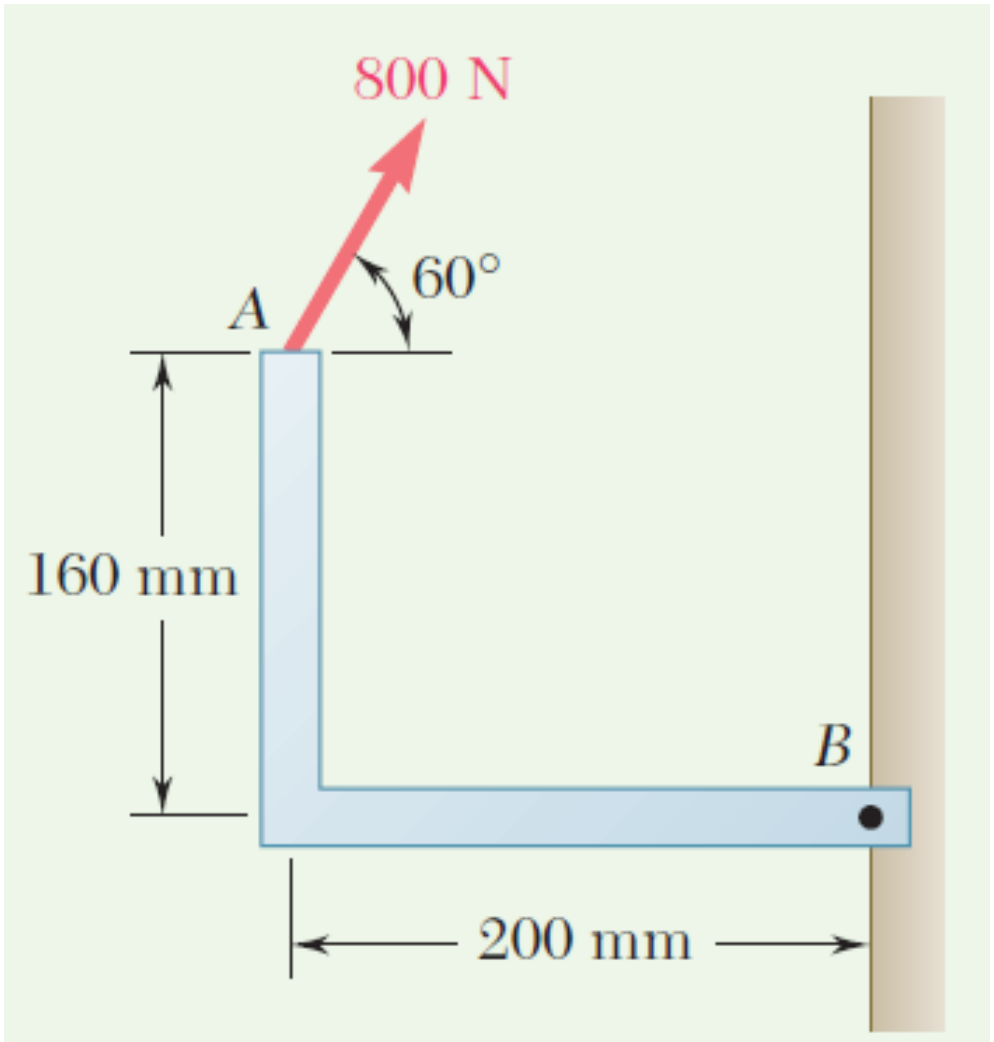
The moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

$$M_o = F\,x\,d = F_x . X_1 - F_Y . X_2$$



# Sample Problem 3.2

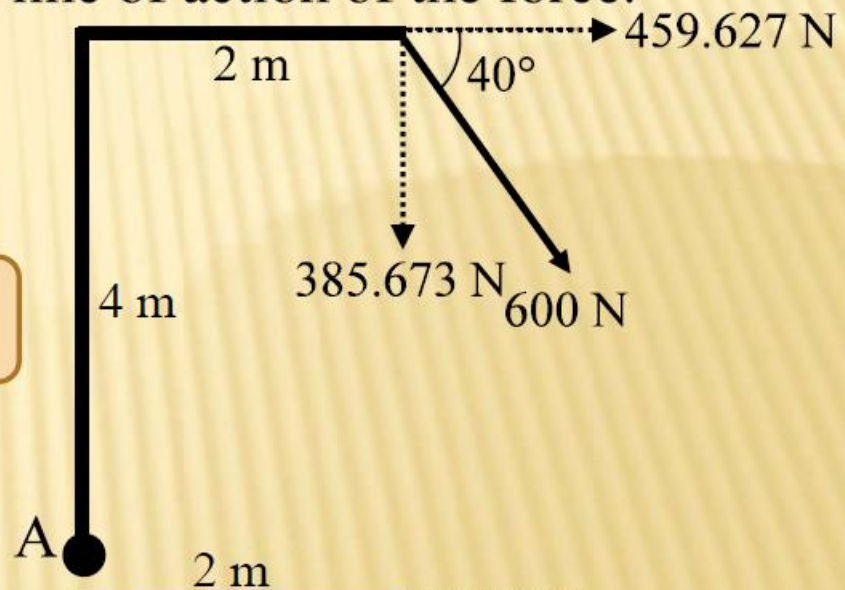
A force of 800 N acts on a bracket as shown. Determine the moment of the force about *B*.



# Moment of a force

**Example:** For the structure shown in figure below, calculate the moment at **A** using:

1. Varignon's theorem. 2. The perpendicular distance to the line of action of the force.




**1. Varignon's theorem.**

$$(MR)_A = \sum M_A = \sum (F\,.d)$$

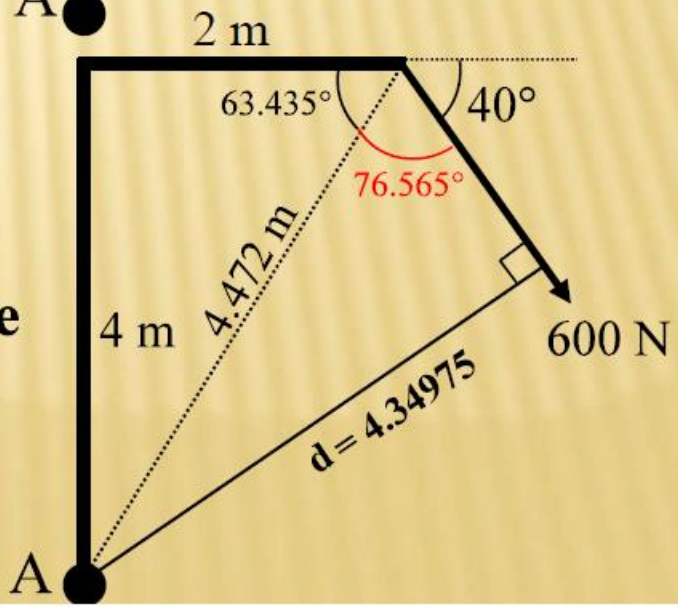
$$M_A = (459.627x\,4) + (385.673\,x\,2) = 2609.854\,N.m$$

**Clockwise**

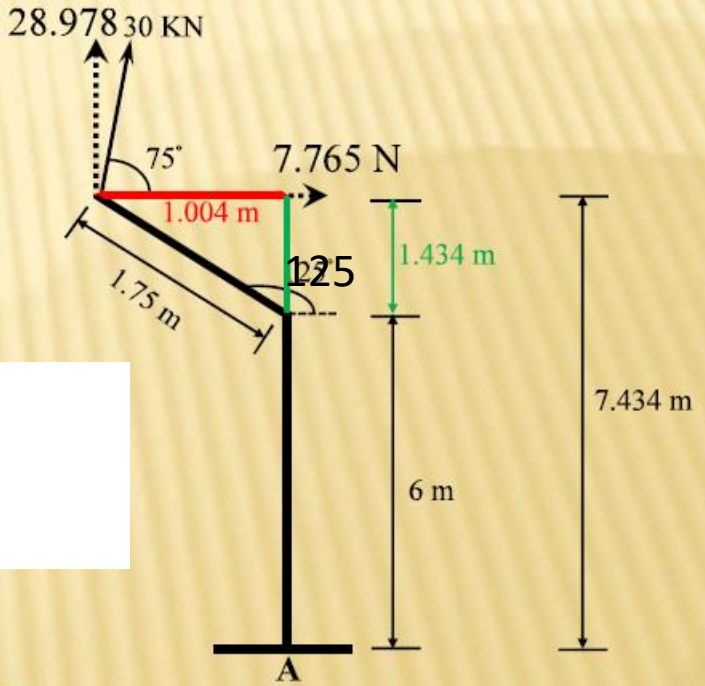
**2. The perpendicular distance to the line of action of the force.**

$$M_A = F\,x\,d$$

$$M_A = 600\,x\,4.34975 = 2609.852\,N.m$$ **Clockwise**



Example: For the structure shown in figure below, calculate the moment at **A** using:
**1.** Varignon's theorem. **2.** The perpendicular distance to the line of action of the force.




**1. Varignon's theorem.**

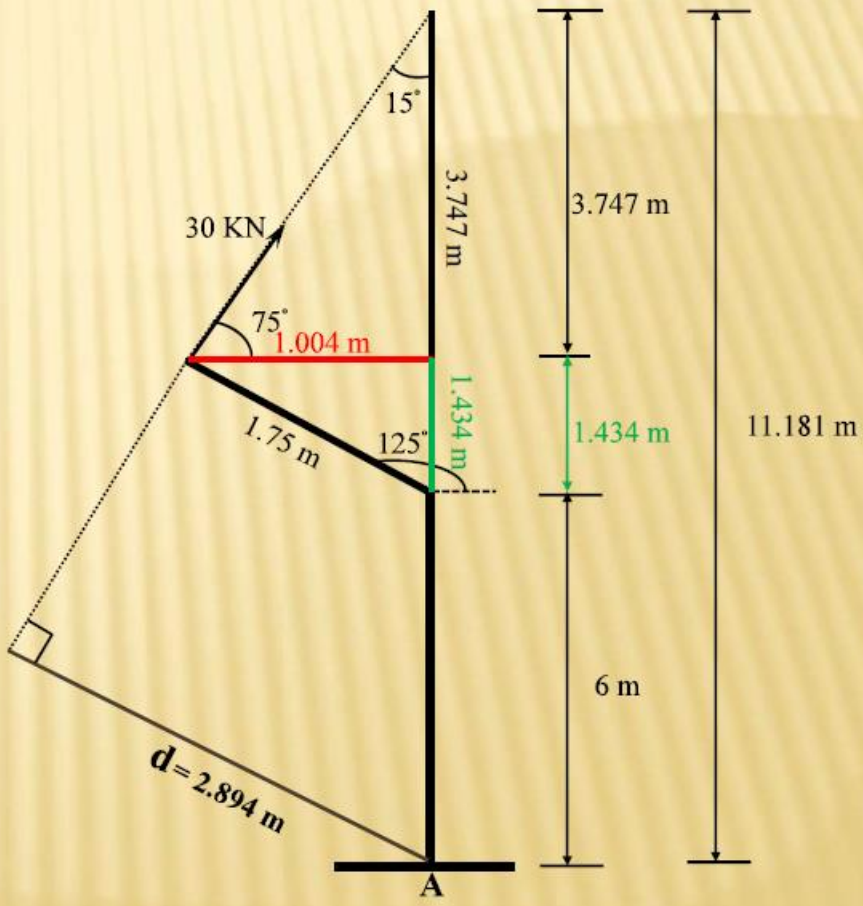
$$(MR)_A = \sum M_A = \sum (F \,.d)$$

$$M_A = (7.765\ x\ 7.434) + (28.978x\ 1.004) = 86.82\ kN.m$$ **Clockwise**

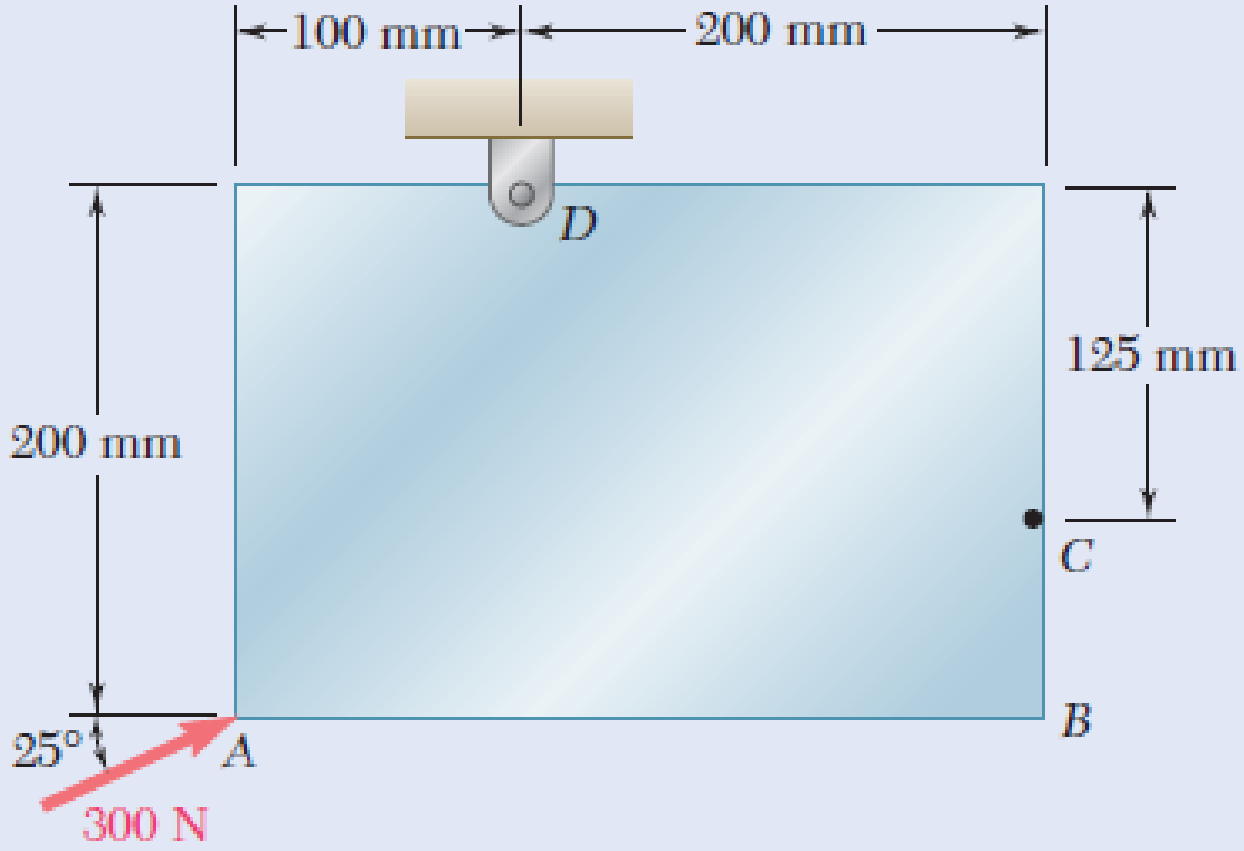
**2. The perpendicular distance to the line of action of the force.**

$$M_A = F\ x\ d$$

$$M_A = 30\ x\ 2.894 = 86.82\ kN.m$$ **Clockwise**



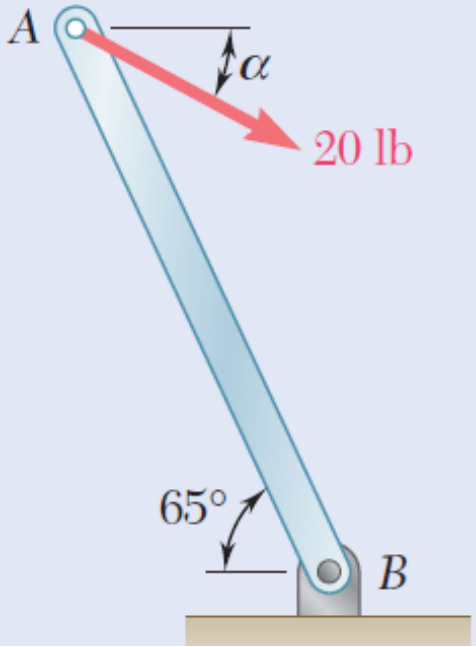
**3.5** A 300-N force is applied at $A$ as shown. Determine ($a$) the moment of the 300-N force about $D$, ($b$) the magnitude and sense of the horizontal force applied at $C$ that creates the same moment about $D$, ($c$) the smallest force applied at $C$ that creates the same moment about $D$.




**3.5** ($a$) 41.7 N·m ↰. ($b$) 334 N. ($c$) 176.8 N ∠ 58.0°.

**3.6** A 20-lb force is applied to the control rod $AB$ as shown. Knowing that the length of the rod is 9 in. and that $\alpha = 25°$, determine the moment of the force about point $B$ by resolving the force into horizontal and vertical components.

**3.7** A 20-lb force is applied to the control rod $AB$ as shown. Knowing that the length of the rod is 9 in. and that $\alpha = 25°$, determine the moment of the force about point $B$ by resolving the force into components along $AB$ and in a direction perpendicular to $AB$.

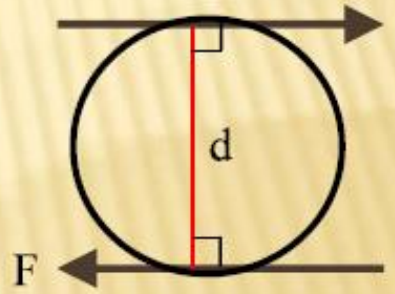



**Answer** $M_B = 115.7$ lb.in

# Moment of a Couple

**Couple:** A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.

$$C = F \; x \; d$$




- The resultant of forces = 0
- A couple will however tend to rotate the body it is acting on

## Difference between Moment and Couple

| Moment | Couple |
|---|---|
| **Depends** upon the axis of rotation and the point of application of the force | It is **independent** upon the axis of rotation and the point of application of the forces |

**Units Of moment of a couple**

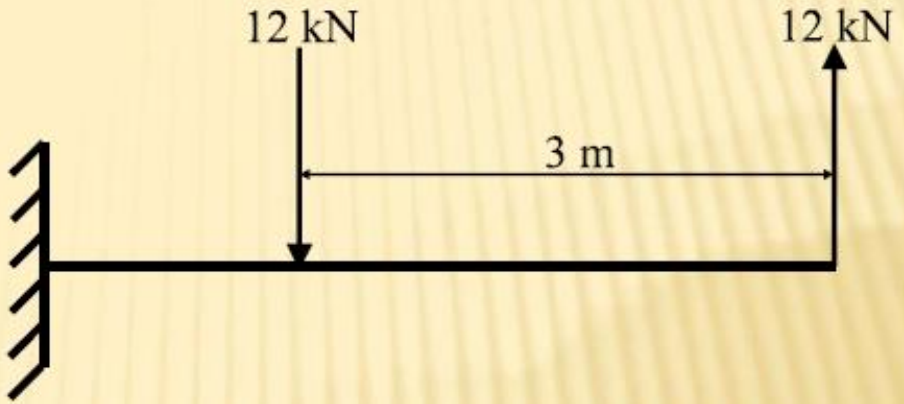
Unit force x Unit length

| | | |
|---|---|---|
| N | mm | $= N.mm$ |
| kN | m | $= kN.m$ |

Example: Determine the magnitude of the couple from the two forces, as shown in figure.

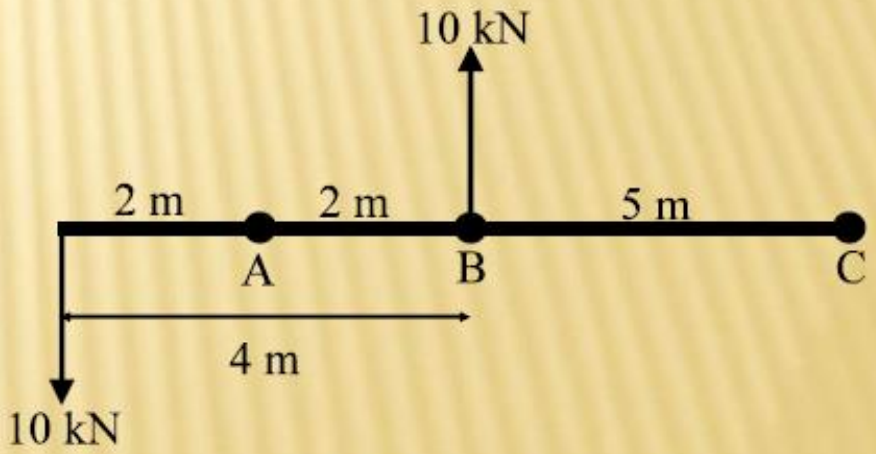
$$C = F\ x\ d$$

$$C = 12\ x\ 3 = 36\ kN.m$$ Counter Clockwise




**Properties of Moment of a Couple**

❖ The couple moment is unaffected by the pivot location



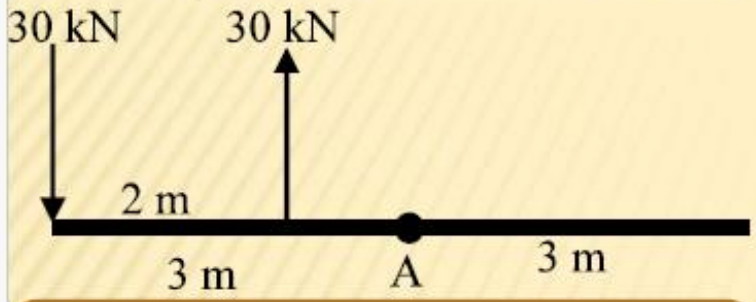

$$M_A = -(10x\ 2) - (10\ x\ 2\ ) = -\ 40\ kN.m$$ Counter Clockwise

$$M_B = -(10x\ 4) + (10\ x\ 0\ ) = -\ 40\ kN.m$$ Counter Clockwise

$$M_C = (10x\ 5) - (10\ x\ 9\ ) = -\ 40\ kN.m$$ Counter Clockwise

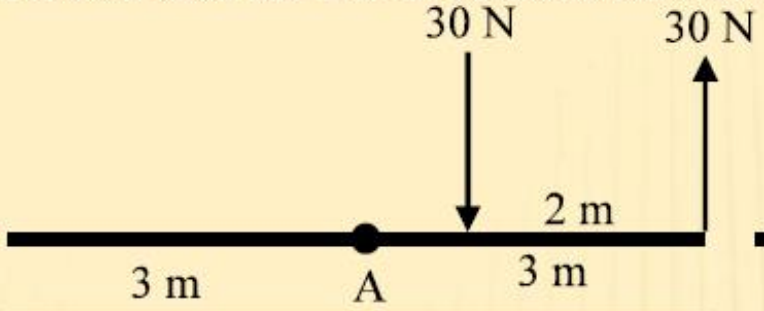
$$C = 10\ x\ 4\ =\ 40\ kN.m$$ Counter Clockwise

❖Couple can be shifted and still have the same moment about a given point



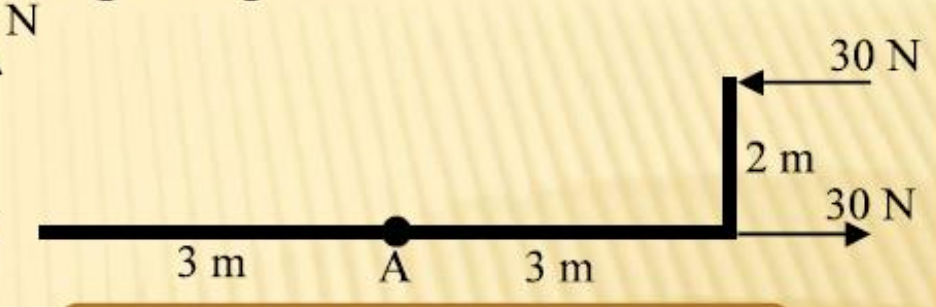

$$C = 30\ x\ 2\ = 60\ kN.m$$

**Counter Clockwise**




$$C = 30\ x\ 2\ = 60\ kN.m$$

**Counter Clockwise**

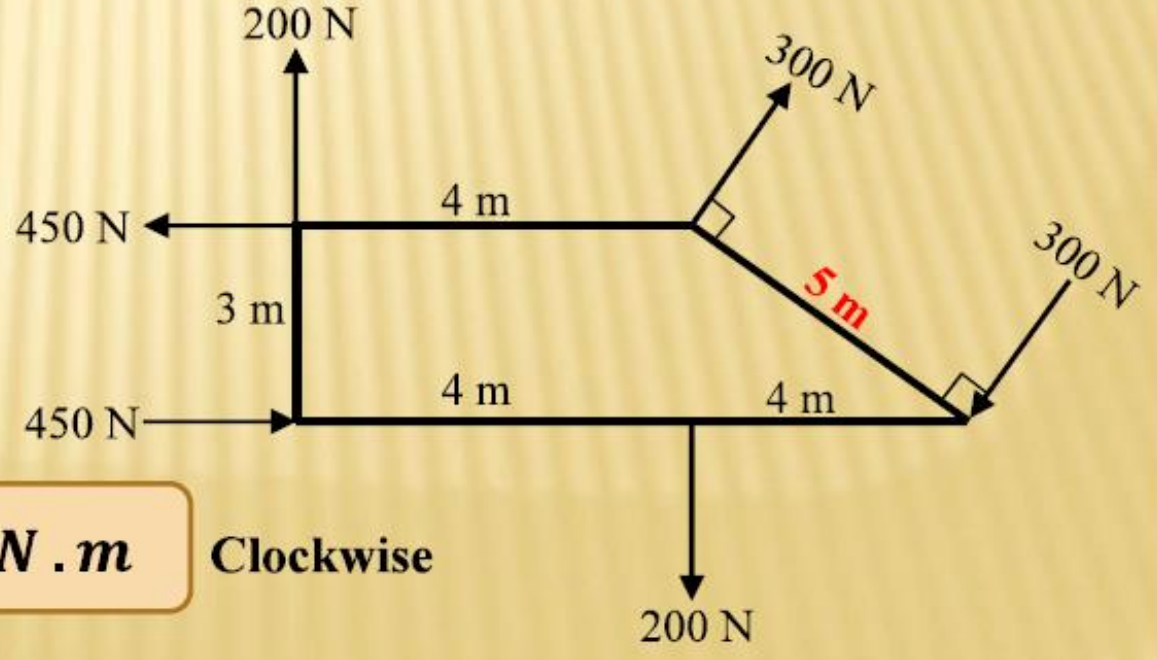



$$C = 30\ x\ 2\ = 60\ kN.m$$

**Counter Clockwise**

**Resultant Couple :** When the system of couples can be reduced to a single a couple

**Example:** Calculate the resultant of the moment of a couple of couples act on the plate as shown in figure.




$$C = \sum F\ x\ d$$

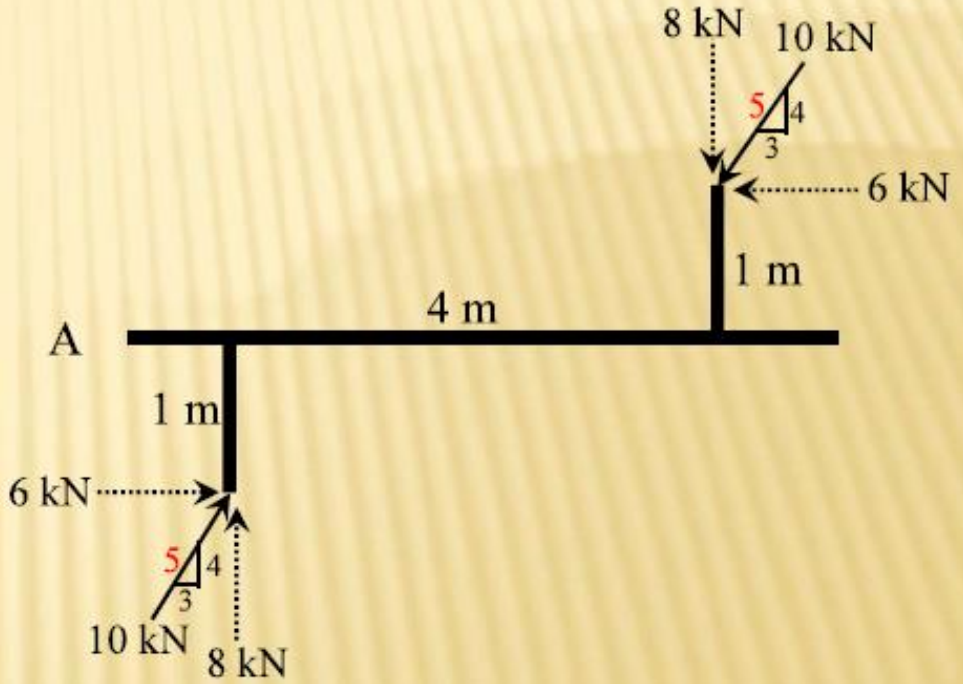
$$C = (300\ x\ 5) + (200\ x\ 4) - (\ 450\ x\ 3) = 950N\ .m$$

**Clockwise**

**Example:** Calculate the moment of a couple of the structure as shown in figure.

$$C = \sum F \; x \; d$$

$$C = (8 \; x \; 4) - (6 \; x \; 2) = 20 \; kN \; . m$$ **Clockwise**



**3.71** Two parallel 40-N forces are applied to a lever as shown. Determine the moment of the couple formed by the two forces (*a*) by resolving each force into horizontal and vertical components and adding the moments of the two resulting couples, (*b*) by using the perpendicular distance between the two forces, (*c*) by summing the moments of the two forces about point *A*.

Answere: Mc=6.194 N.m

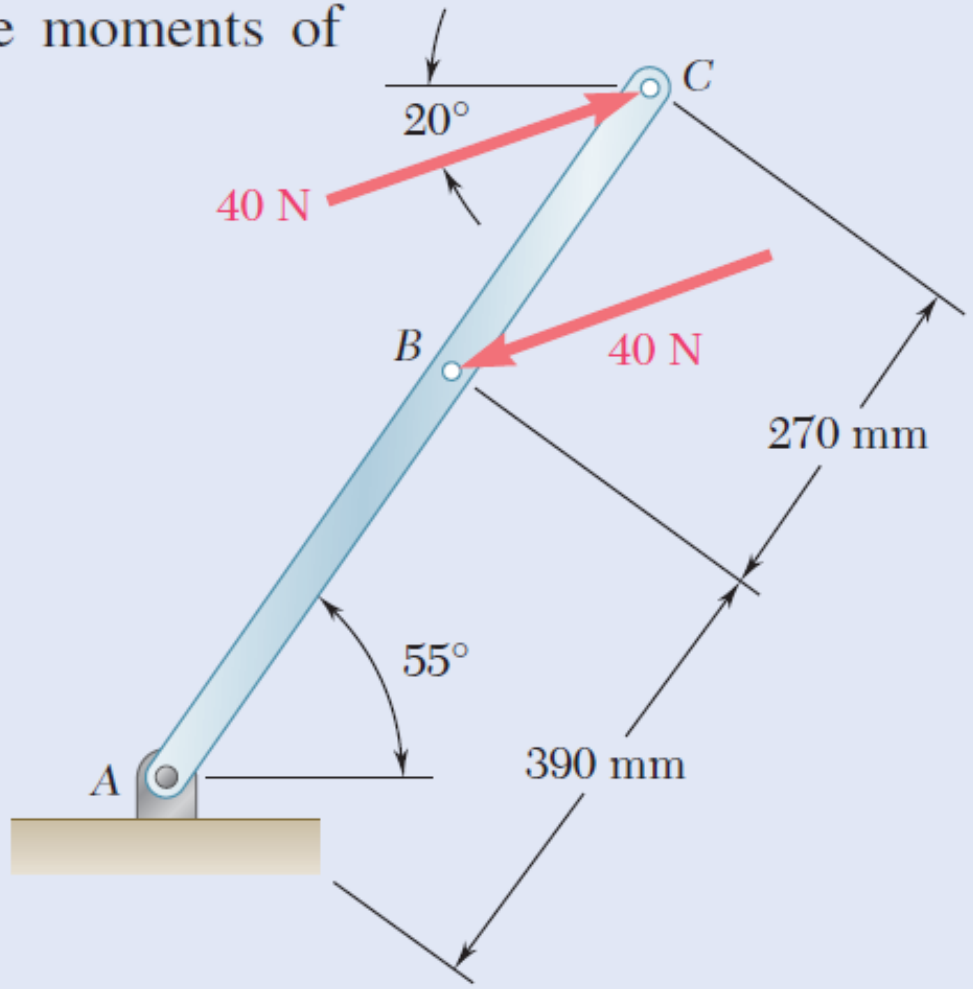
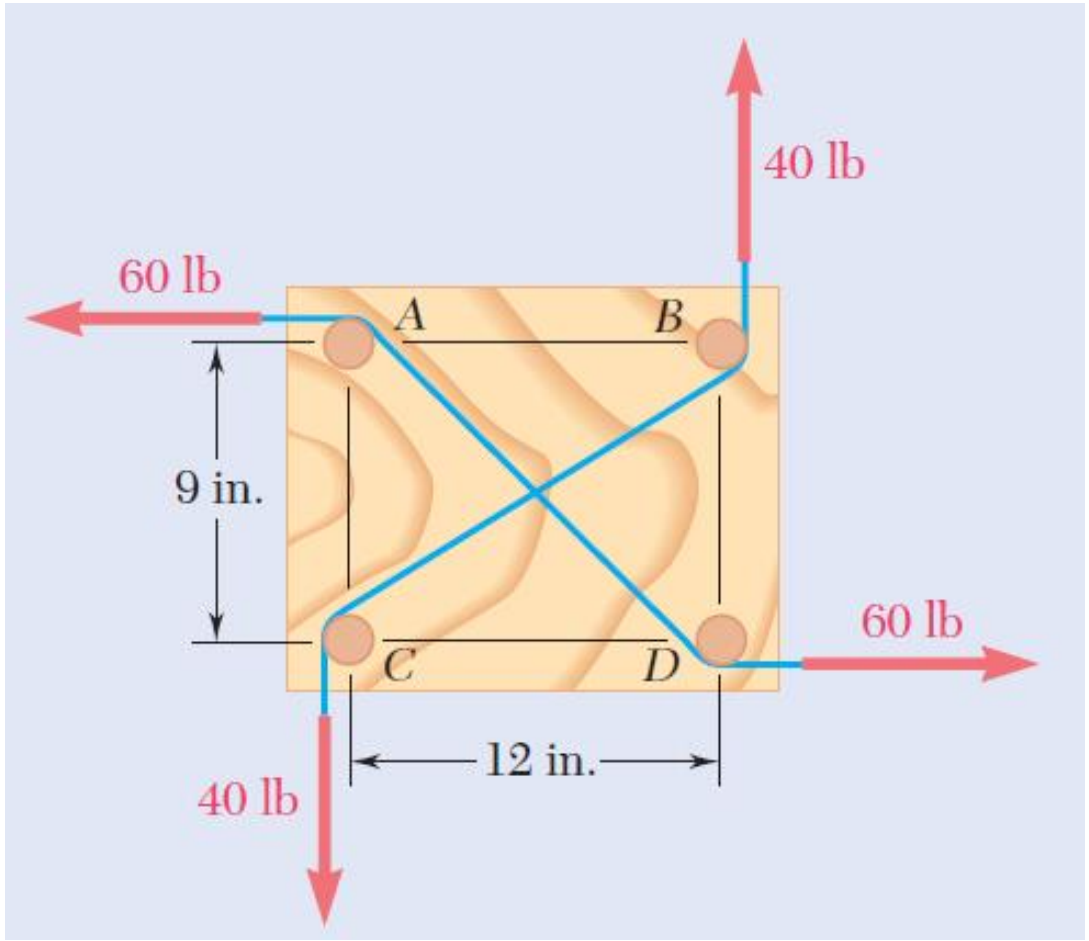



Fig. P3.71

**3.73** Four pegs of the same diameter are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. Determine the diameter of the pegs knowing that the resultant couple applied to the board is 1132.5 lb·in. counterclockwise.

Dimeter = 1.125 in



**3.82** The tension in the cable attached to the end $C$ of an adjustable boom $ABC$ is 560 lb. Replace the force exerted by the cable at $C$ with an equivalent force-couple system (*a*) at $A$, (*b*) at $B$.

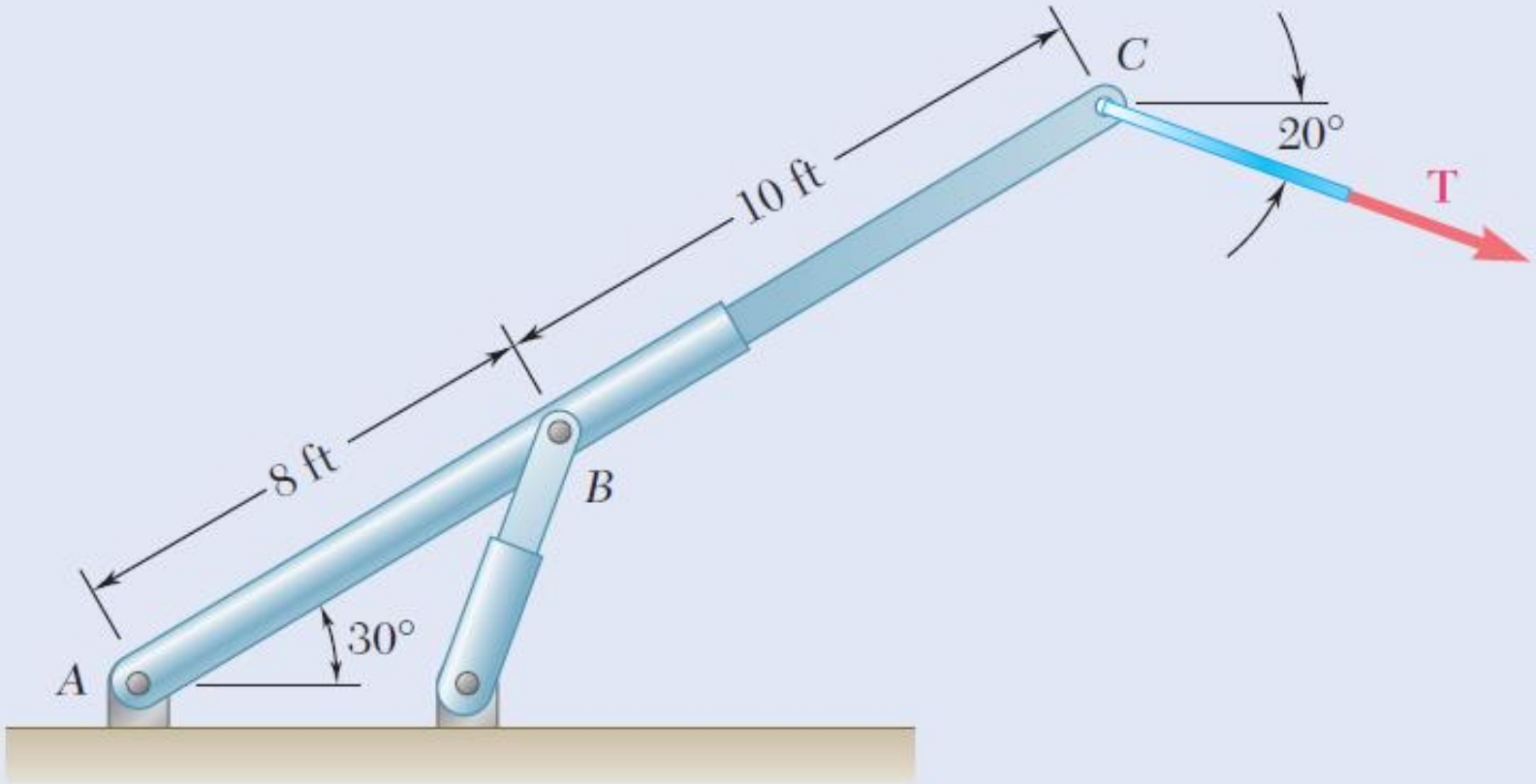



Fig. P3.82

**3.82** (*a*) $\mathbf{F}_A = 560$ lb ◹ 20.0°; $\mathbf{M}_A = 7720$ lb·ft ↻.
(*b*) $\mathbf{F}_B = 560$ lb ◹ 20.0°; $\mathbf{M}_B = 4290$ lb·ft ↻.

**3.83** A dirigible is tethered by a cable attached to its cabin at $B$. If the tension in the cable is 1040 N, replace the force exerted by the cable at $B$ with an equivalent system formed by two parallel forces applied at $A$ and $C$.

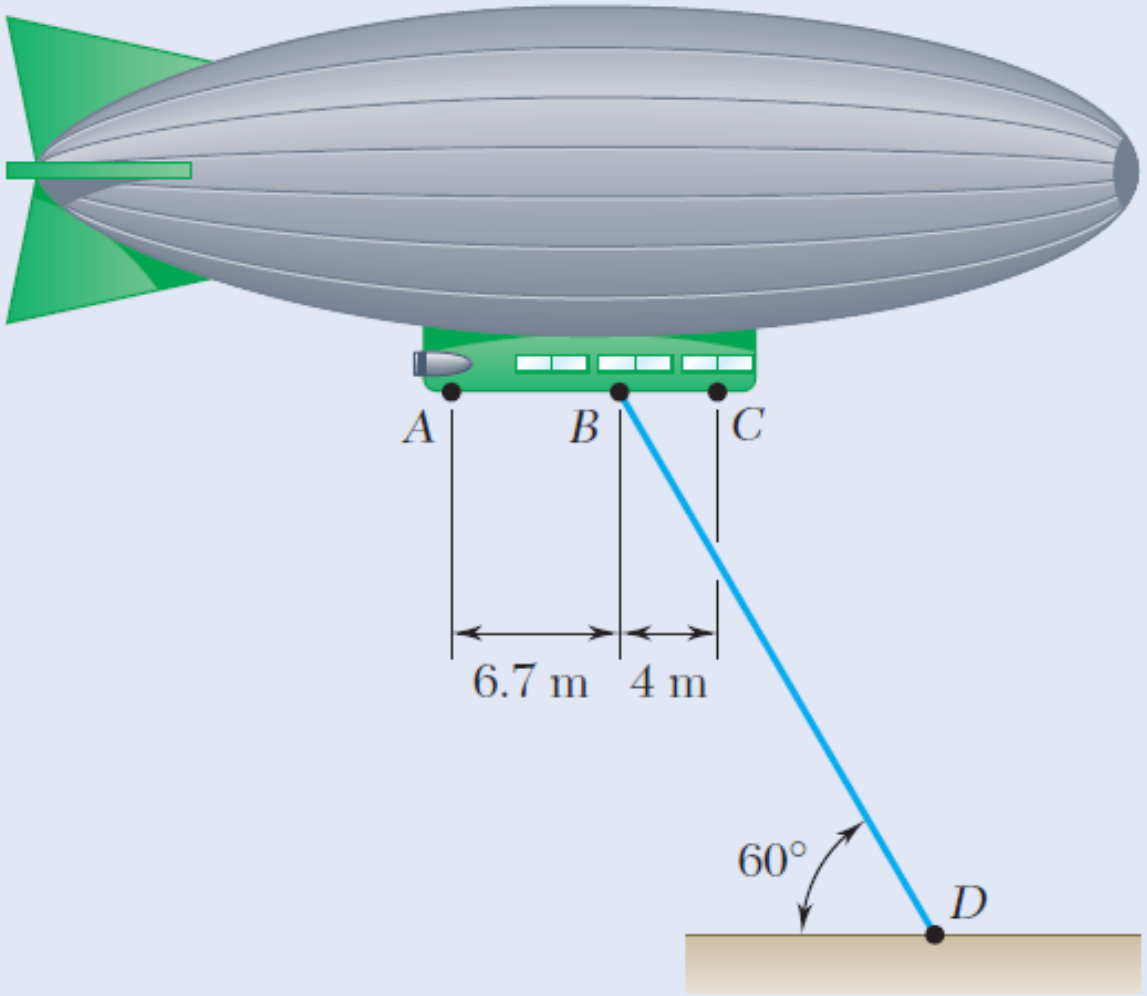
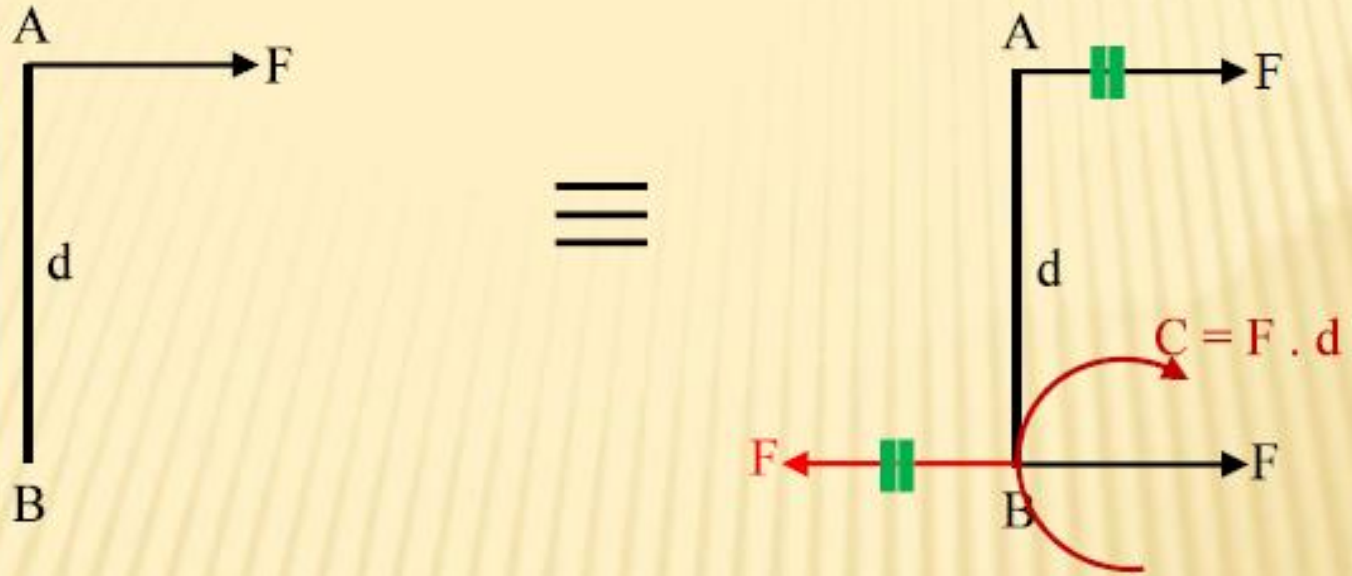



Fig. P3.83

3.83 $\mathbf{F}_A = 389\ \text{N} \measuredangle 60.0°$; $\mathbf{F}_C = 651\ \text{N} \measuredangle 60.0°$.

# Translating of forces

For some reason we want to **move** A force (F) acting on a body from point (A) to anther point (B).



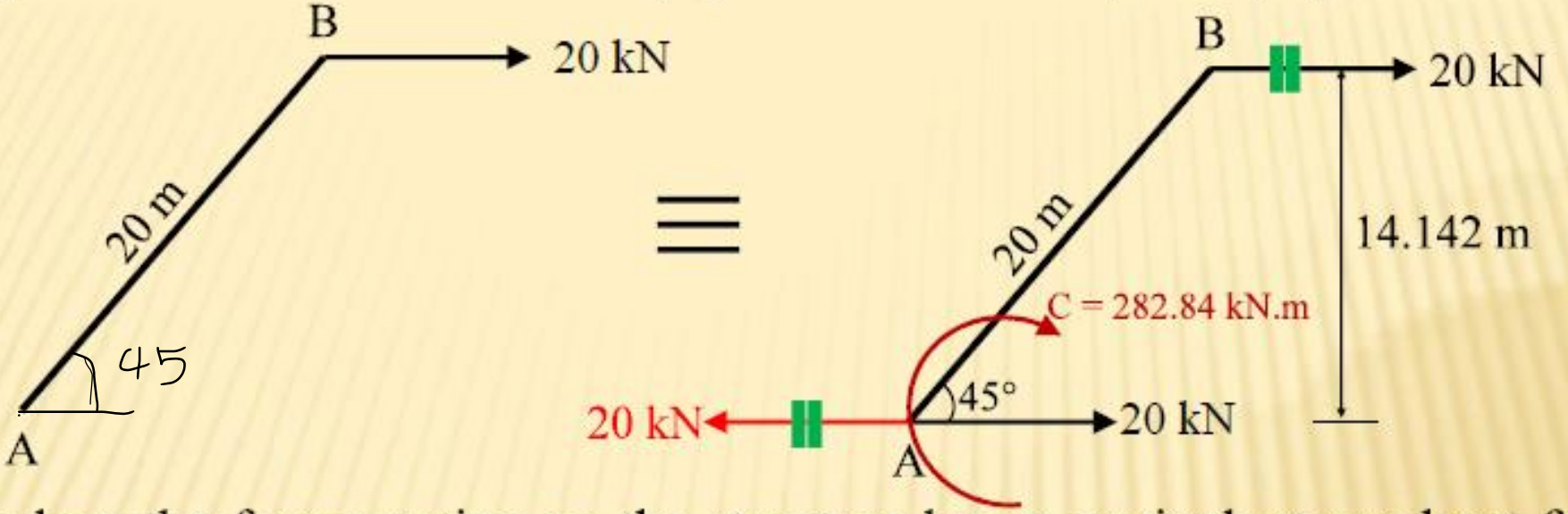

Any force **F** acting on a body can be moved to an arbitrary point provided that a couple is added whose moment is equal to the moment of **F** about the point.

A single force **F** acting at point may be replaced by:

**(a)**An equal and similarly directed force acting through any point and.

**(b)** A couple **C = F . d.**

**Example:** Replace the force F = 20 kN at **(B)** to force and couple at **(A)**, as shown in figure.




**Example:** Replace the forces acting on the structure by an equivalent resultant force and couple moment at **A**.

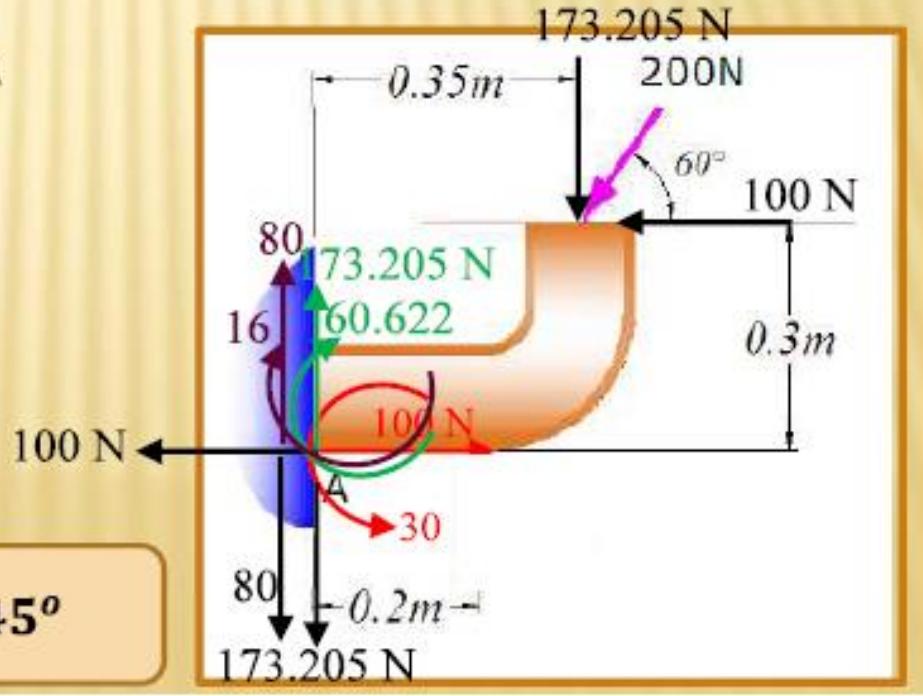
$$C = 60.622 + 16 - 30 = 46.622\ N.m$$

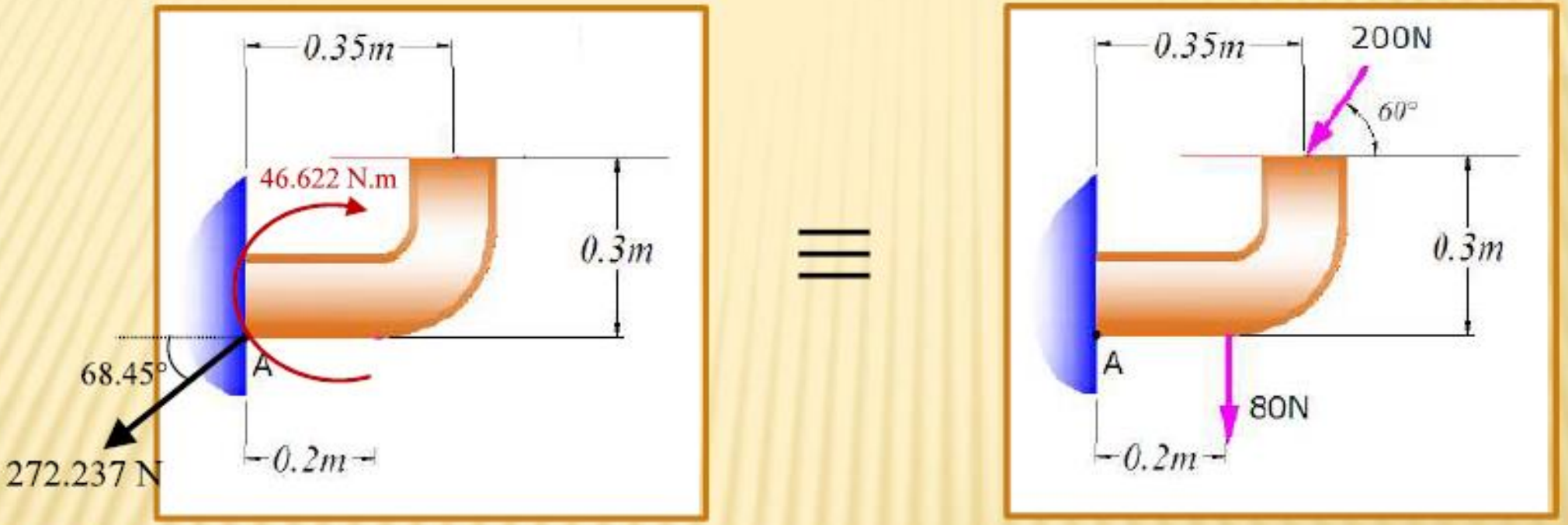
$$Fx = -100\ N \leftarrow$$

$$Fy = -80 - 173.205 = -253.205 \downarrow$$

$$R = \sqrt{F_x^{\,2} + F_y^{\,2}} = 272.237\ N$$

$$\theta = tan^{-1}\left(\frac{F_y}{F_x}\right) = 68.45^o$$




0.35m
46.622 N.m
0.3m
68.45°
A
272.237 N
0.2m
≡
0.35m
200N
60°
0.3m
A
80N
0.2m

**Example:** Replace the force and the moment act on the beam shown in figure with a **resultant force** and **moment** at **A.**

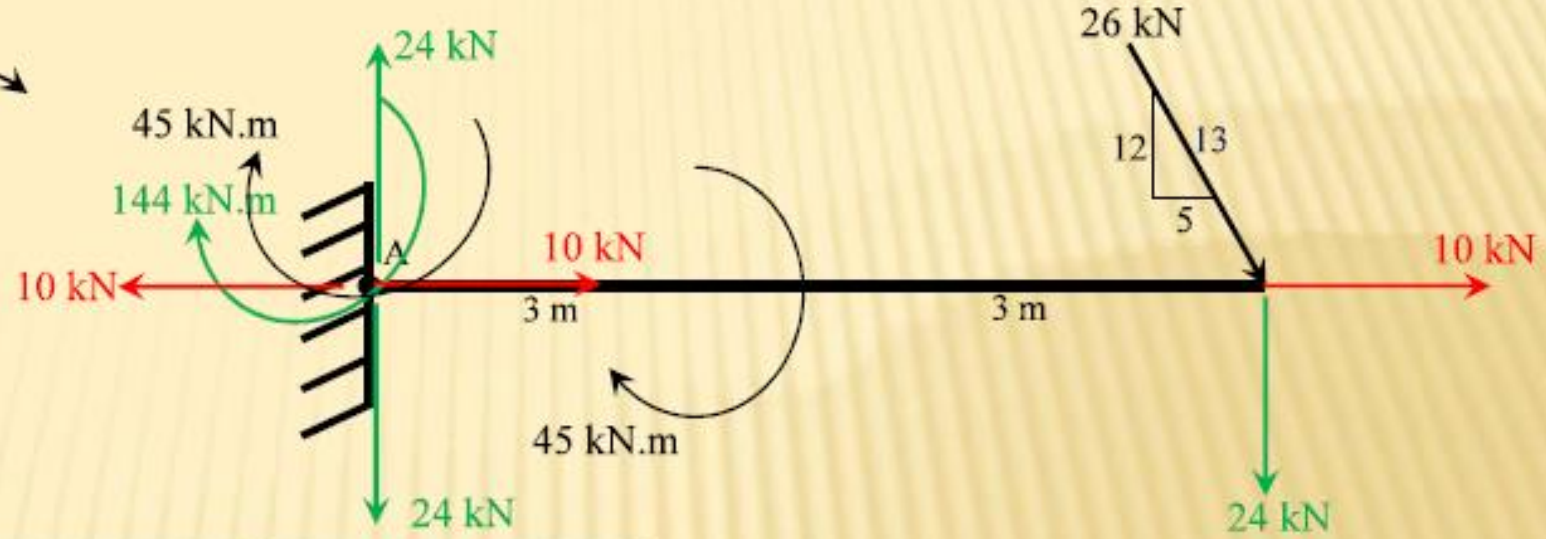
$$C = 45 + 144 = 189\ kN.m$$

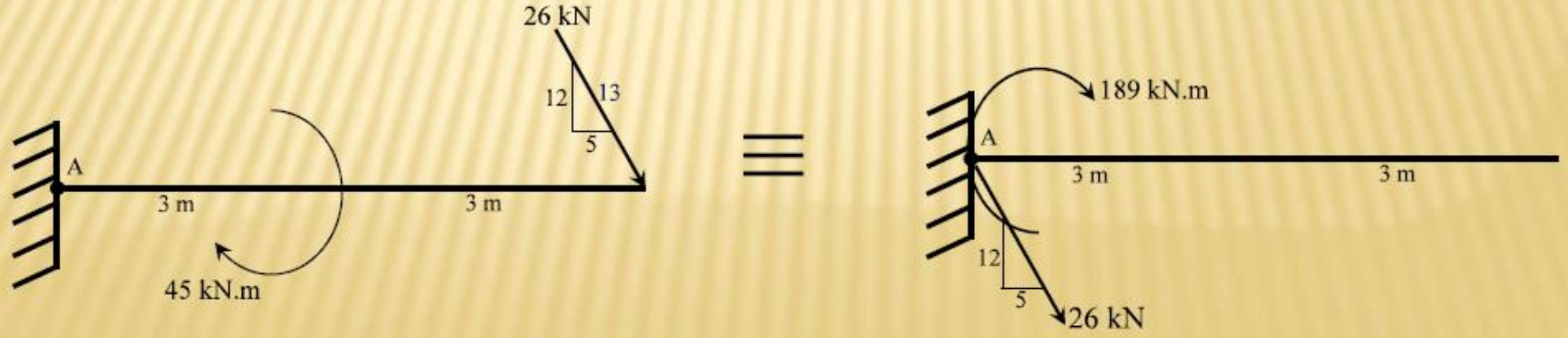
$$Fx = 10\ kN\ \rightarrow$$

$$Fy = -24\ kN\ \downarrow$$

$$R = \sqrt{F_x{}^2 + F_y{}^2} = 26\ kN$$

$$\theta = tan^{-1}(\frac{F_y}{F_x}) = 67.38^o$$

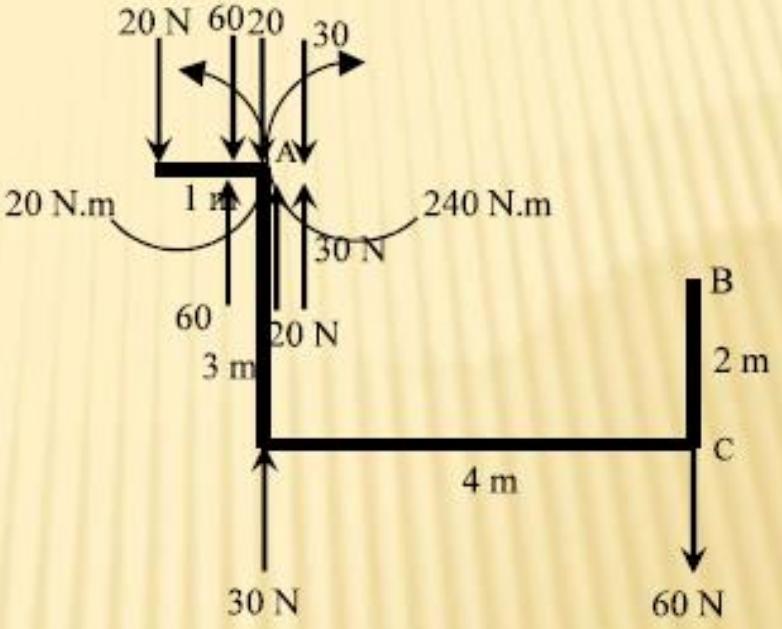


**Example:** Replace the system of forces acting on the frame in figure below by a **resultant force** at **A** and a **couple** acting **horizontally** throughout **B** and **C**.

$$C_A = 240 - 20 = 220\ N.m$$

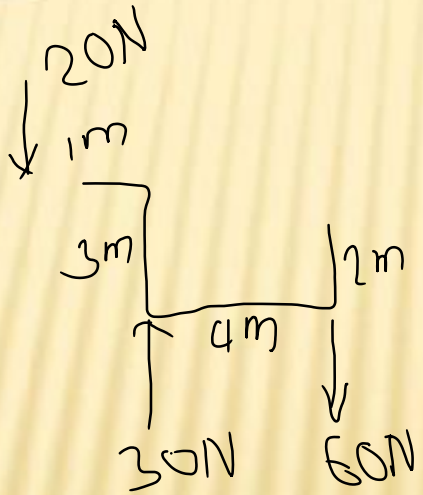
$$F = 30 - 20 - 60 = -50\ N \downarrow$$

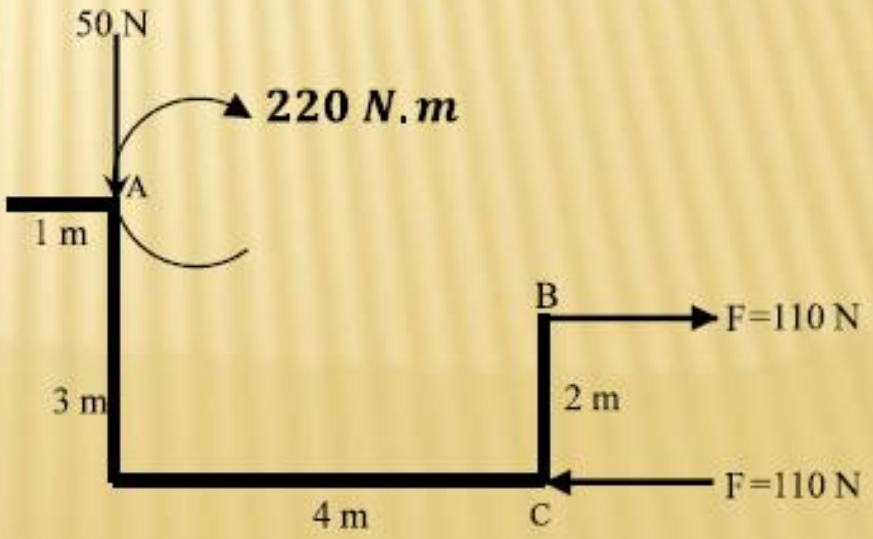



$$C = F.d$$

$$220 = F.2$$

$$F = 110\ N$$



## ➢ Parallel force system



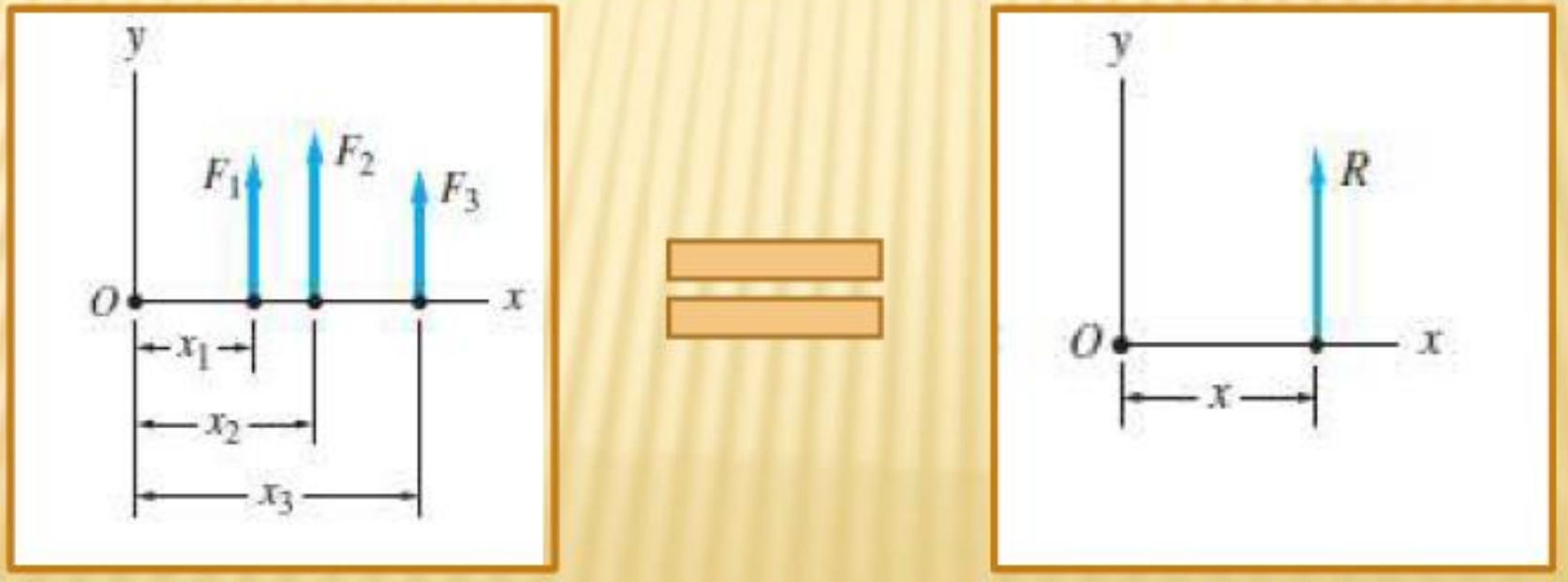
$$R = \sum F = F1 + F2 + F3 + \ldots.$$

$$R.x = \sum M_o \Longrightarrow x = \frac{\sum M_o}{R}$$

R : The resultant.

X : Perpendicular distance from the moment point to the R.

$\sum M_o$ : Algebraic sum of the moment of the forces with respect to O.




**Parallel force system** **Resultant**

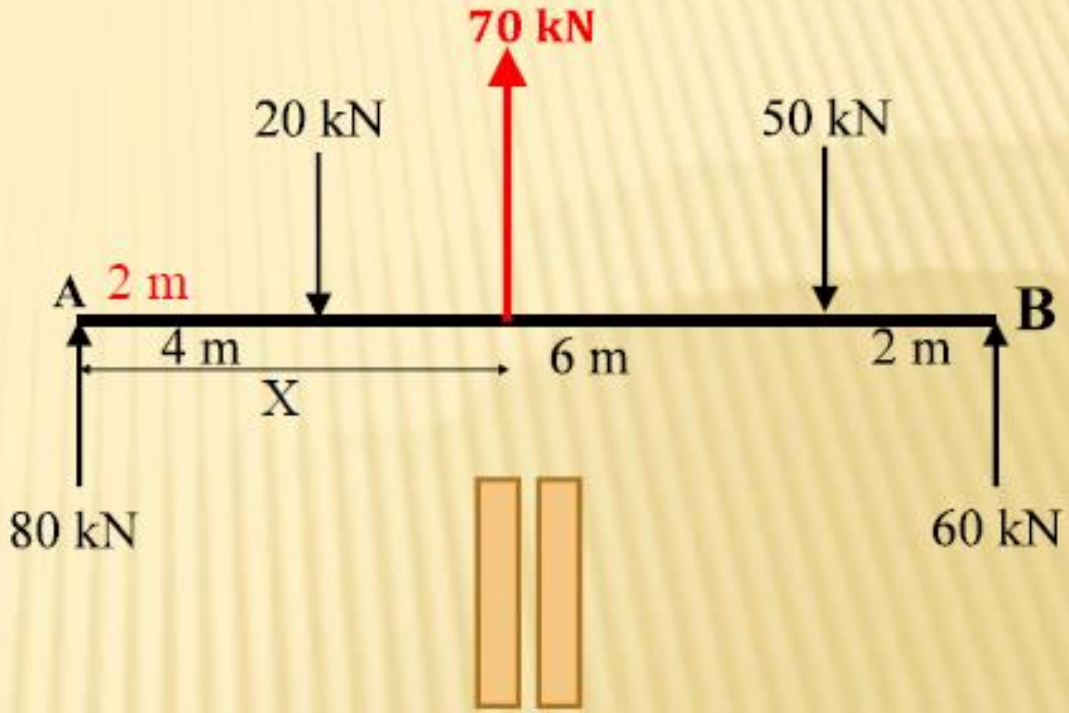
**Example:** Determine the resultant of the system of parallel forces shown in figure and locate it with respect to point A.

$$R = \sum F_y = 80 + 60 - 20 - 50 = 70\ kN \uparrow$$

$$R.x = \sum M_A$$

$$-70.x = 20\,(4) + 50(10) - 60\,(12)$$

$$x = \frac{\sum M_A}{R} = \frac{-140}{-70} = 2\ \text{m}$$





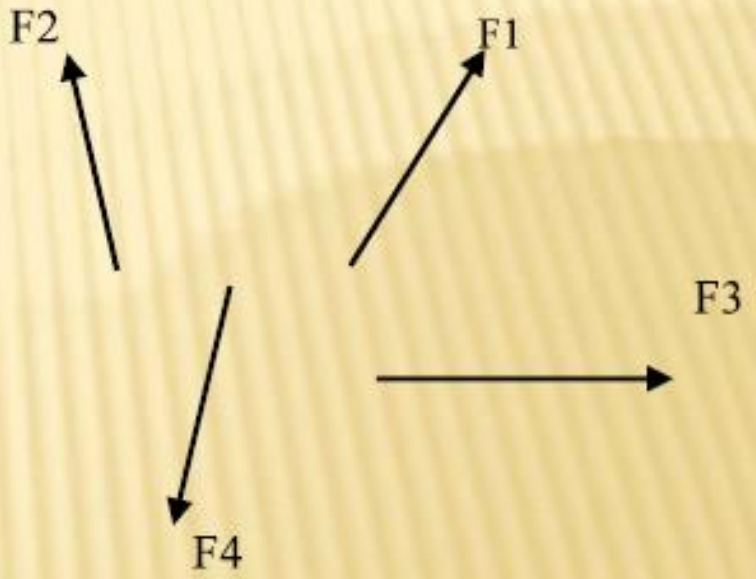


**H.W.**
Calculate the location of the resultant respect to point B.

# Resultant

## ➢ Non concurrent force system

The resultant of non-concurrent force system is defined according to magnitude, inclination, and position.




$$Rx = \sum F_x \qquad Ry = \sum F_y$$

$$R = \sqrt{R_x^{\,2} + R_y^{\,2}} \qquad \theta = tan^{-1}\left(\frac{R_y}{R_x}\right)$$

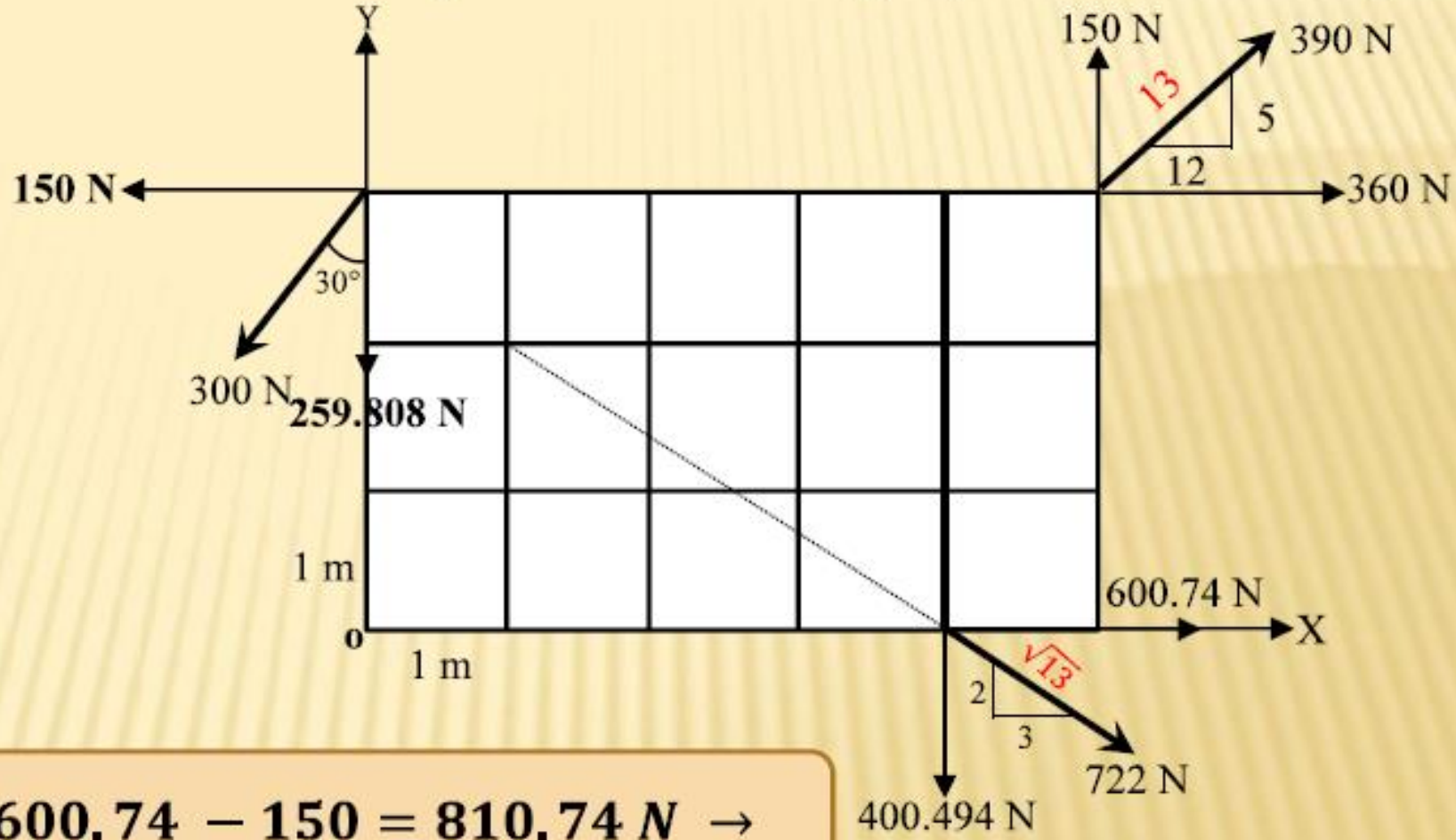
$$R.x = \sum M_o \Longrightarrow x = \frac{\sum M_o}{R}$$

R : The resultant.

X : Perpendicular distance from the moment point to the R.

$\sum M_o$ : Algebraic sum of the moment of the forces with respect to O.

**Example:** Determine the resultant of the force system shown in figure, locate it with respect to O, and its x and y intercepts.




$$Rx = \sum F_x = 360 + 600.74 - 150 = 810.74\ N \rightarrow$$

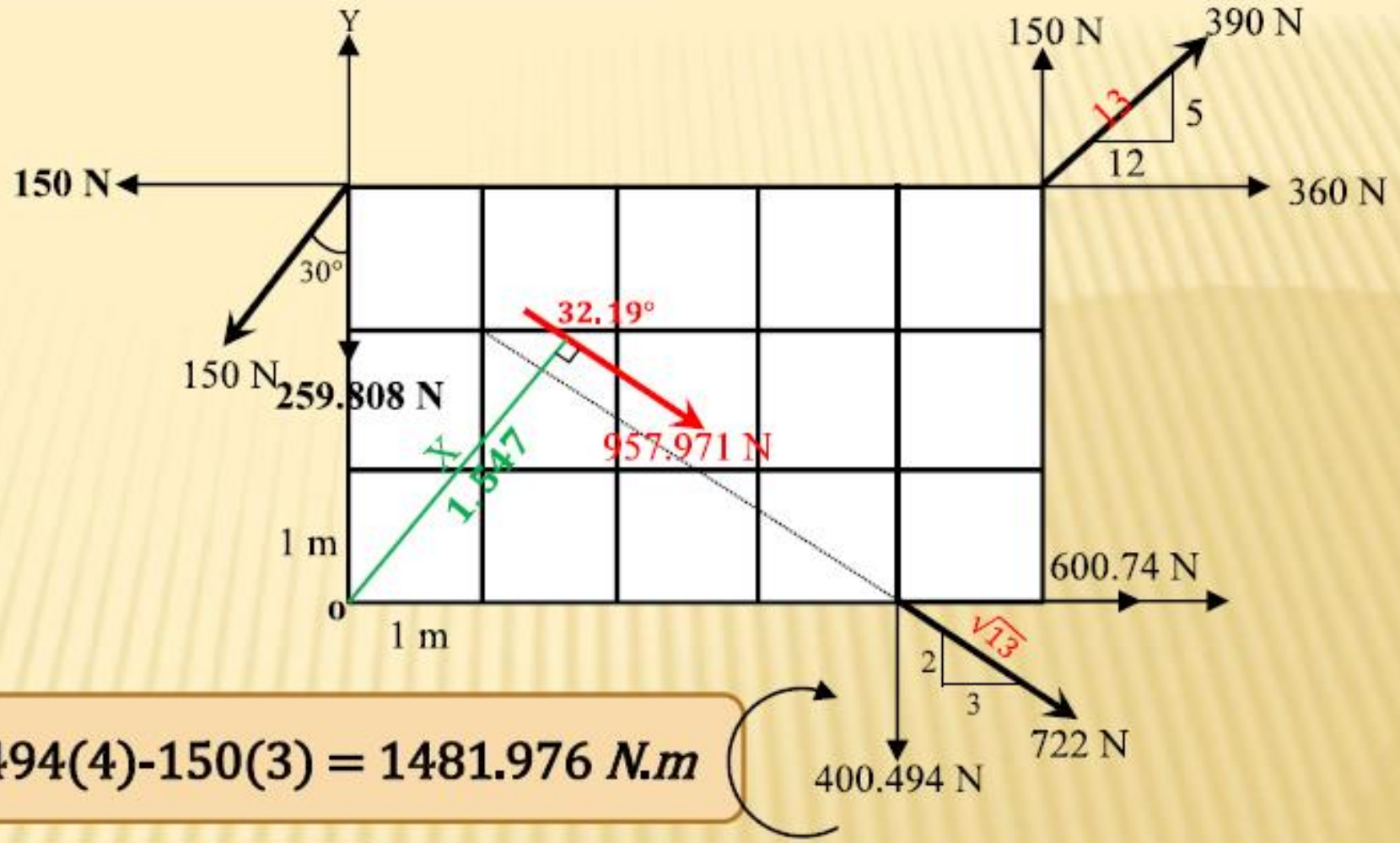
$$Ry = \sum F_y = 150 - 400.494 - 259.808 = -510.302\ N \downarrow$$

$$R = \sqrt{R_x{}^2 + R_y{}^2} = 957.971\ N$$

$$\theta = tan^{-1}\left(\frac{R_y}{R_x}\right) = 32.19°$$



$$(R.x)_o = \sum M_o$$




$$\sum M_o = 360(3)-150(5)+400.494(4)-150(3) = 1481.976\ N.m$$

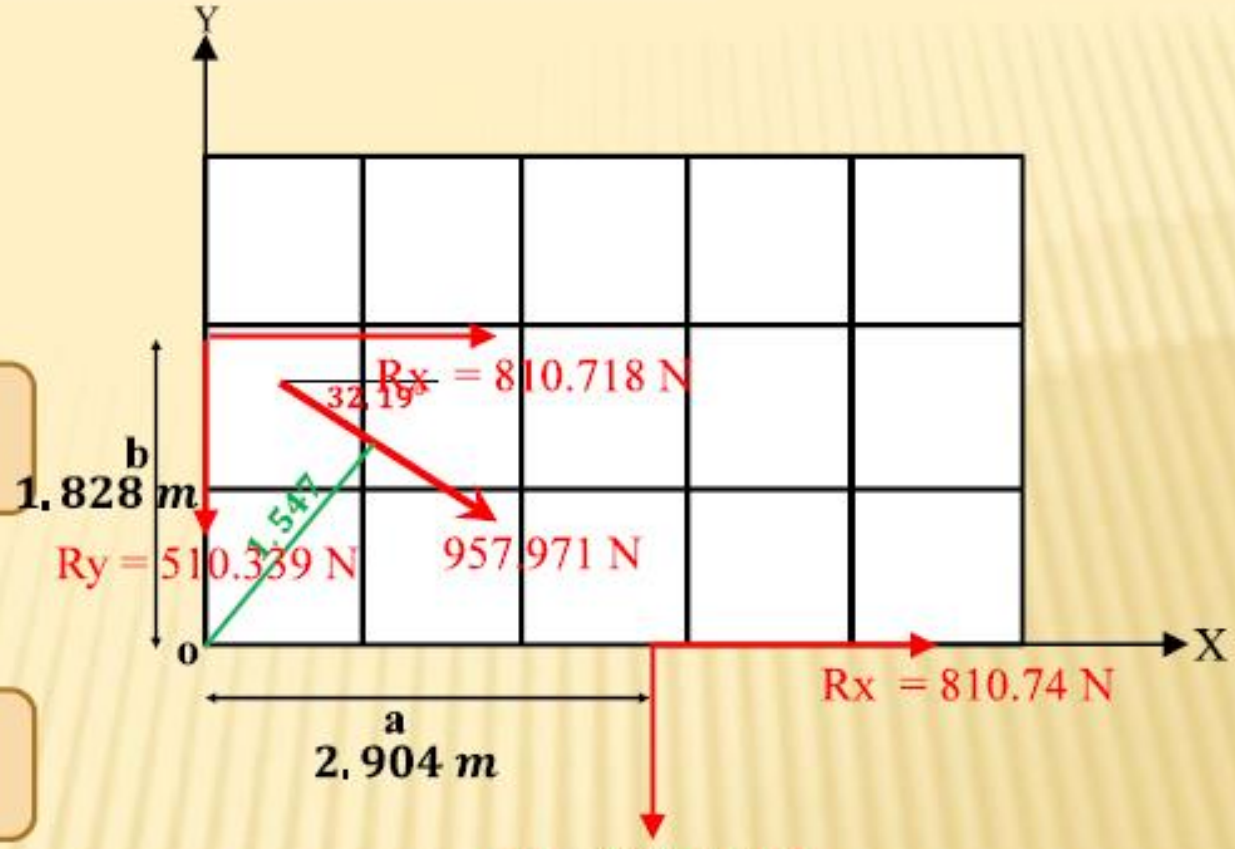
$$x = \frac{\sum M_o}{R} = \frac{1481.976}{957.971} = 1.547\ m$$

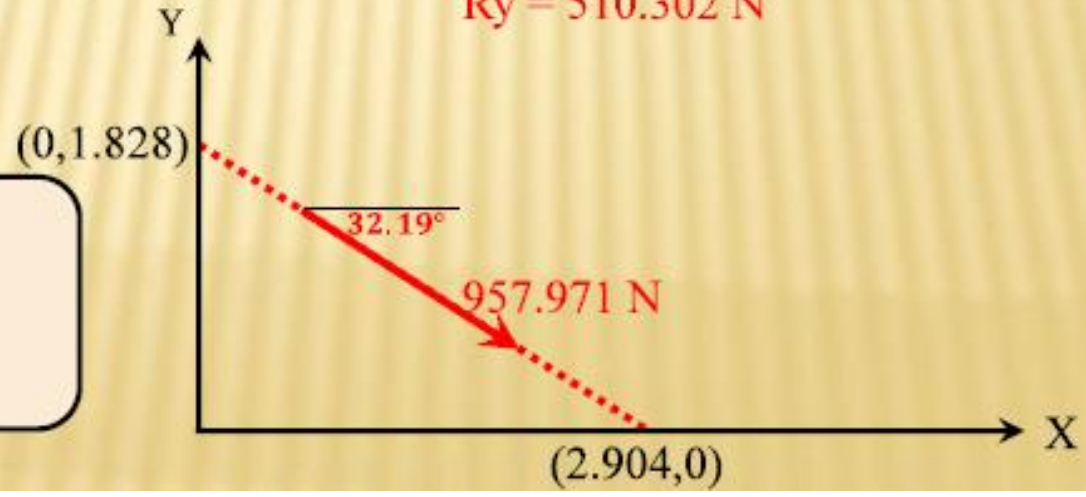
$$Ry.a = \sum M_o$$

$$a = \frac{\sum M_o}{Ry} == \frac{1481.976}{510.302} 2.904\ m$$

$$Rx.b = \sum M_o$$

$$b = \frac{\sum M_o}{Rx} = \frac{1481.976}{810.74} = 1.828\ m$$




$$Thus, R = 957.971\ N\ , \theta_X = 32.19°$$

**The intercepts of R are: (2.904,0) and (0,1.828)**

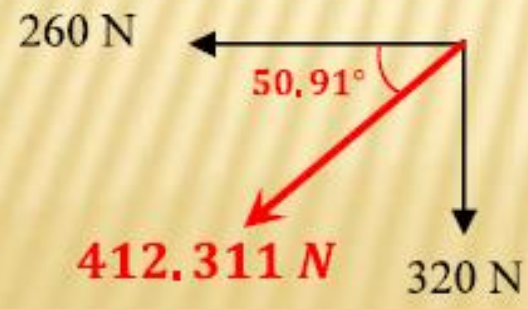
Example: determine the resultant for the force system in figure below, and locate it with respect to point A.

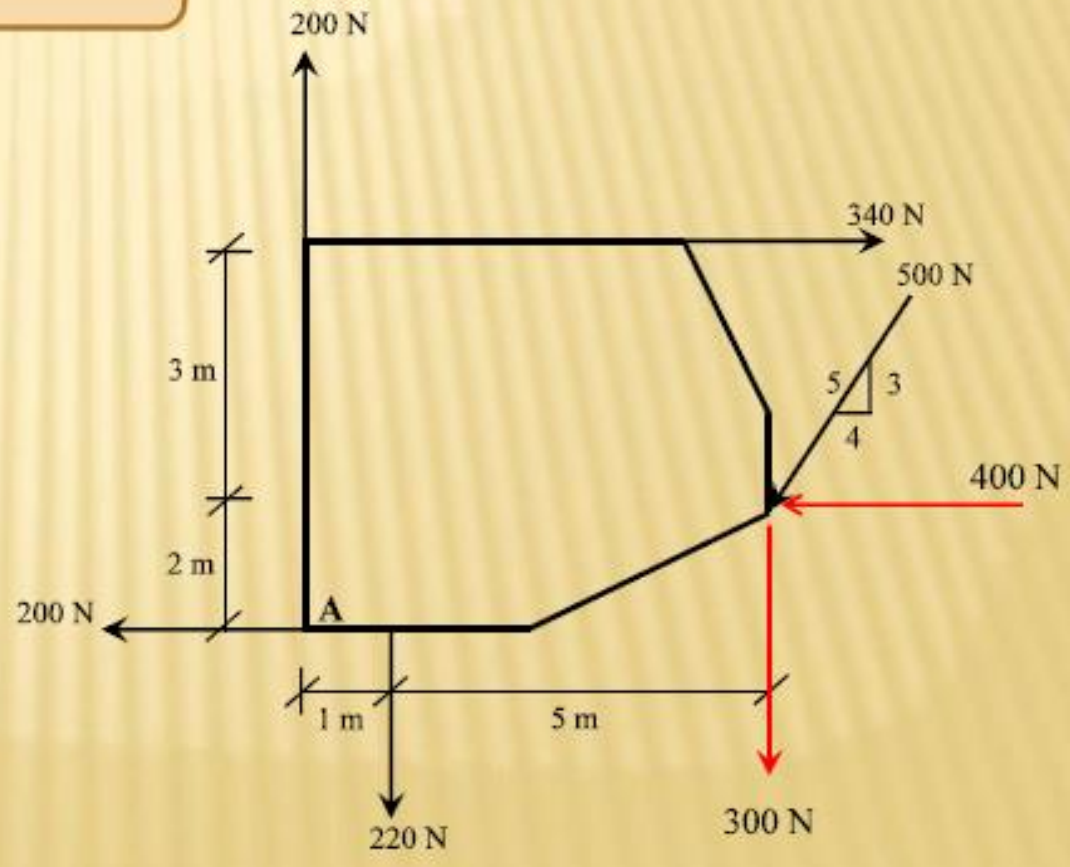
$$Rx = \sum F_x = 340 - 200 - 400 = -260\ N \leftarrow$$

$$Ry = \sum F_y = 200 - 220 - 300 = 320\ N \downarrow$$

$$R = \sqrt{R_x^{\,2} + R_y^{\,2}} = 412.311\ N$$

$$\theta = tan^{-1}\left(\frac{R_y}{R_x}\right) = 50.91°$$



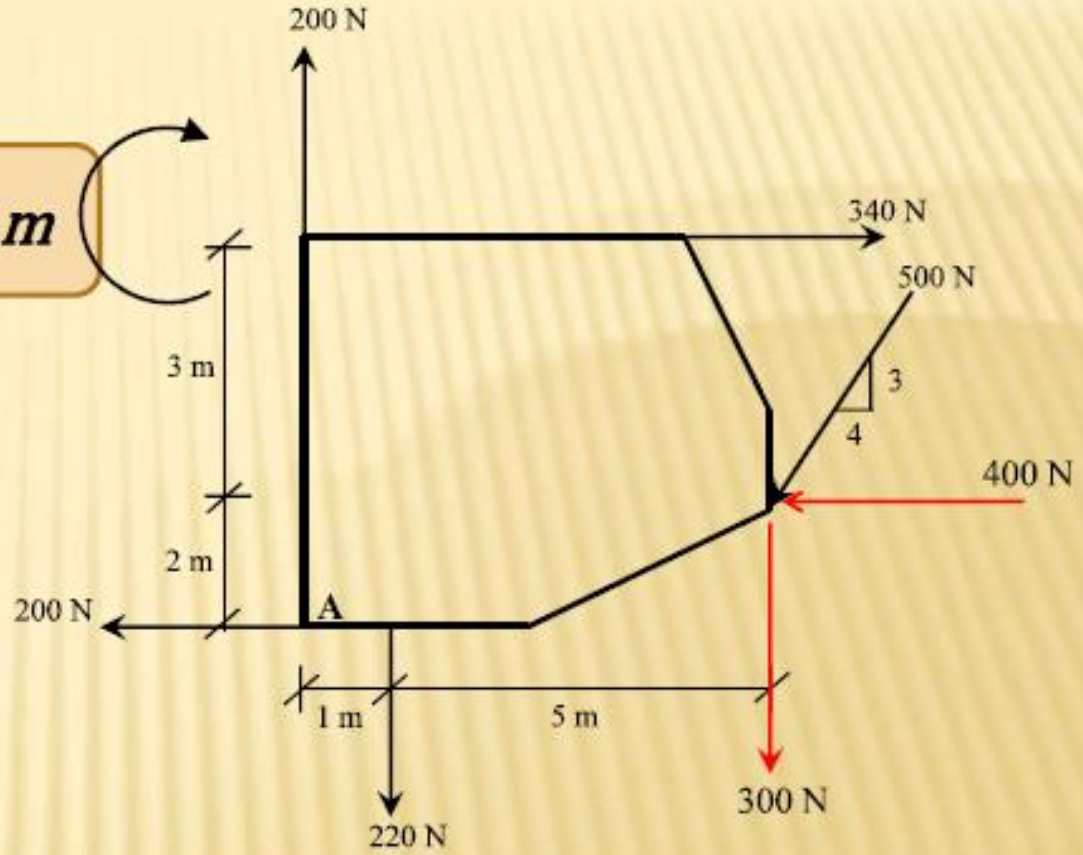



$$(R.x)_A = \sum M_A$$

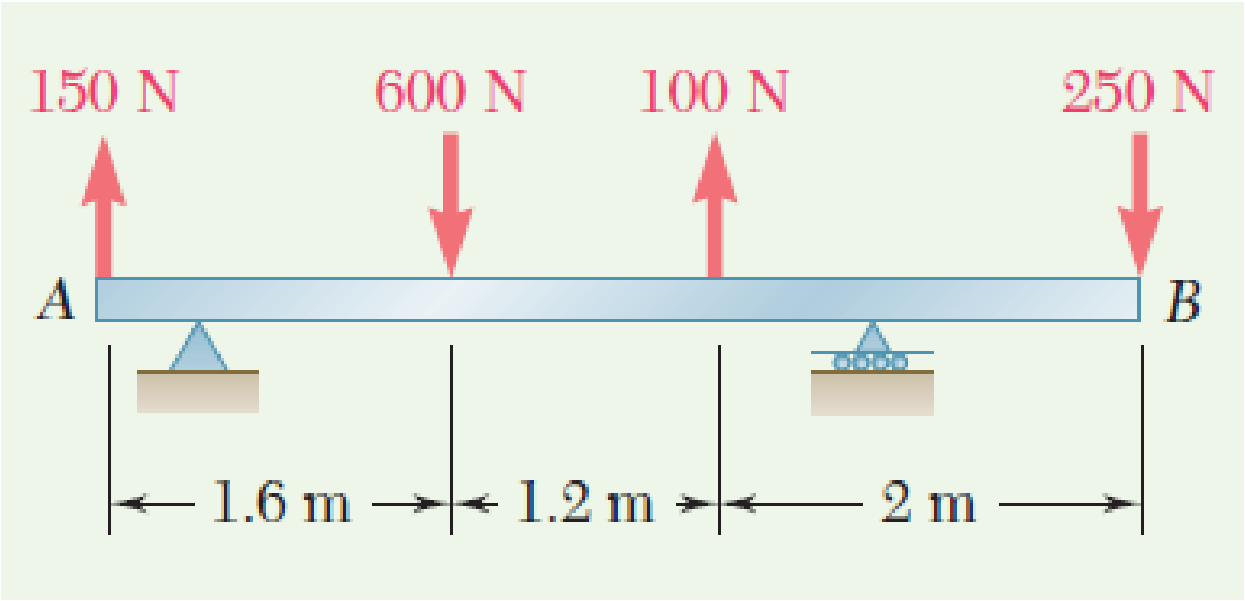
$$\sum M_A = 220(1)+300(6)-400\,(2)+340(5) = 2920\ N.m$$

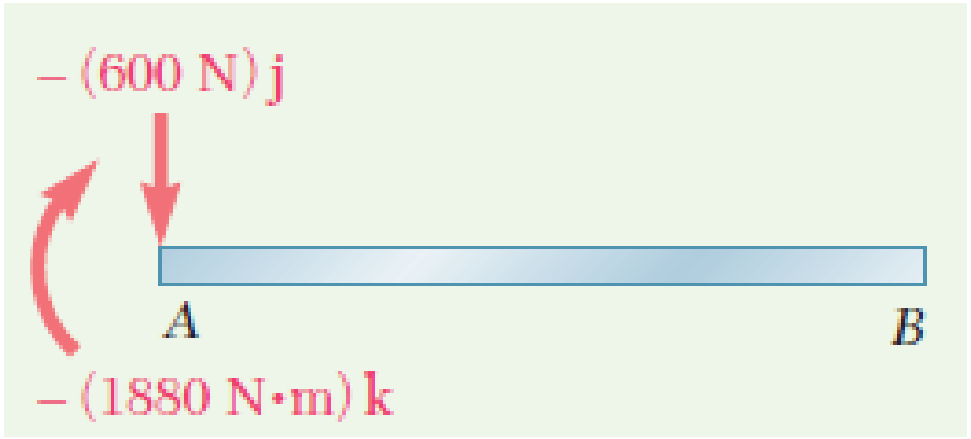
$$x = \frac{\sum M_A}{R} = \frac{2920}{412.311} = 7.08\ m$$



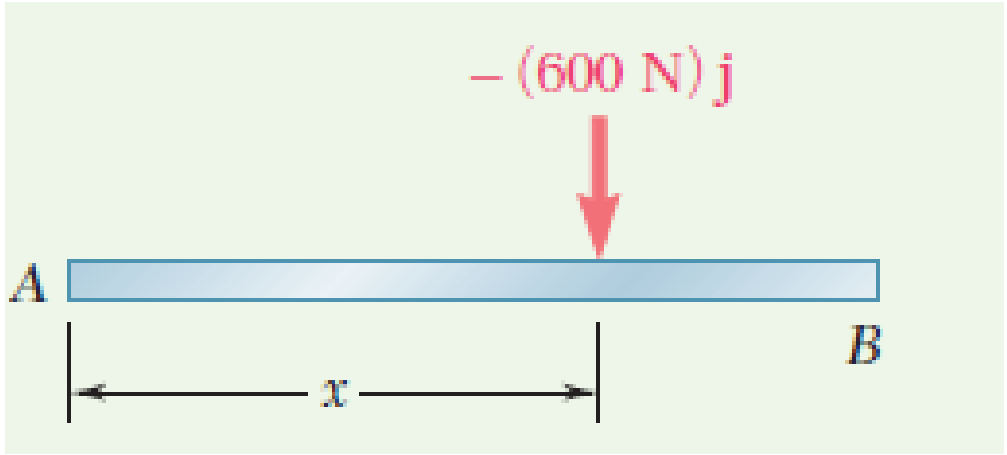



A 4.80-m-long beam is subjected to the forces shown. Reduce the given system of forces to (*a*) an equivalent force-couple system at *A*, (*b*) an equivalent force-couple system at *B*, (*c*) a single force or resultant. *Note:* Since the reactions at the supports are not included in the given system of forces, the given system will not maintain the beam in equilibrium.




– (600 N) j
A
B
– (1880 N·m) k


– (600 N) j
A
B
(1000 N·m) k


– (600 N) j
A
B
x

## Equilibrium

A body is said to be in equilibrium if the resultant of the force system that acts on the body vanishes.

Equilibrium means that both the resultant force and the resultant couple are zero.

**Equations of equilibrium are:**

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_o = 0$$

## Free-Body Diagram of a Body

The first step in equilibrium analysis is to identify all the forces that act on the body. This is accomplished by means of a free-body diagram.

**Free-body diagram (FBD)** of a body is a sketch of the body showing all forces that act on it. The term free implies that all supports have been removed and replaced by the forces (reactions) that they exert on the body.

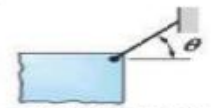
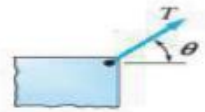
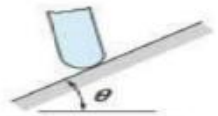
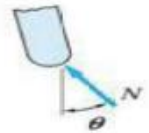
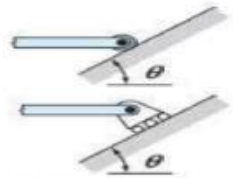
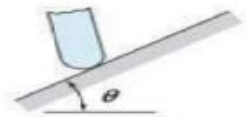
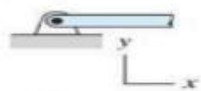
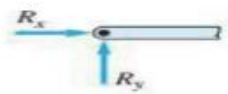
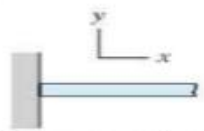
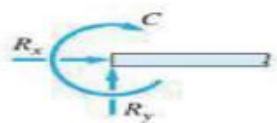
**The following is the general procedure for constructing a free-body diagram:**

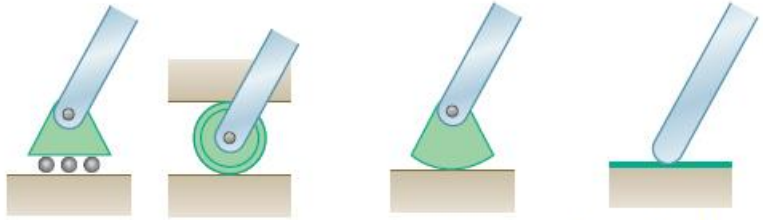
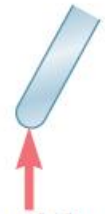
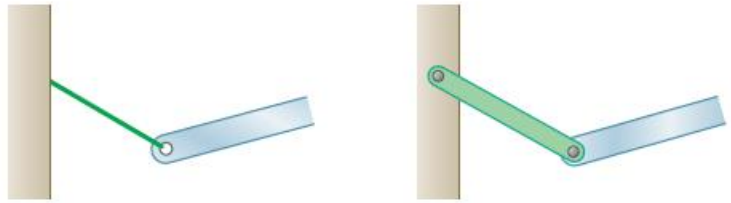
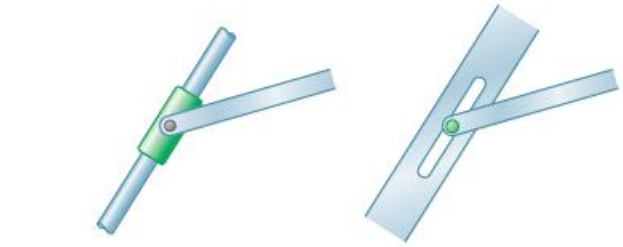
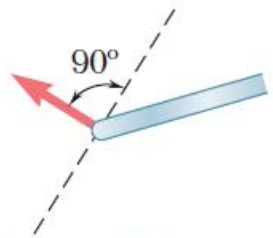
**1.** A sketch of the body is drawn assuming that all supports (surfaces of contact, supporting cables, etc.) have been removed.

**2.** All applied forces are drawn and labeled on the sketch. The weight of the body is considered to be applied force acting at the center of gravity.

**3.** The support reactions are drawn and labeled on the sketch. If the sense of a reaction is unknown, it should be assumed. The solution will determine the correct sense: A positive result indicates that the assumed sense is correct, whereas a negative result means that the correct sense is opposite to the assumed sense.

**4.** All relevant angles and dimensions are shown on the sketch.

| Support | Reaction(s) | Description of reaction(s) | Number of unknowns |
|---|---|---|---|
| (a)  Flexible cable of negligible weight |  | Tension of unknown magnitude $T$ in the direction of the cable | One |
| (b)  Frictionless surface (single point of contact) |  | Force of unknown magnitude $N$ directed normal to the surface | One |
| (c)  Roller support |  | Force of unknown magnitude $N$ normal to the surface supporting the roller | One |
| (d)  Surface with friction (single point of contact) |  | Force of unknown magnitude $N$ normal to the surface and a friction force of unknown magnitude $F$ parallel to the surface | Two |
| (e)  Pin support |  | Unknown force **R** | Two |
| (f)  Built-in (cantilever) support |  | Unknown force **R** and a couple of unknown magnitude $C$ | Three |

| Support or Connection | Reaction | Number of Unknowns |
|---|---|---|
|  Rollers; Rocker; Frictionless surface |  Force with known line of action perpendicular to surface | 1 |
|  Short cable; Short link |  Force with known line of action along cable or link | 1 |
|  Collar on frictionless rod; Frictionless pin in slot |   Force with known line of action perpendicular to rod or slot | 1 |



This rocker bearing supports the weight of a bridge. The convex surface of the rocker allows the bridge to move slightly horizontally.



Links are often used to support suspended spans of highway bridges.



Force applied to the slider exerts a normal force on the rod, causing the window to open.

| | | |
|---|---|---|
| Frictionless pin or hinge; Rough surface | or; $\alpha$; Force of unknown direction | 2 |
| Fixed support | or; $\alpha$; Force and couple | 3 |

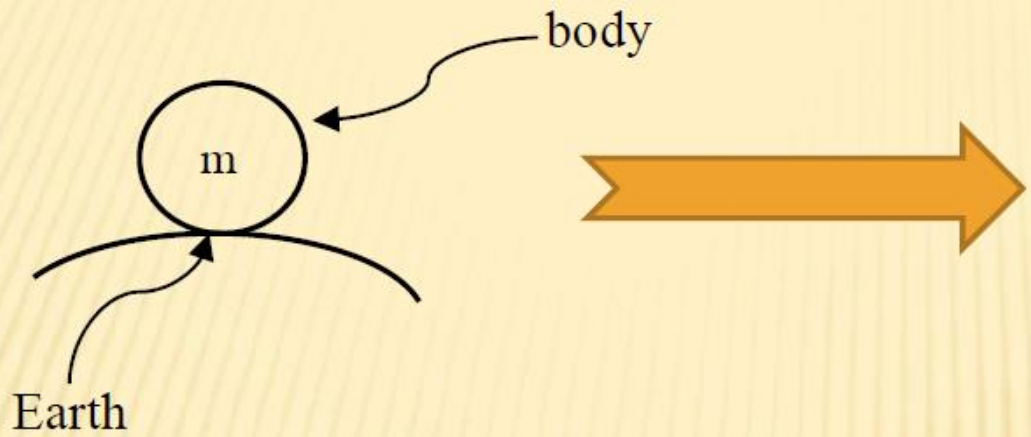
**Fig. 4.1** Reactions of supports and connections in two dimensions.



Pin supports are common on bridges and overpasses.



This cantilever support is fixed at one end and extends out into space at the other end.



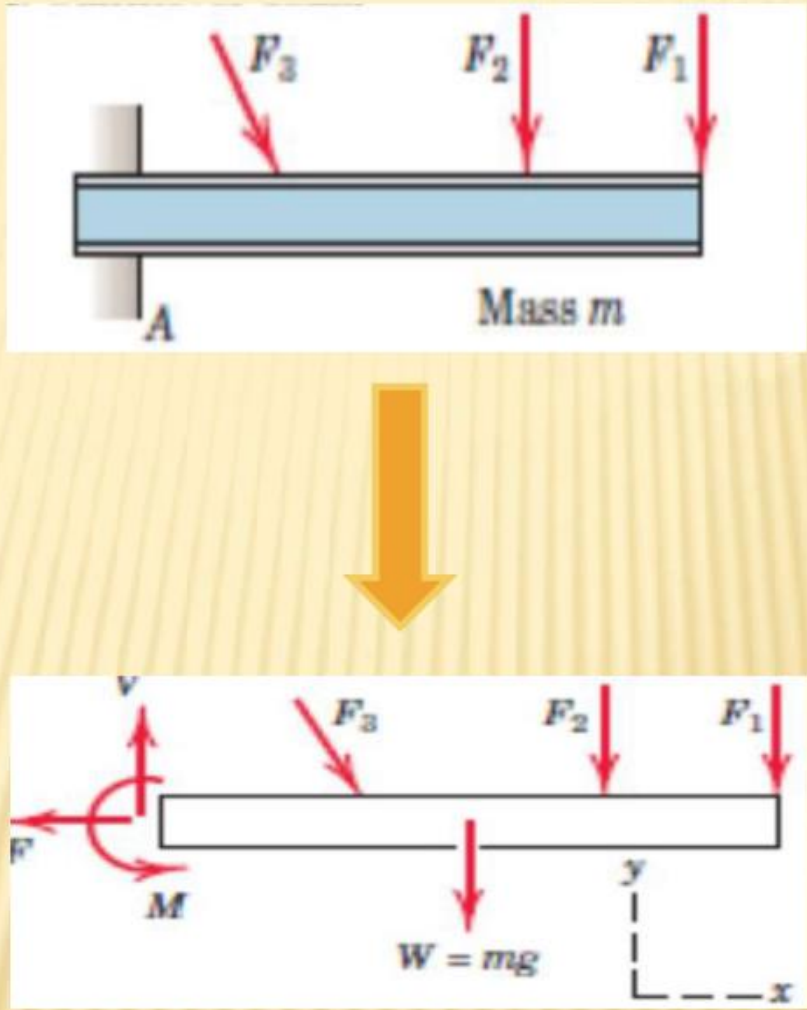




**FBD**

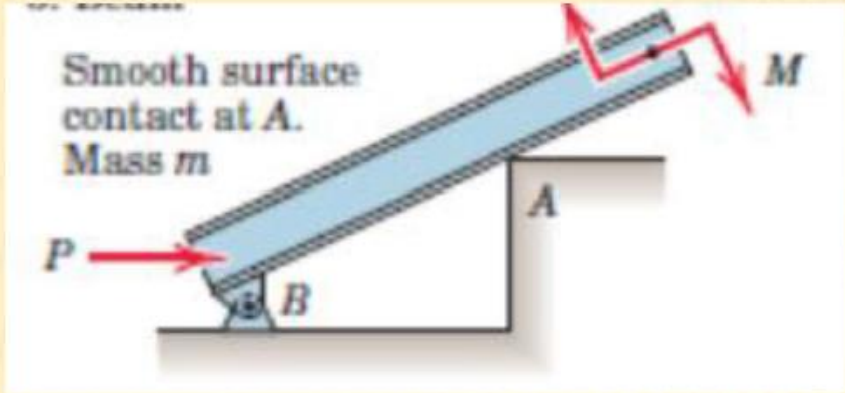
Toward the center of the earth throughout the center of mass

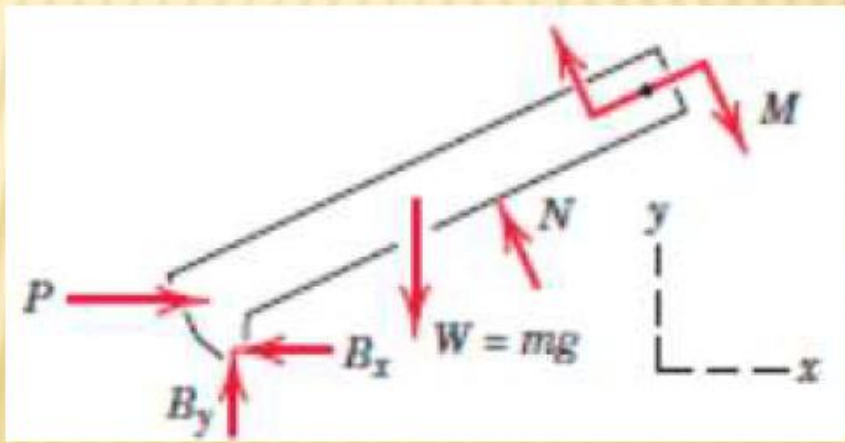
**Example**: Draw the Free Body Diagram (FBD) for the structure shown in figure.




**FBD**

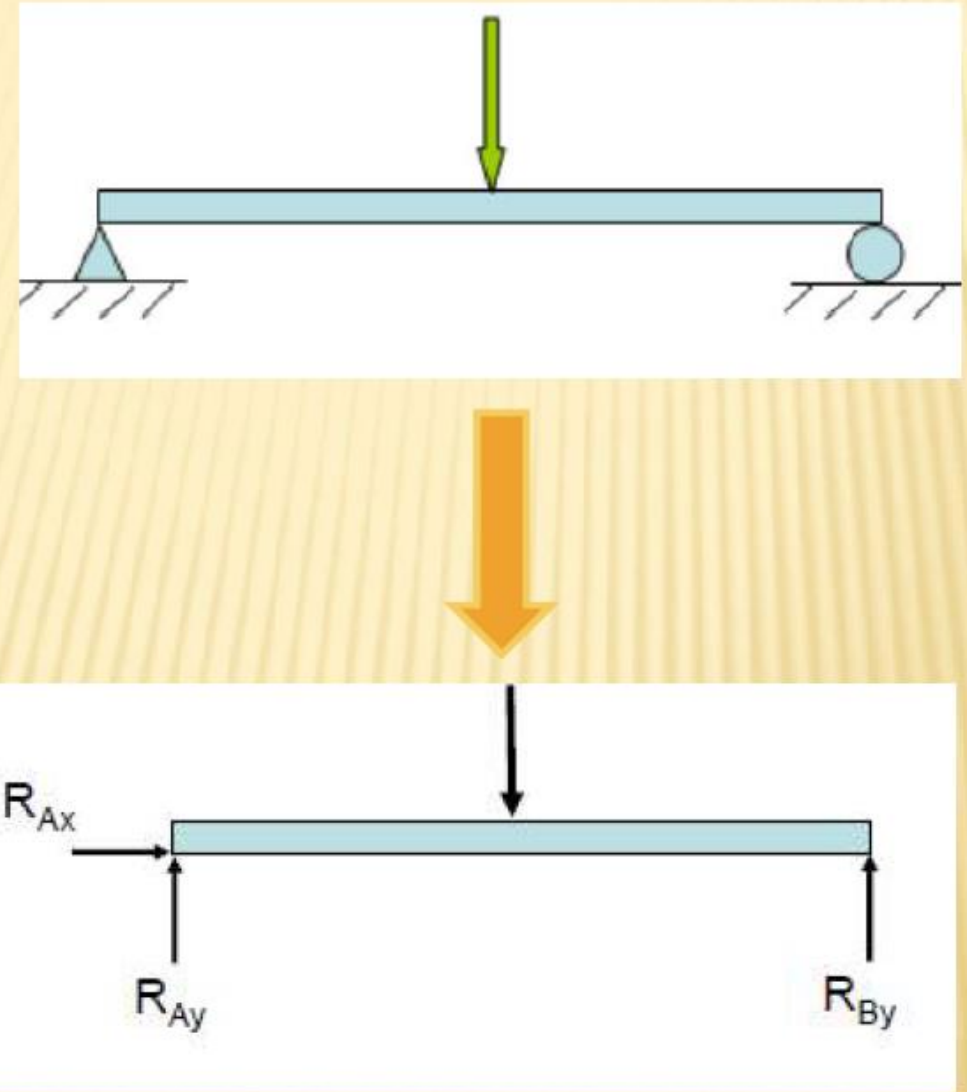
**Example**: Draw the Free Body Diagram (FBD) for the structure shown in figure.



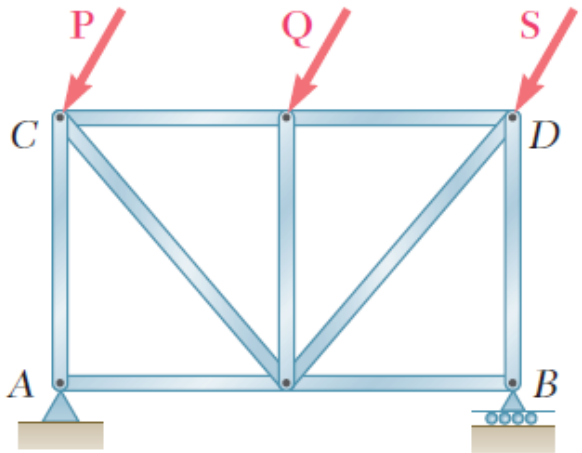




**FBD**

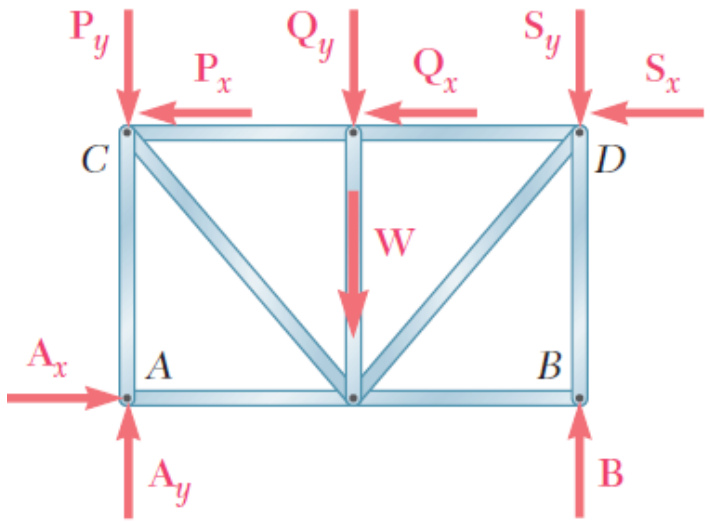
**Example**: Draw the Free Body Diagram (FBD) for the simply supported beam shown in figure.




**FBD**




(a)

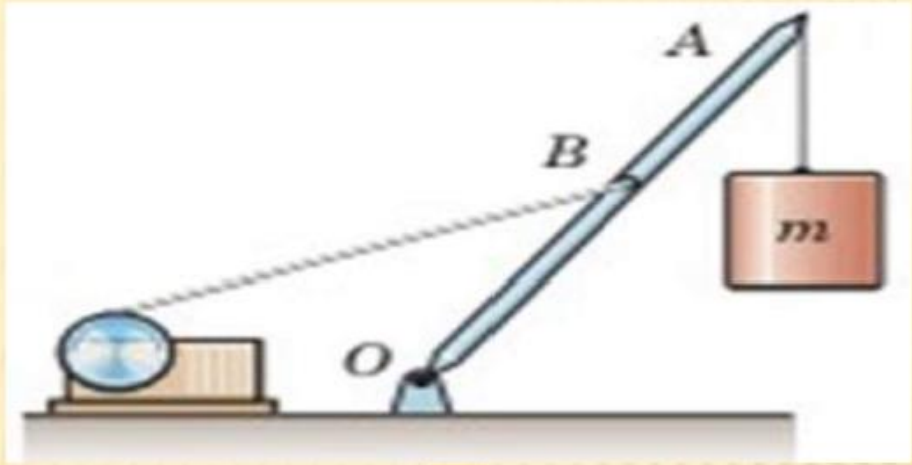


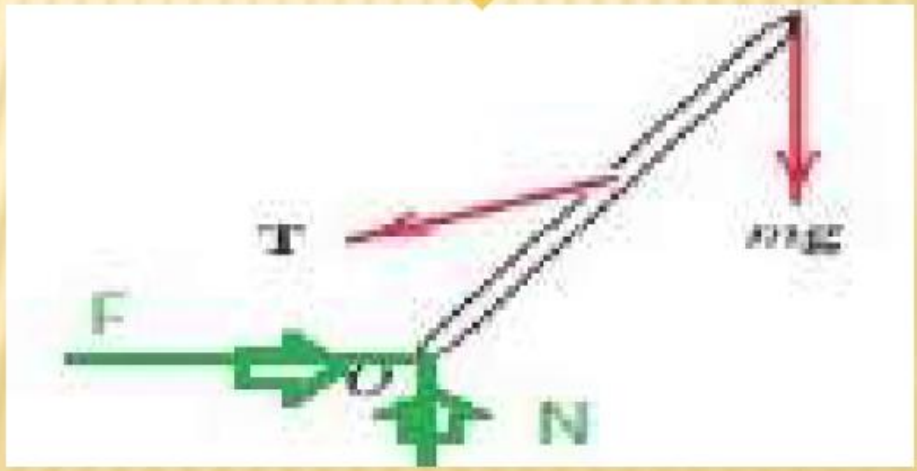

(b)

**Fig. 4.2** *(a)* A truss supported by a pin and a roller; *(b)* free-body diagram of the truss.

**Example**: Draw the Free Body Diagram (FBD) for the structure shown in figure.

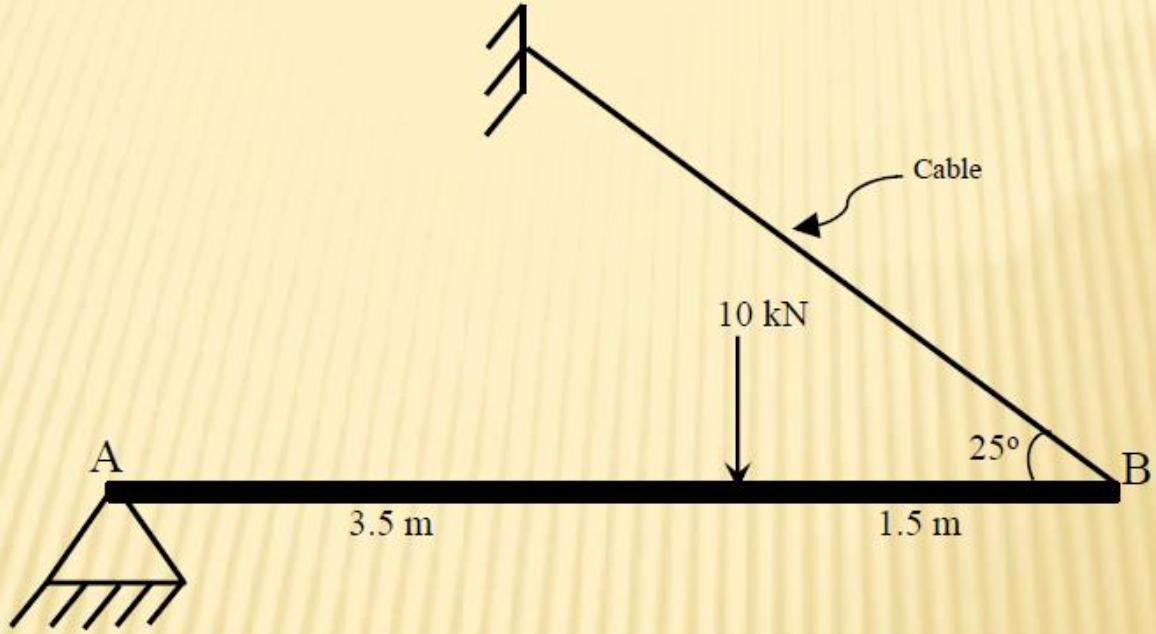
Negligible mass
O is hinge support
Cable at B

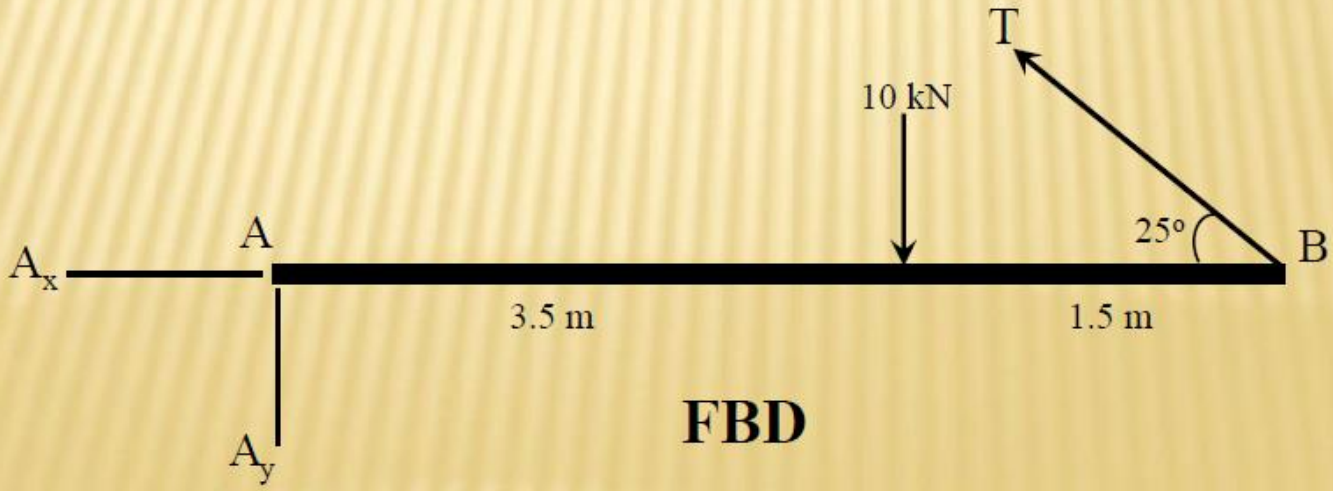


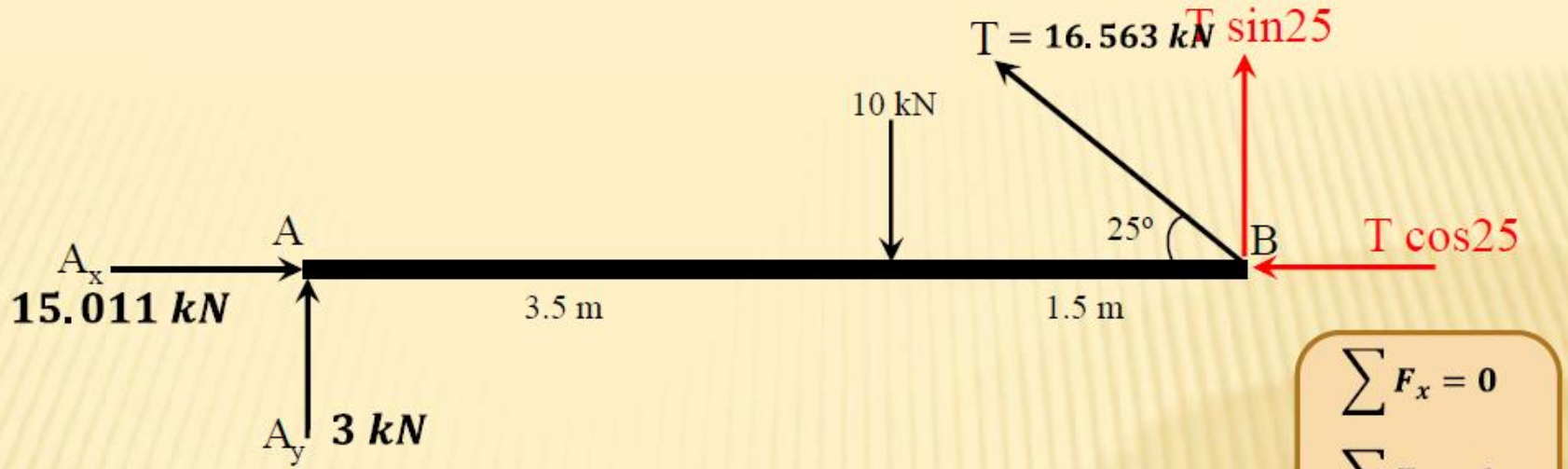

**FBD**

**Example**: Determine the magnitude **T** of the tension in the supporting cable and the magnitude of the force on the pin (hinge) support at **A** for the beam shown in figure. Neglect the weight of the beam.




**FBD**




$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$

$$\sum M_A = 0$$

$$10 \times 3.5 - T \sin 25\,(5) = 0 \Rightarrow T = 16.563\ kN$$

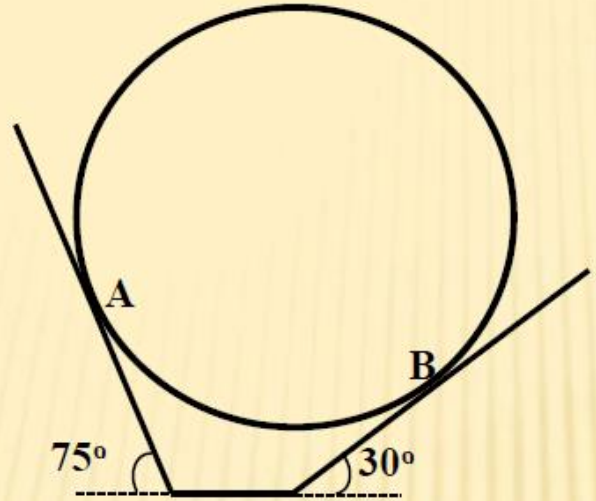
$$\sum F_y = 0$$

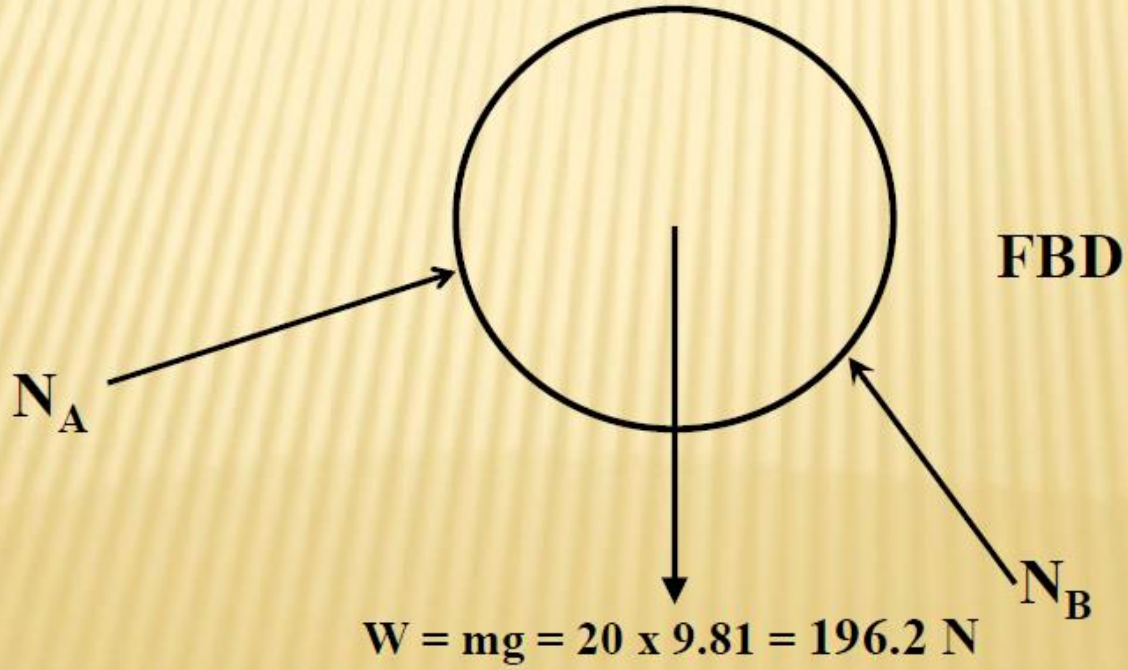
$$T \sin 25 - 10 + A_y = 0 \Rightarrow A_y = 3\ kN\ \uparrow$$

$$\sum F_x = 0$$

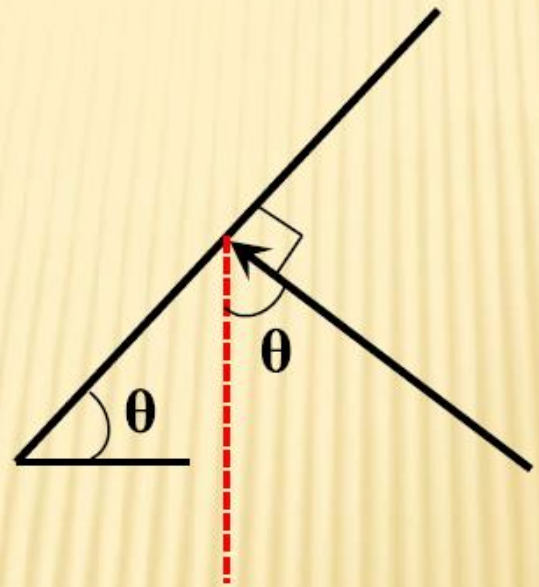
$$-T \cos 25 + A_x = 0 \Rightarrow A_x = 15.011\ kN\ \rightarrow$$

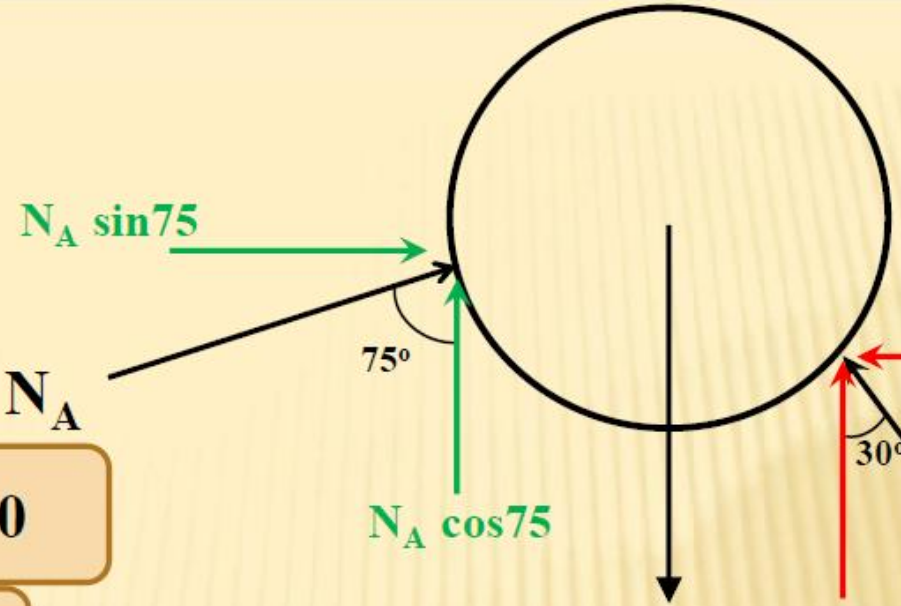
**Example:** The 20 kg homogeneous smooth sphere rests on the two inclines as shown in figure. Determine the contact forces at **A** and **B**.



العمود على السطح المائل يصنع مع الشاقول زاوية بقدر زاوية السطح المائل






$$\sum F_x = 0$$

$$N_A \sin 75 - N_B \sin 30 = 0$$

$$N_B = \frac{\sin 30}{\sin 75} N_A \quad \cdots\cdots (1)$$

$$\sum F_y = 0$$

$$N_A \cos 75 + N_B \cos 30 - 196.2 = 0 \quad \cdots\cdots (2)$$

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$

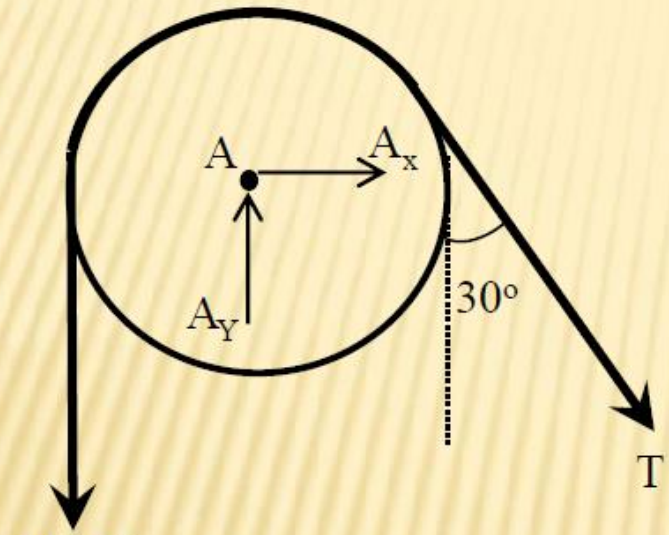
Substitute (1) in (2)

$$N_A \cos 75 + \frac{\sin 30}{\sin 75} N_A \cos 30 - 196.2 = 0$$

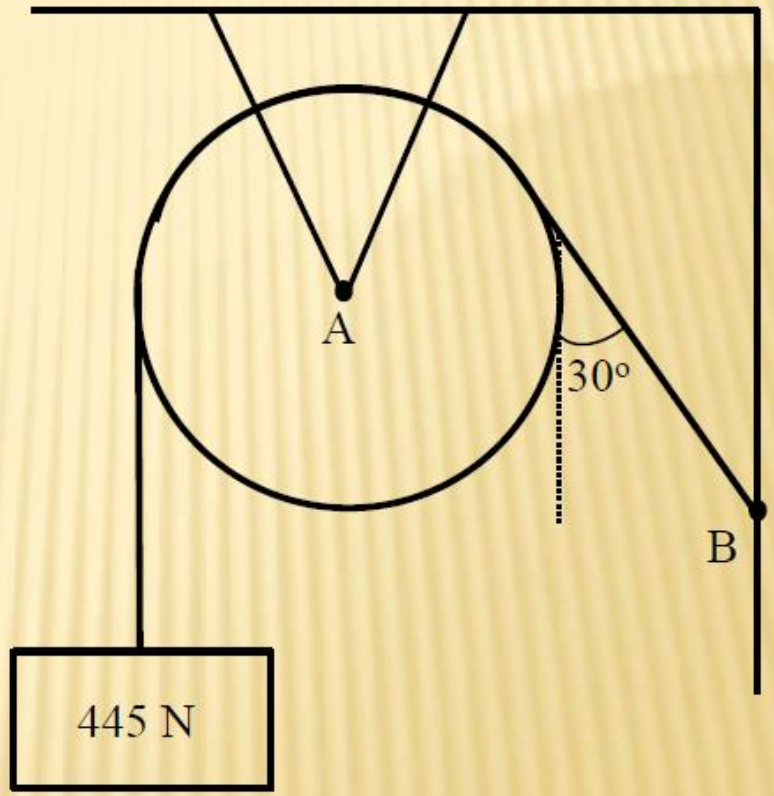
$$N_A = 101.561 \text{ N}$$

$$N_B = 196.2 \text{ N}$$

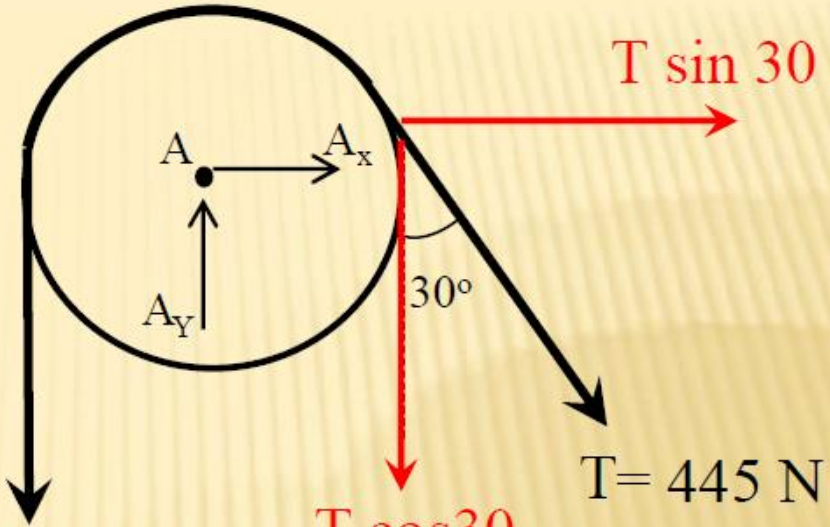
**Example:** The cord shown supports a force of 445 N and warps over frictionless pulley. Determine the tension in the cord, and the reactions at pin A. The diameter of the pulley is 304.8 mm.




**FBD**



$$\sum F_x = 0 , \sum F_y = 0 \ , \sum M = 0$$




$$\sum M_A = 0$$

$$-445\,(152.4) + T\,(152.4) = 0 \Rightarrow T = 445\ N$$

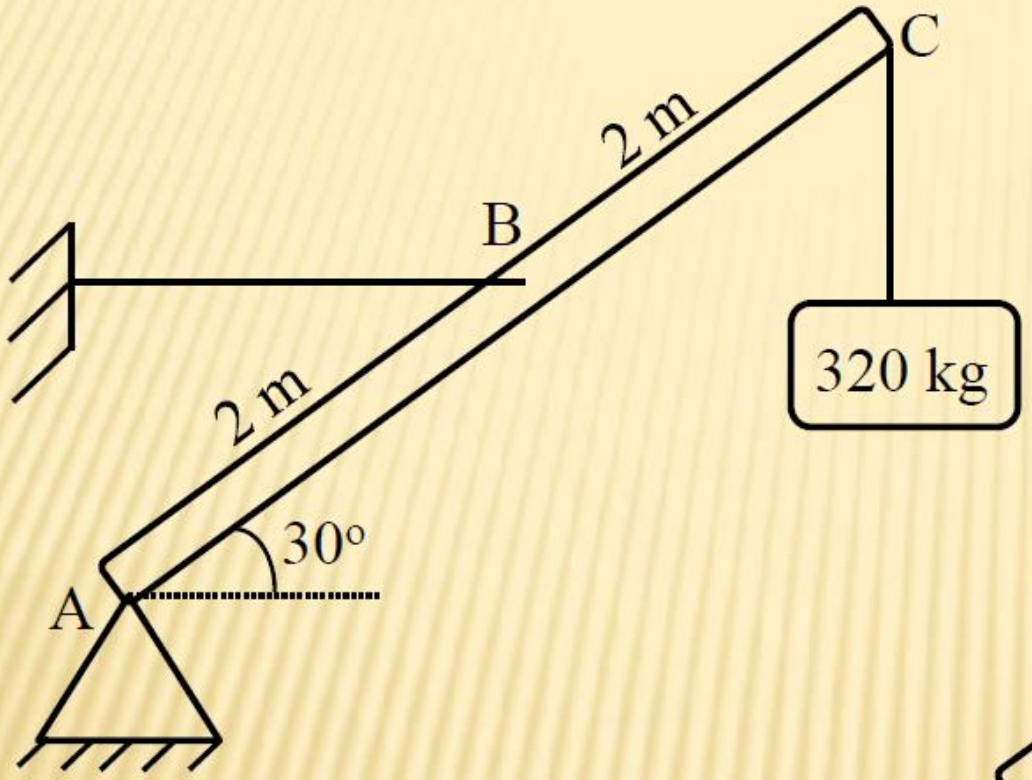
$$\sum F_x = 0$$

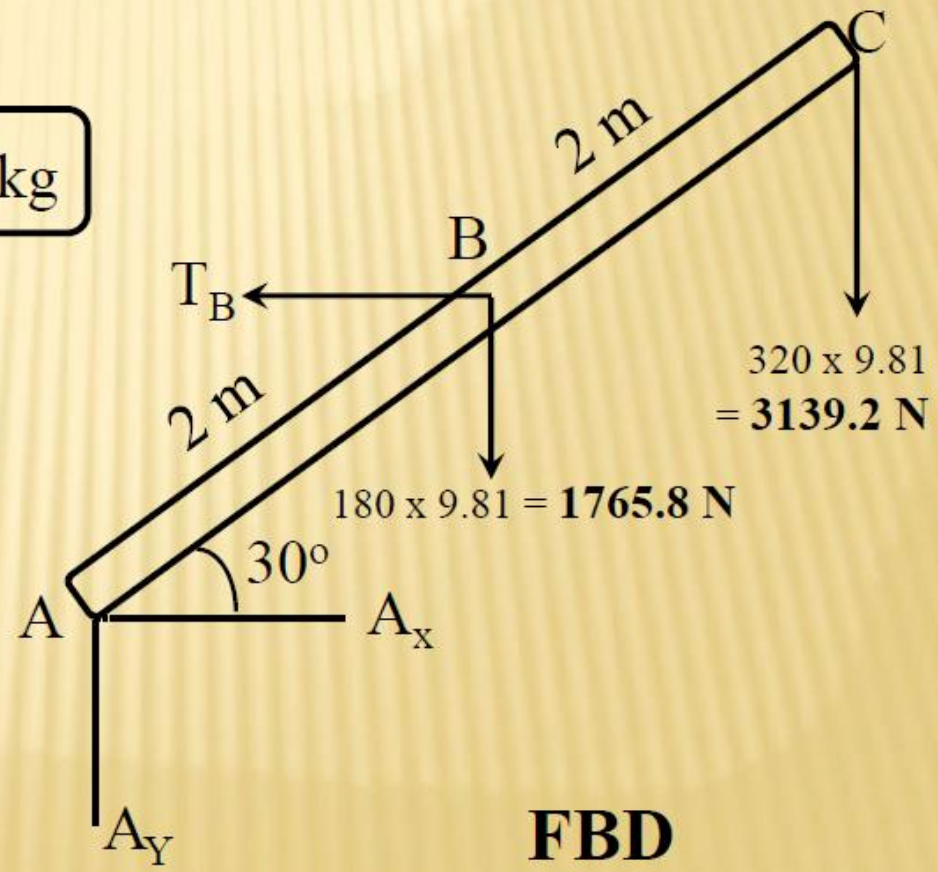
$$T \sin 30 + A_x = 0 \Rightarrow A_x = -222.5\ N \leftarrow$$

$$\sum F_y = 0$$

$$-T \cos 30 - 445 + A_y = 0 \Rightarrow A_y = 830.381\ N \uparrow$$

**Example:** The 180 kg uniform boom **ABC**, supported by a horizontal cable at B and a pin at **A**, carries a 320 kg load at **C**. Determine the force in the cable and the magnitude of the pin reactions.



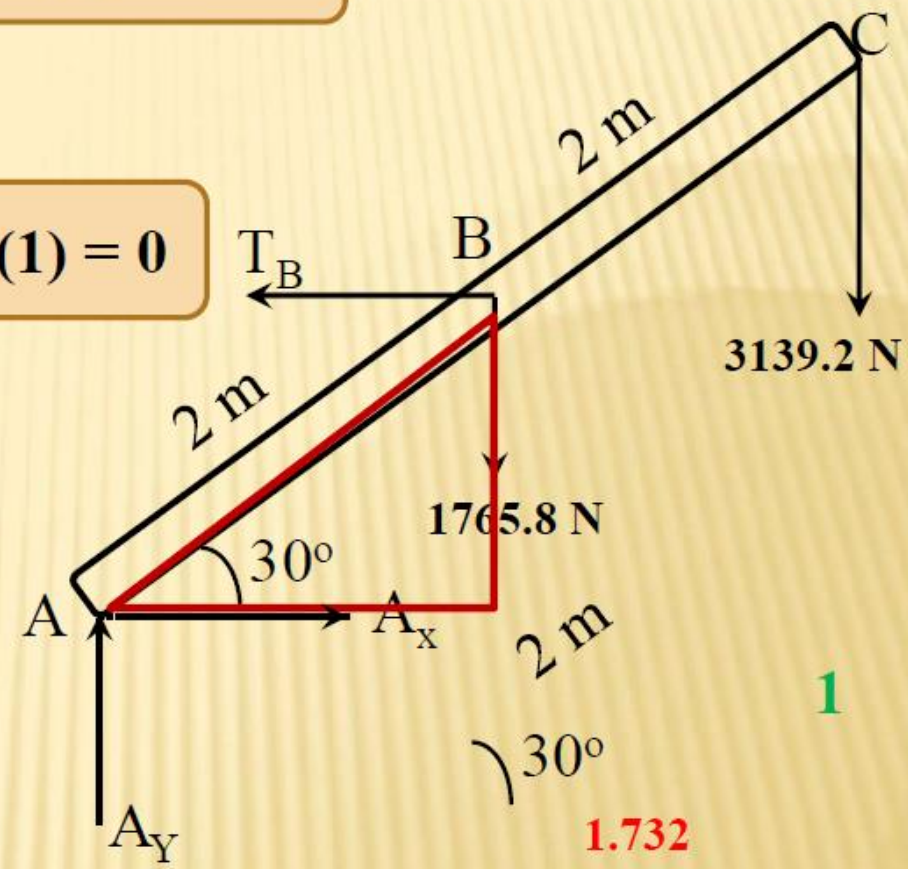




**FBD**

$$\sum F_x = 0 , \sum F_y = 0 \ , \sum M = 0$$

$$\sum M_A = 0$$

$$1765.8(1.732) + 3139.2\,(3.464) - T_B\,(1) = 0$$




$$T_B = 13932.554\ N$$

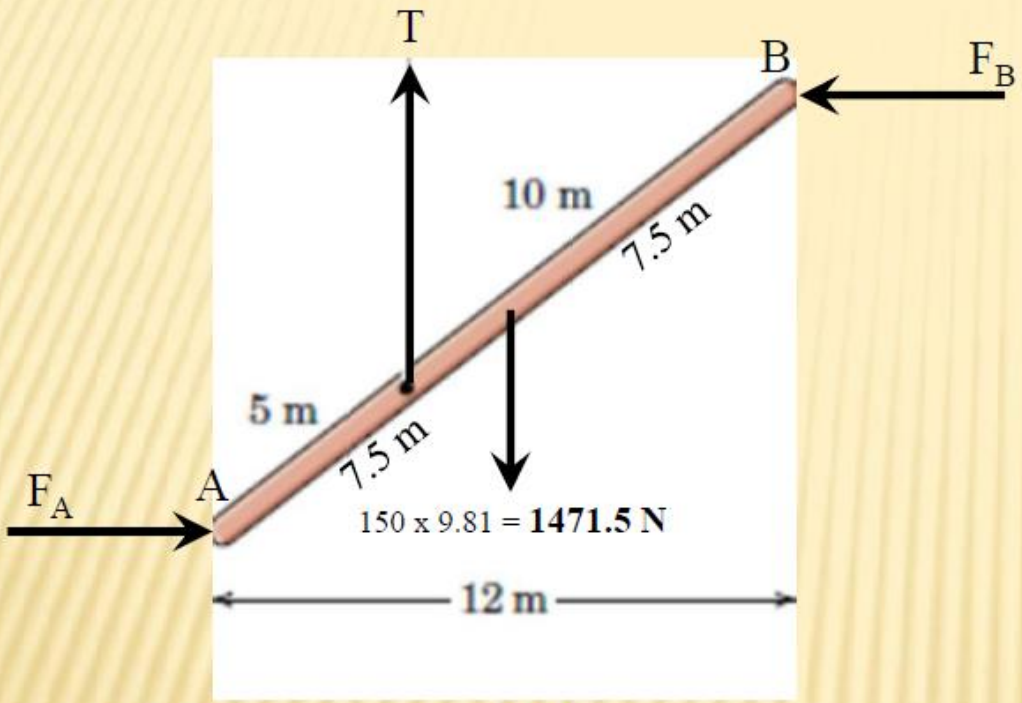
$$\sum F_x = 0$$

$$-13932.554 + A_x = 0 \quad \Rightarrow \quad A_x = 13932.554N \rightarrow$$

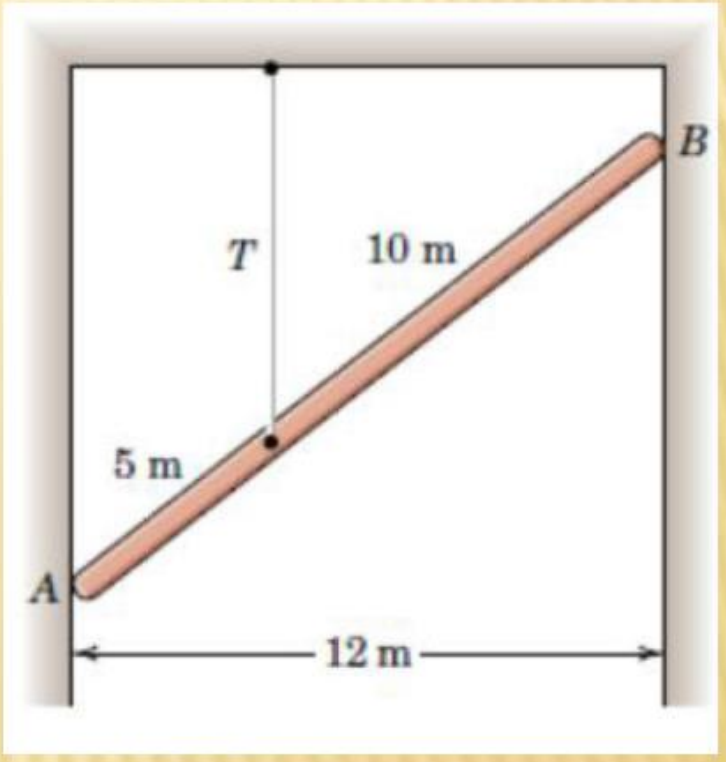
$$\sum F_y = 0$$

$$-1765.8 - 3139.2 + A_y = 0 \quad \Rightarrow \quad A_y = 4905\ N \uparrow$$

**Example**: The uniform 15 m pole has a mass of 150 kg and is supported by its smooth ends against the vertical walls and by the tension **T** in the vertical cable. Compute the reactions at **A** and **B**.




**FBD**



$$\sum F_x = 0 , \sum F_y = 0 \ , \sum M = 0$$

$$\sum F_y = 0$$

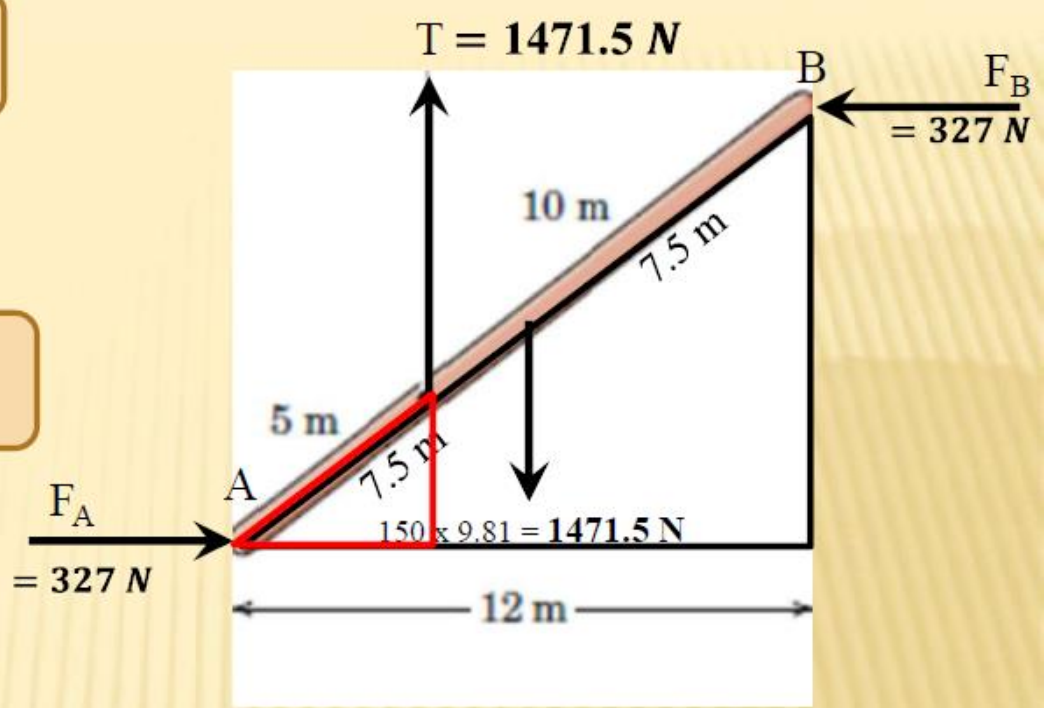
$$T - 1471.5 = 0$$

$$T = 1471.5\ N$$

$$\sum M_A = 0$$

$$1471.5(6) - 1471.5\,(4) - F_B\,(9) = 0$$

$$F_B = 327\ N \qquad F_A = 327\ N$$




5 m

36.87o

4 m

15 m

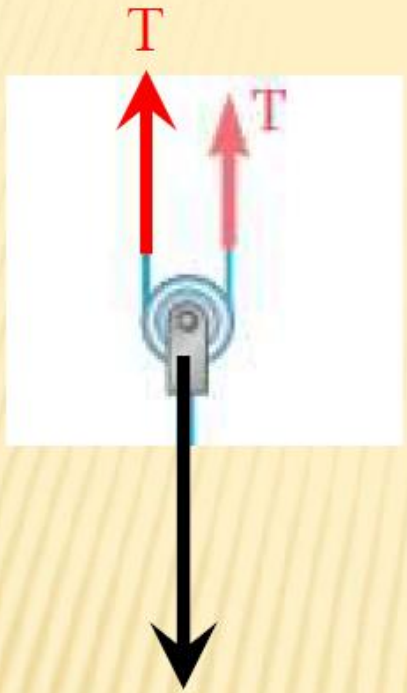
9 m

36.87o

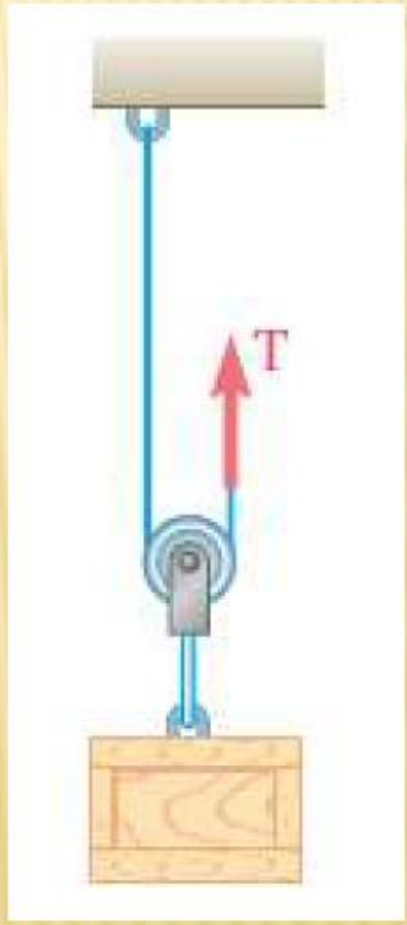
12 m

**Example**: 250 kg crate is supported by several rope-and-pulley arrangements as shown. Determine the tension in the rope.

**FBD**




250 x 9.81 = **2452.5 N**

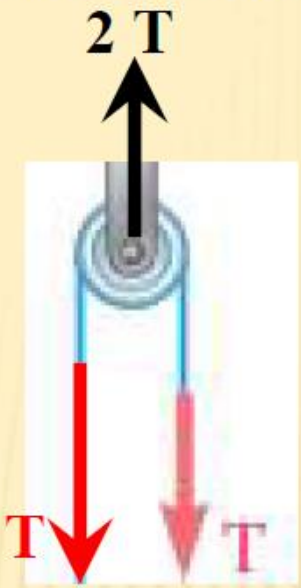


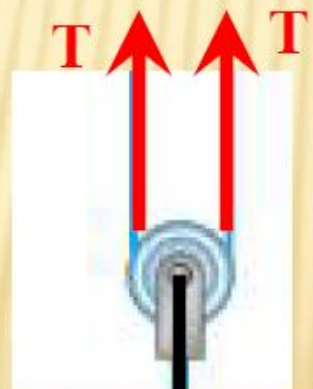

$$\sum F_y = 0$$

$$T + T - 2452.5 = 0$$

$$T = 1226.25\ N$$

**Example**: 250 kg crate is supported by several rope-and-pulley arrangements as shown. Determine the tension in the rope.



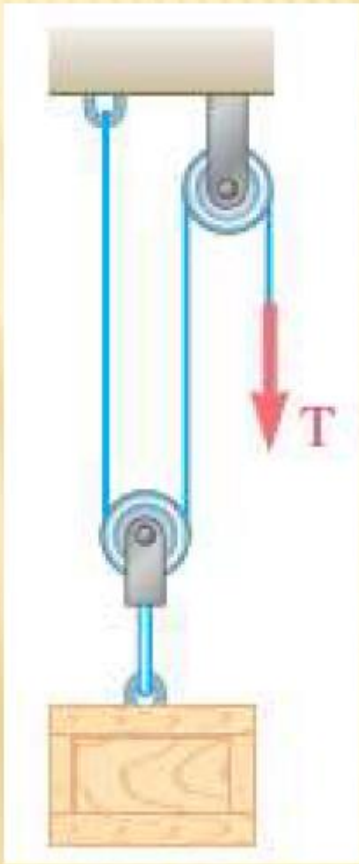




250 x 9.81 = **2452.5 N**

$$\sum F_y = 0$$

$$T + T - 2452.5 = 0$$

$$T = 1226.25\ N$$

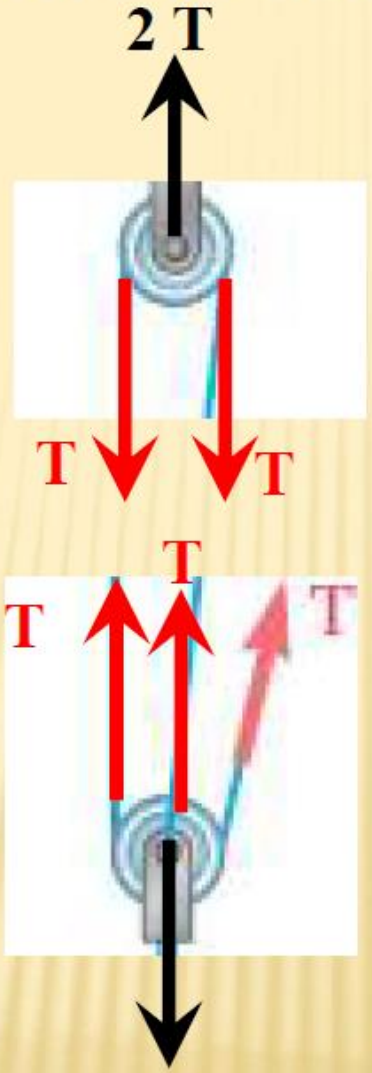


**Example**: 250 kg crate is supported by several rope-and-pulley arrangements as shown. Determine the tension in the rope.

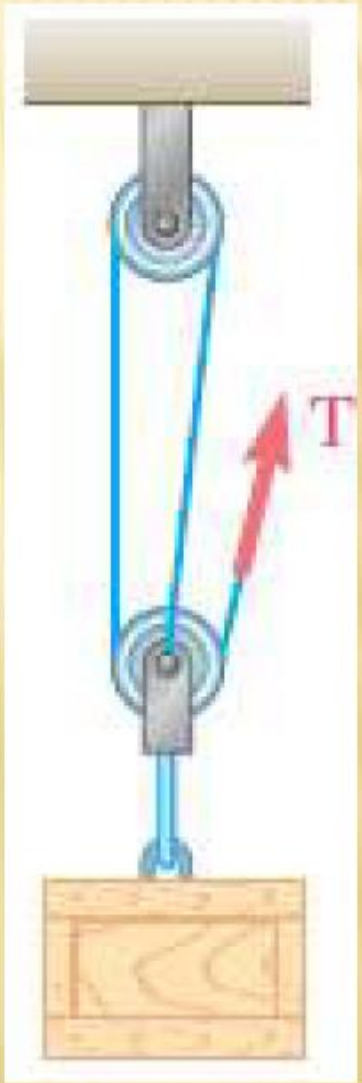
$$\sum F_y = 0$$

$$T + T + T - 2452.5 = 0$$

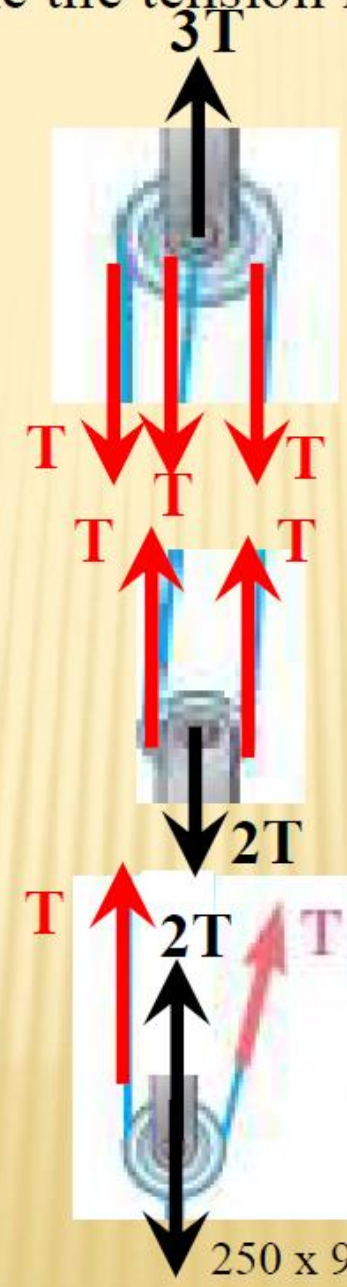
$$T = 817.5\ N$$

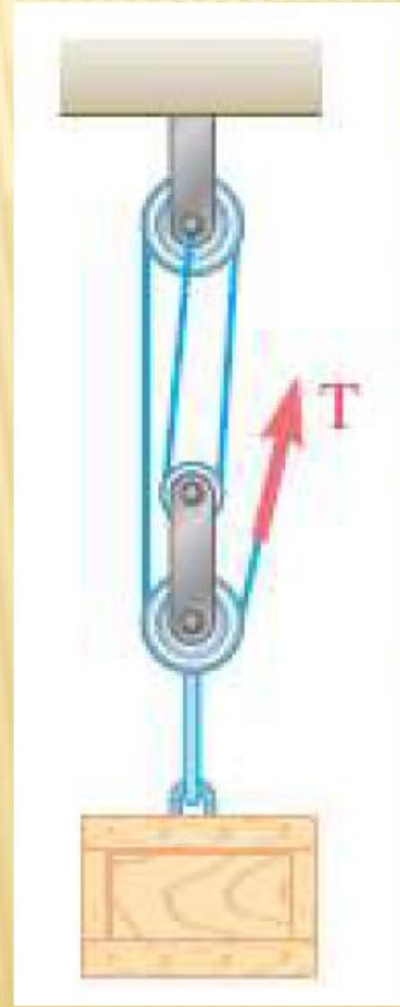



250 x 9.81 = **2452.5** N



**Example**: 250 kg crate is supported by several rope-and-pulley arrangements as shown. Determine the tension in the rope.



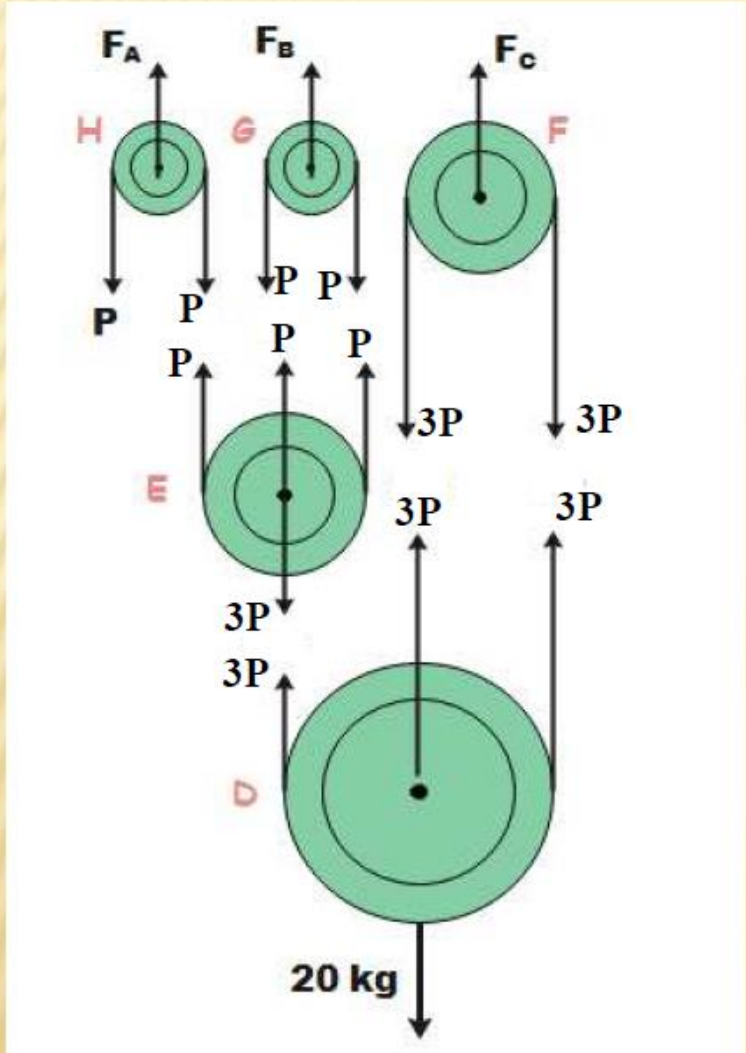



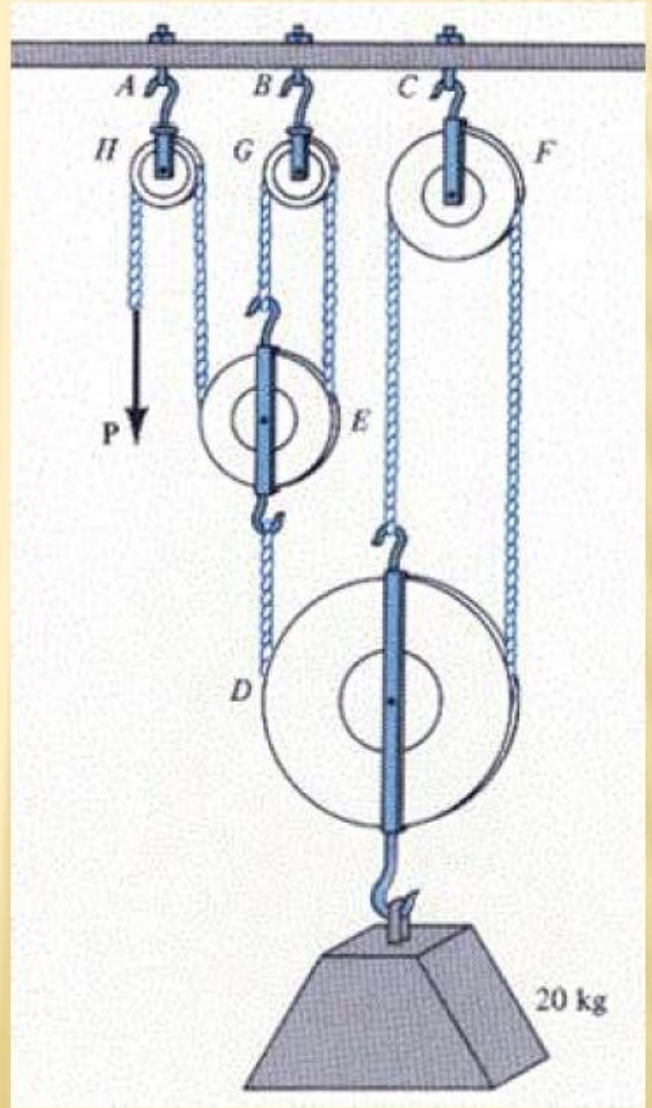

$$\sum F_y = 0$$

$$T + 2T + T - 2452.5 = 0$$

$$T = 613.125\ N$$

**Example**: Determine the force **P** needed to support 20 kg mass. Also what are the **reactions** at the supporting hooks **A**, **B** , and **C**.



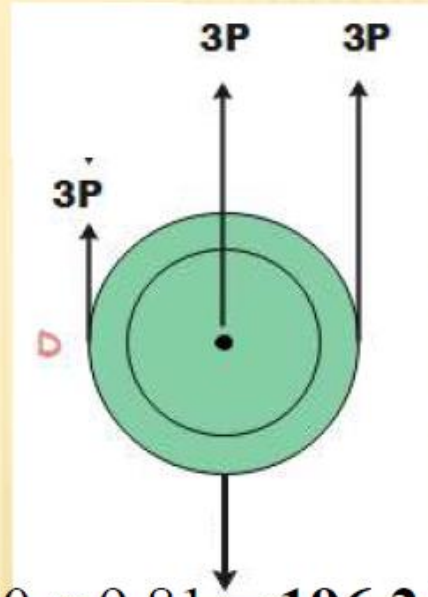



## Pulley D

$$\sum F_y = 0$$

$$3p + 3p + 3p - 196.2 = 0$$

$$p = 21.8\ N$$



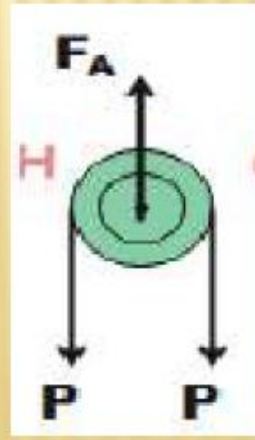

20 x 9.81 = **196.2 N**

## Pulley H

$$\sum F_y = 0$$

$$- p - p + FA = 0$$

$$F_A = 2p = 2\,(21.8) = 43.6\ N$$

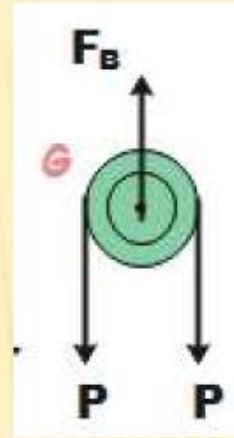


## Pulley G

$$\sum F_y = 0$$

$$- p - p + F_B = 0$$

$$F_B = 43.6\ N$$

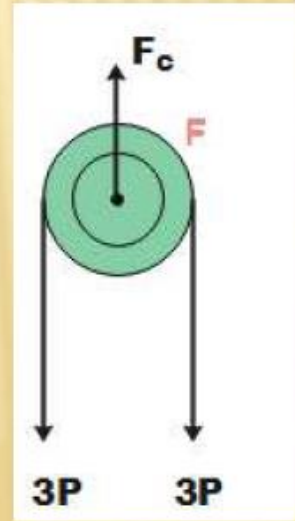



## Pulley F

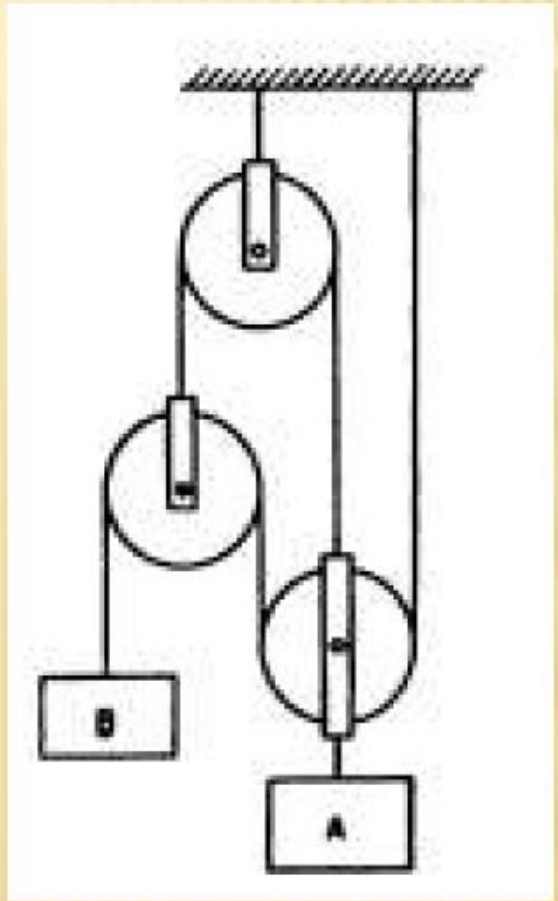
$$\sum F_y = 0$$

$$- 3p - 3p + F_C = 0$$

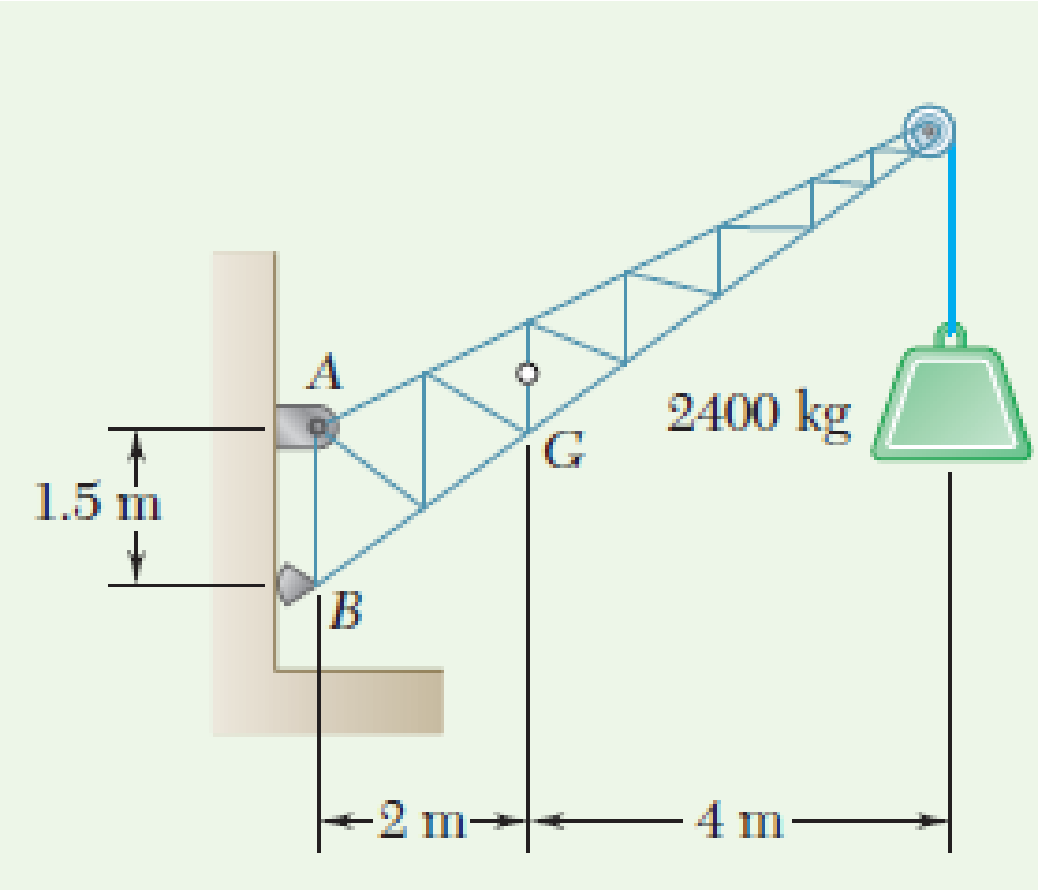
$$F_C = 6p = 6(21.8) = 130.8\ N$$

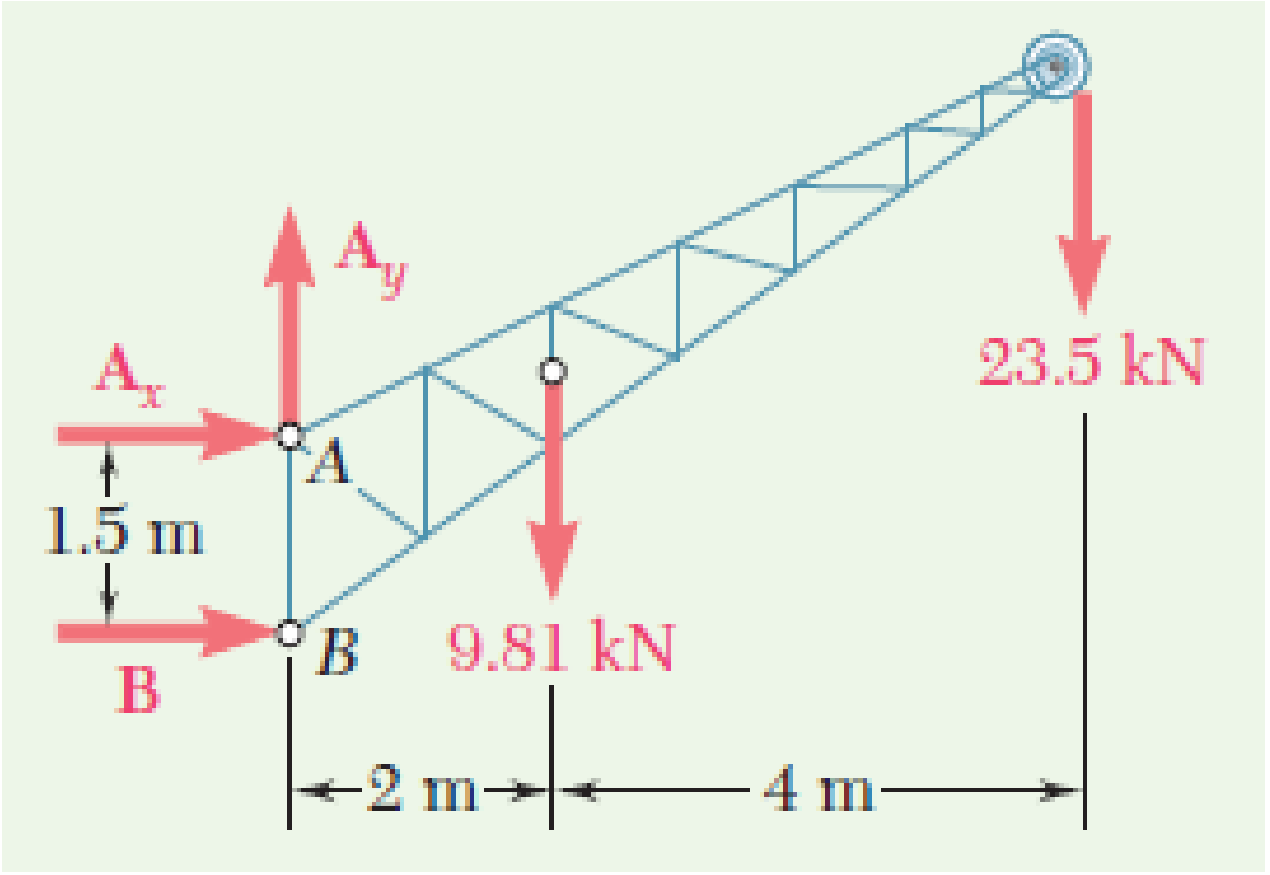


**Example**: The weights of the pulleys in Figure may be neglected. The body A weighs 54 kg. Determine the weight of B necessary to maintain the system in equilibrium.



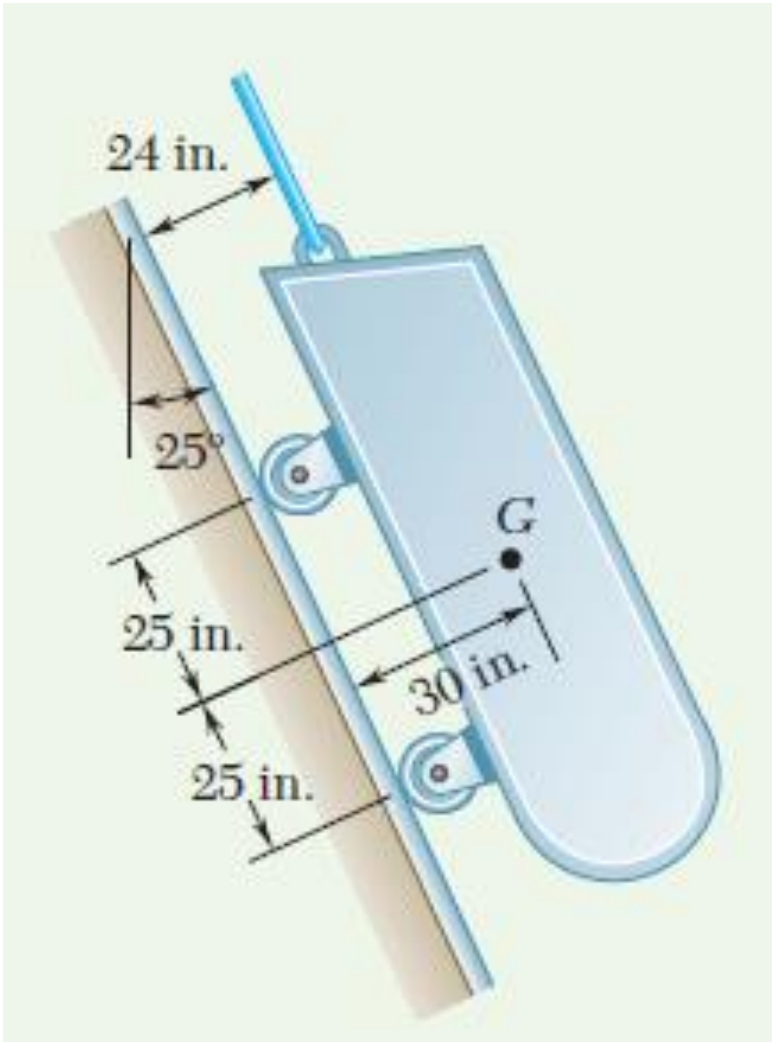
A fixed crane has a mass of 1000 kg and is used to lift a 2400-kg crate. It is held in place by a pin at $A$ and a rocker at $B$. The center of gravity of the crane is located at $G$. Determine the components of the reactions at $A$ and $B$.

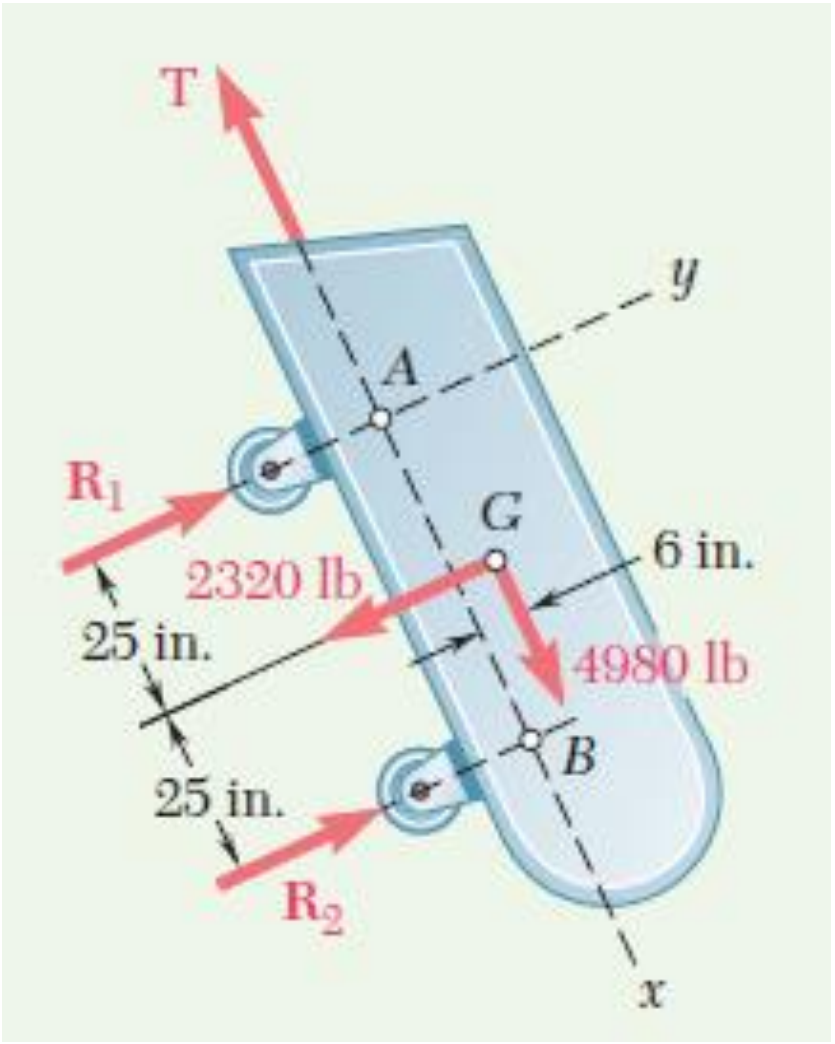



**B** = 107.1 kN →  $A_x$ = 107.1 kN ←  $A_y$ = 33.3 kN ↑

A loading car is at rest on a track forming an angle of 25° with the vertical. The gross weight of the car and its load is 5500 lb, and it acts at a point 30 in. from the track, halfway between the two axles. The car is held by a cable attached 24 in. from the track. Determine the tension in the cable and the reaction at each pair of wheels.



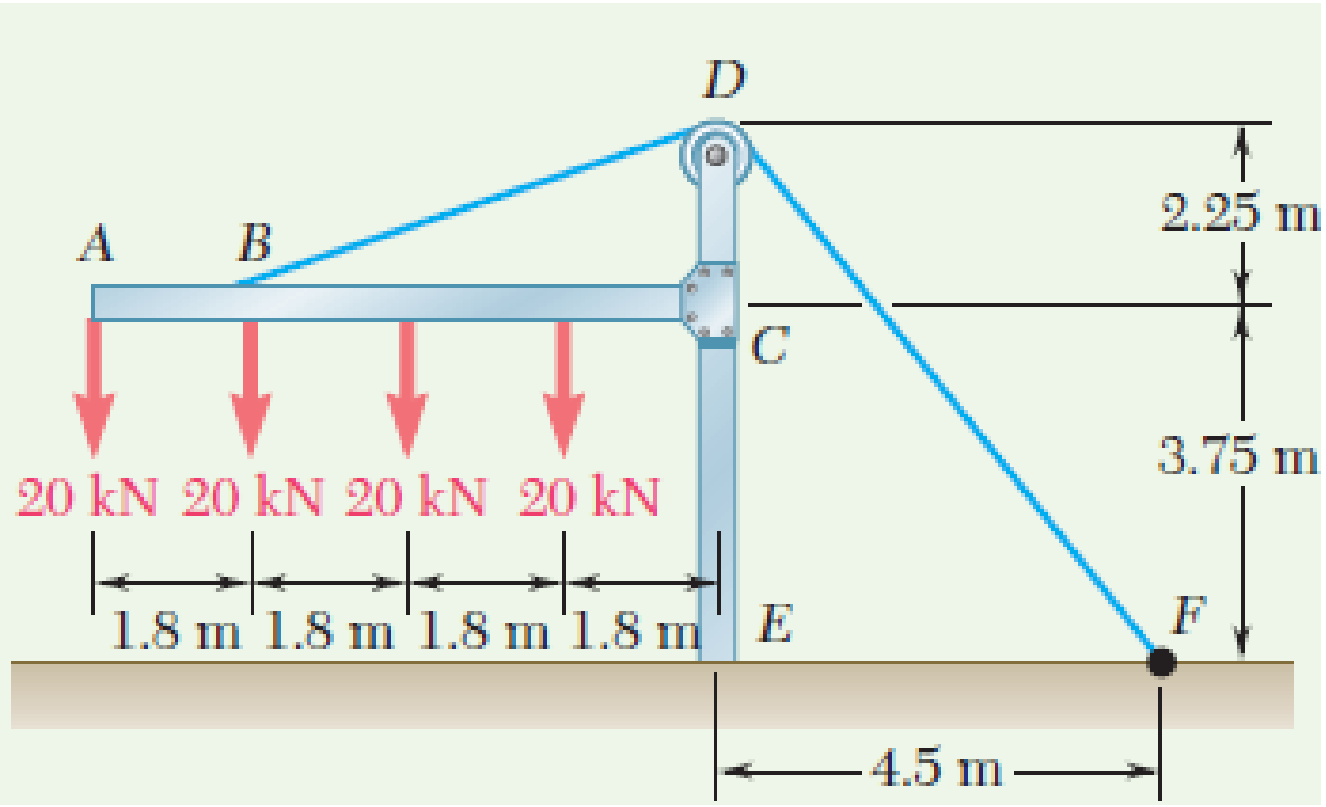



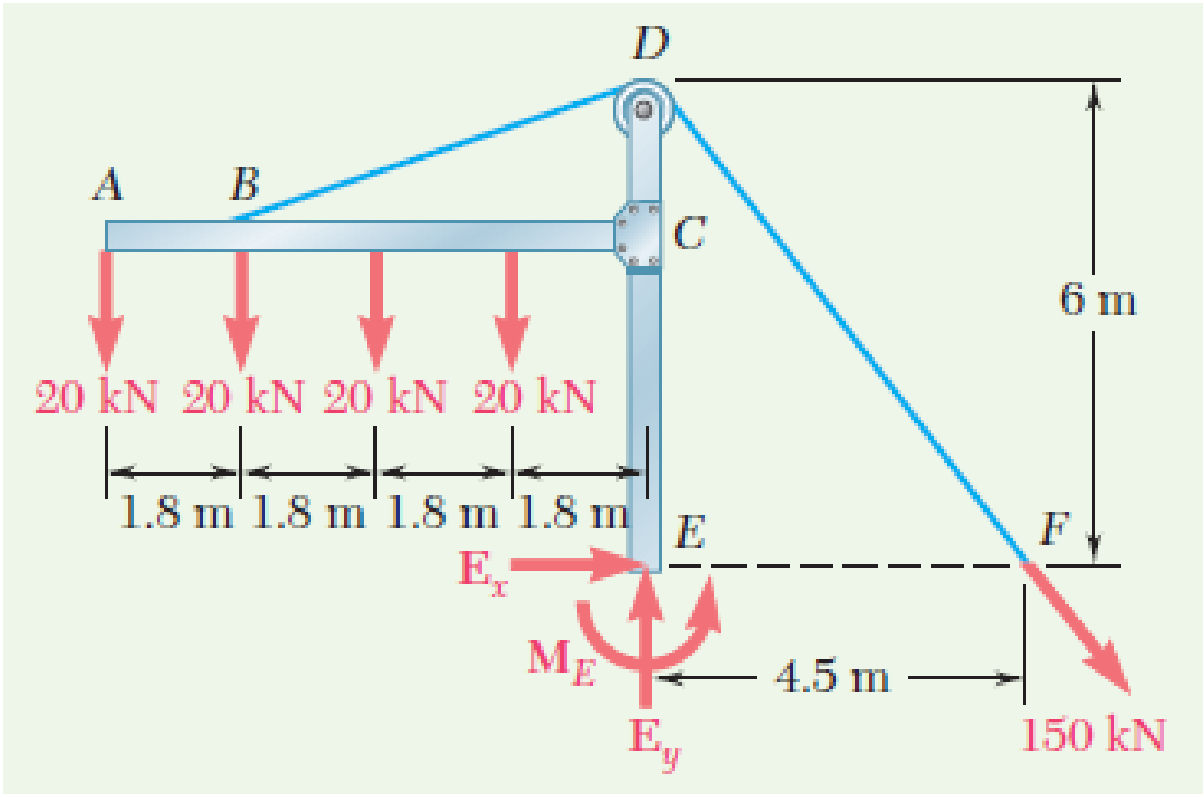

$\mathbf{R}_1 = +562$ lb ↗

$\mathbf{R}_2 = 1758$ lb ↗

$\mathbf{T} = 4980$ lb ↖

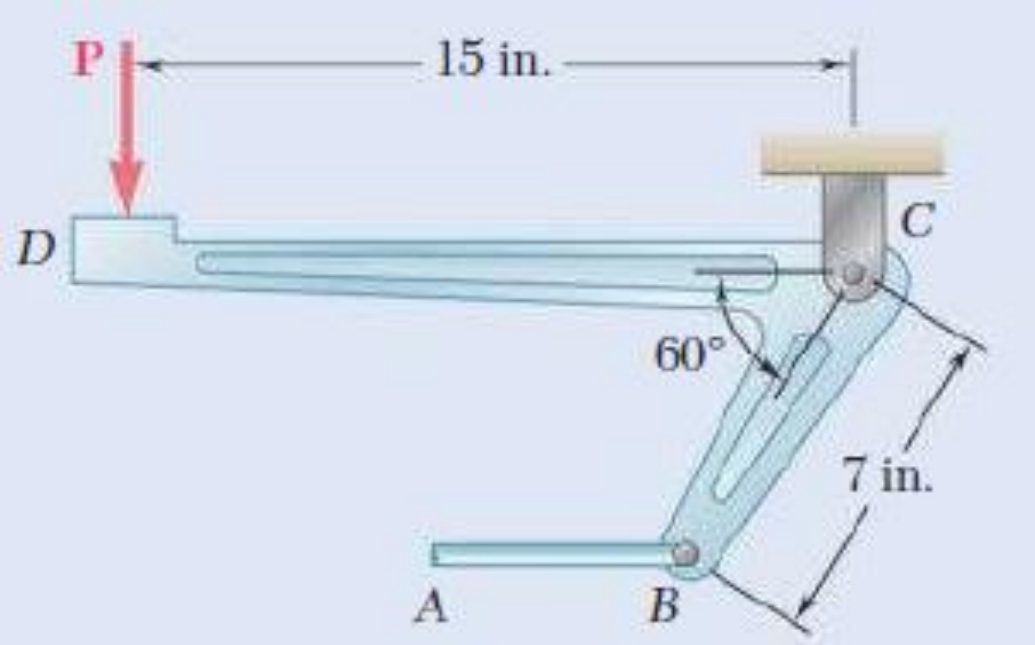
The frame shown supports part of the roof of a small building. Knowing that the tension in the cable is 150 kN, determine the reaction at the fixed end *E*.



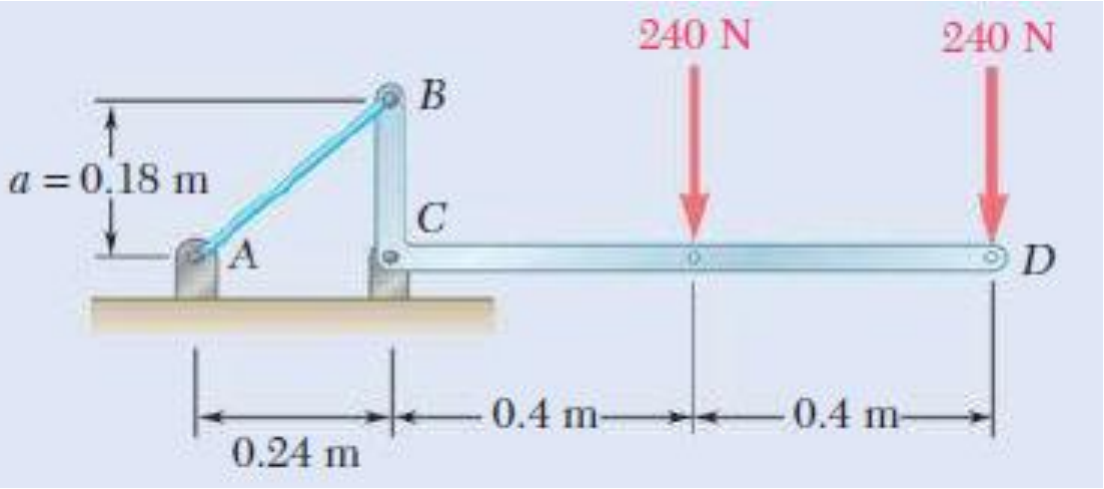

$\mathbf{M}_E = 180.0\ \text{kN·m}\ \curvearrowleft$ $\mathbf{E}_x = 90.0\ \text{kN}\ \leftarrow$ $\mathbf{E}_y = 200\ \text{kN}\ \uparrow$

**4.17** The required tension in cable $AB$ is 200 lb. Determine ($a$) the vertical force **P** that must be applied to the pedal, ($b$) the corresponding reaction at $C$.



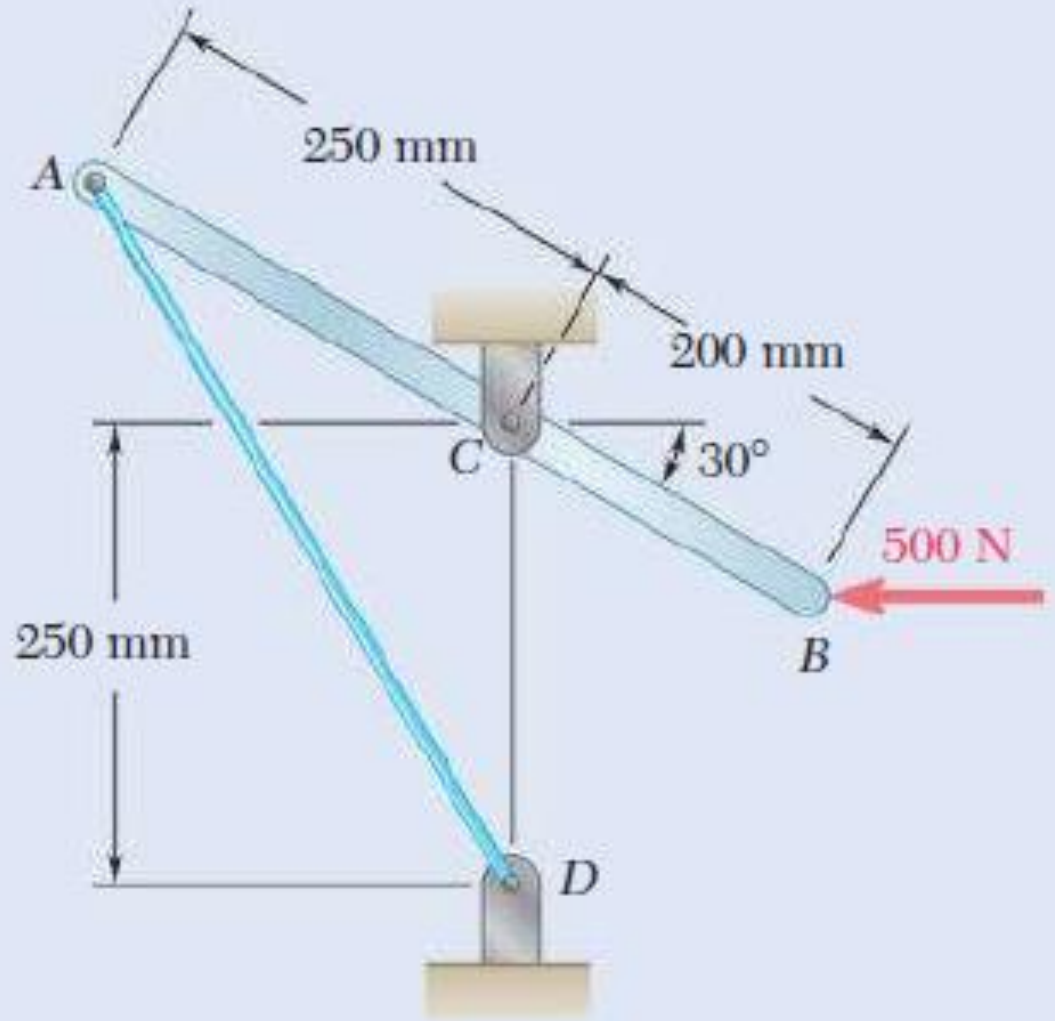

**4.17** ($a$) 80.8 lb↓. ($b$) 216 lb ∠ 22.0°.

**4.19** The bracket $BCD$ is hinged at $C$ and attached to a control cable at $B$. For the loading shown, determine ($a$) the tension in the cable, ($b$) the reaction at $C$.



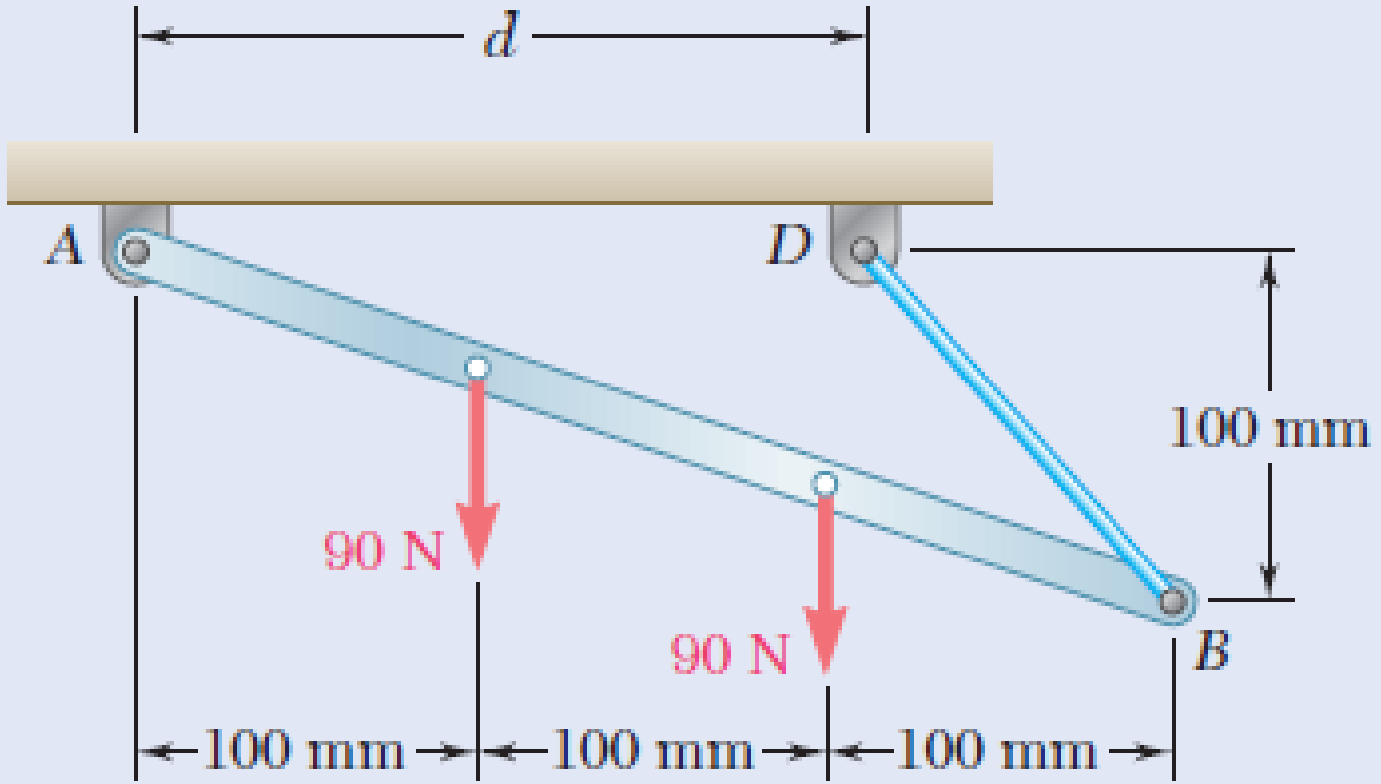

**4.19** ($a$) 2.00 kN. ($b$) 2.32 kN ∠ 46.4°.

**4.22** A lever $AB$ is hinged at $C$ and attached to a control cable at $A$. If the lever is subjected to a 500-N horizontal force at $B$, determine ($a$) the tension in the cable, ($b$) the reaction at $C$.



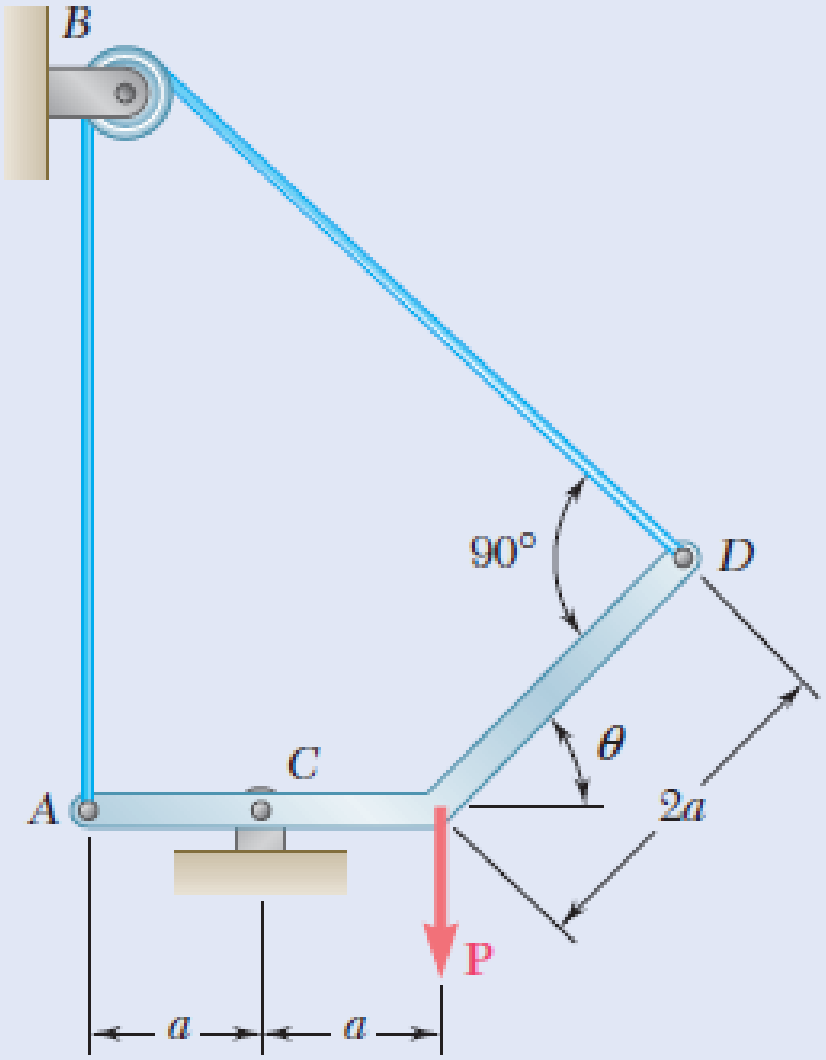

**4.22** ($a$) 400 N. ($b$) 458 N ∠ 49.1°.

**4.25** A rod $AB$, hinged at $A$ and attached at $B$ to cable $BD$, supports the loads shown. Knowing that $d = 200$ mm, determine ($a$) the tension in cable $BD$, ($b$) the reaction at $A$.



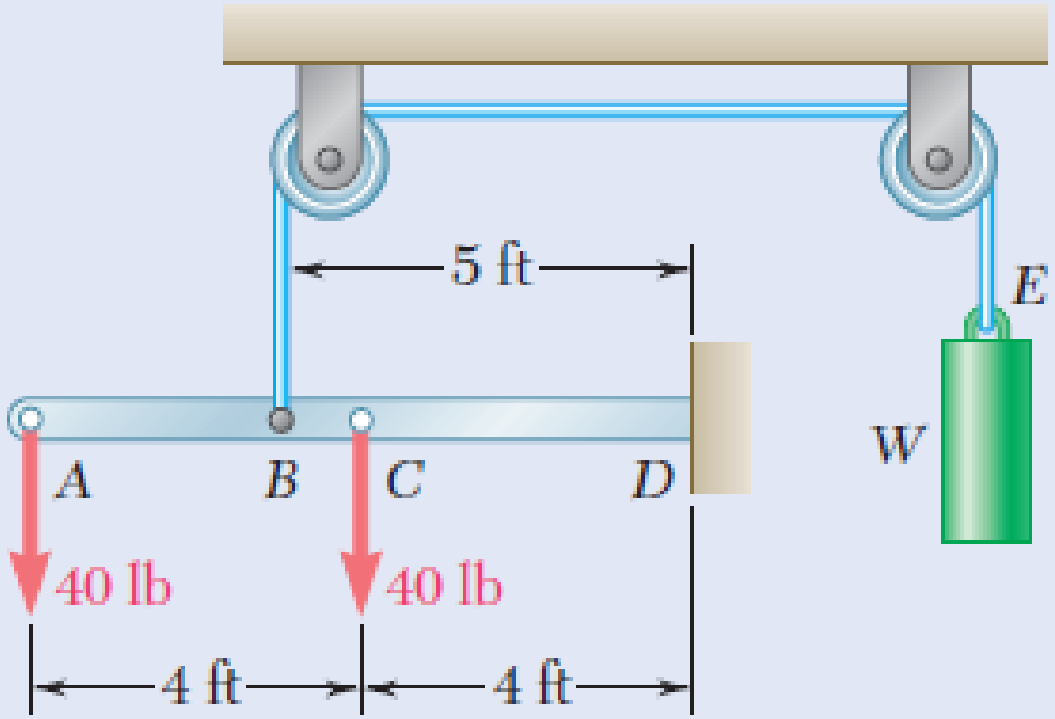

**4.25** ($a$) 190.9 N. ($b$) 142.3 N ∠ 18.43°.

**4.31** Neglecting friction, determine the tension in cable $ABD$ and the reaction at $C$ when $\theta = 60°$.




**4.31** $T = 2P/3$; $C = 0.577P \rightarrow$.

**4.48** Beam $AD$ carries the two 40-lb loads shown. The beam is held by a fixed support at $D$ and by the cable $BE$ that is attached to the counterweight $W$. Determine the reaction at $D$ when (*a*) $W = 100$ lb, (*b*) $W = 90$ lb.




**4.48** (*a*) $\mathbf{D} = 20.0$ lb↓; $\mathbf{M}_D = 20.0$ lb·ft ↰.
(*b*) $\mathbf{D} = 10.00$ lb↓; $\mathbf{M}_D = 30.0$ lb·ft ↲.

**4.51** A uniform rod $AB$ with a length of $l$ and weight of $W$ is suspended from two cords $AC$ and $BC$ of equal length. Determine the angle $\theta$ corresponding to the equilibrium position when a couple **M** is applied to the rod.

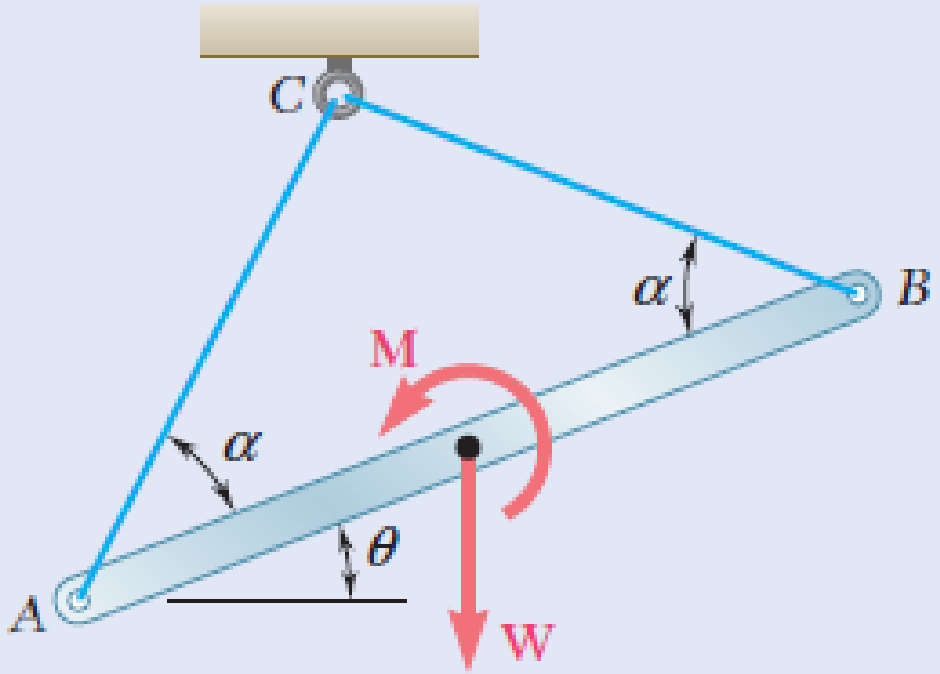



Fig. P4.51

**4.51** $\theta = \sin^{-1}(2M\cot\alpha/Wl)$.

# Analysis of Beams

**Beams** are structural members which offer resistance to bending due to applied loads. Most beams are long prismatic bars, and the loads are usually applied normal to the axes of the bars.

## Types of beams:

1. Simply support beam.



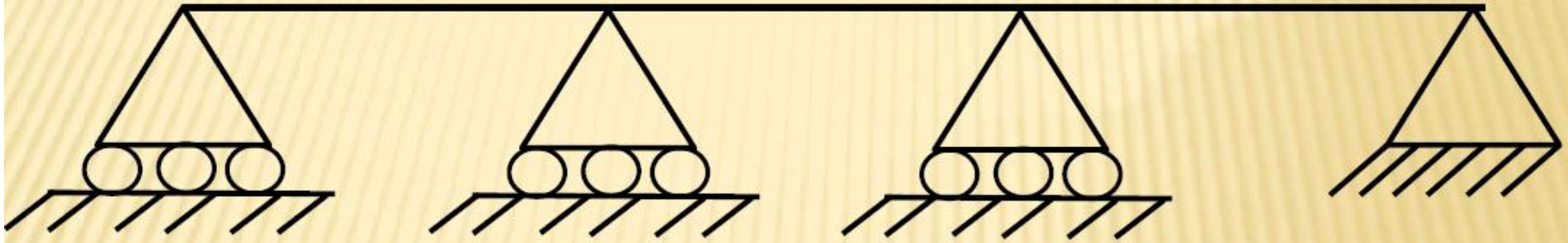
2. Over hanging beam.



3. Cantilever Beam.



4. Continuous Beam.



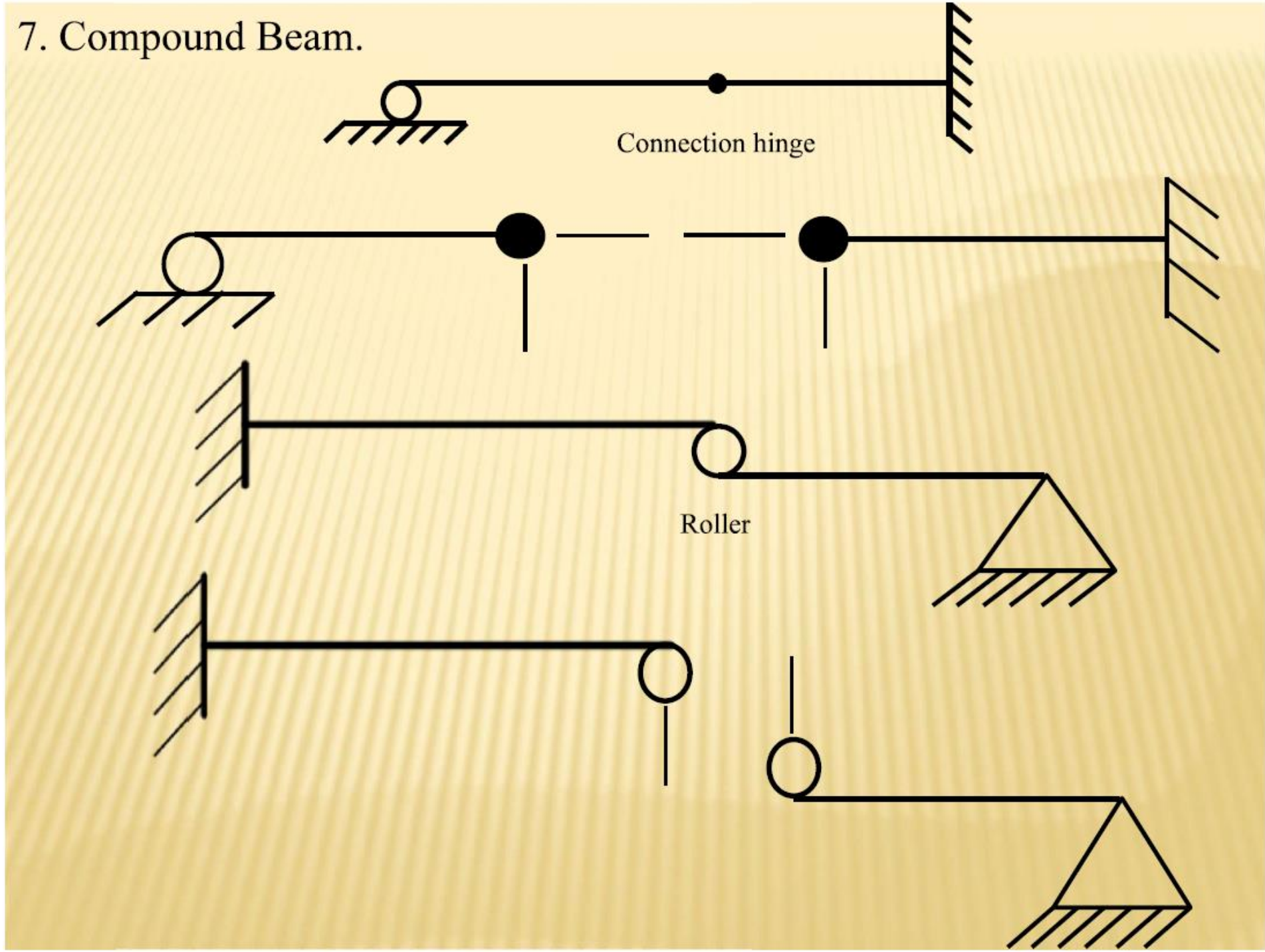
5. Propped beam.



6. Fixed Beam.

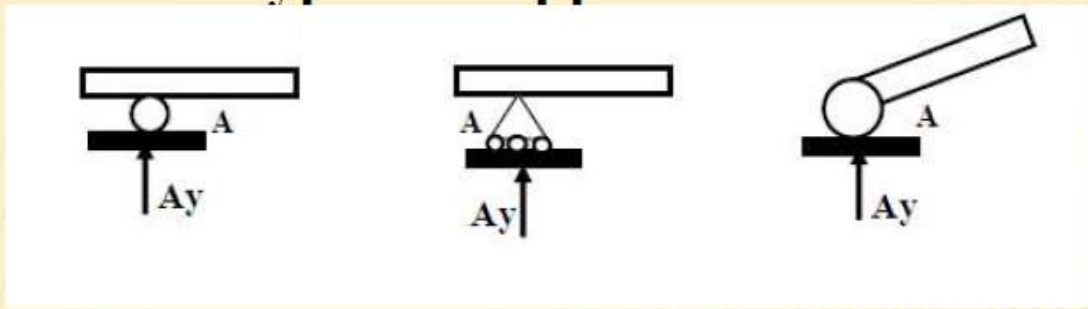


# 7. Compound Beam.

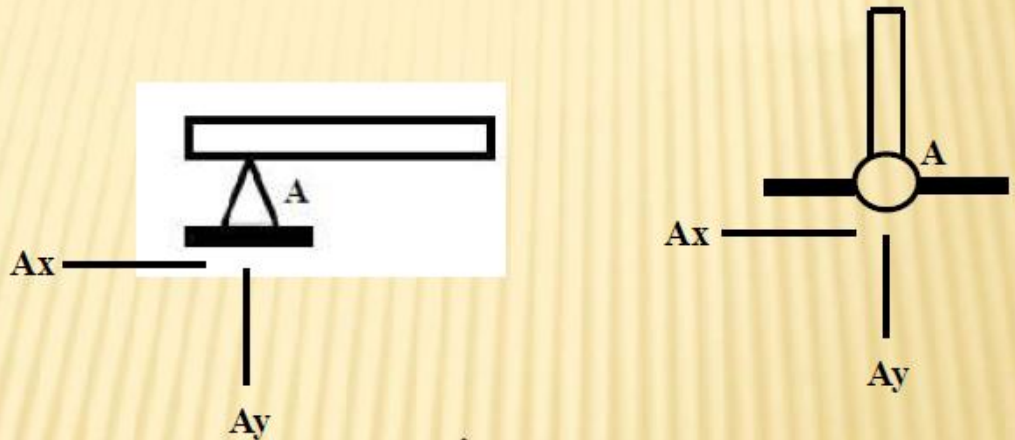


# Types of supports:

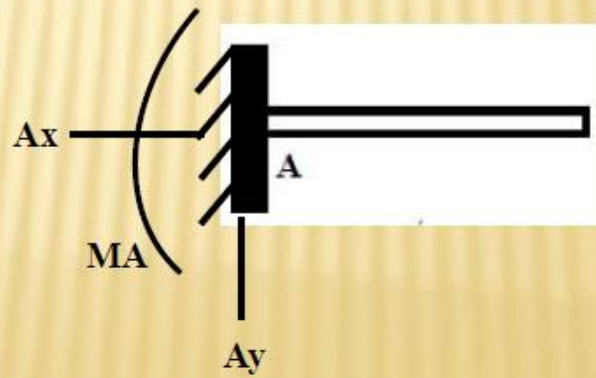
## Roller Support:




## Hinge Support:

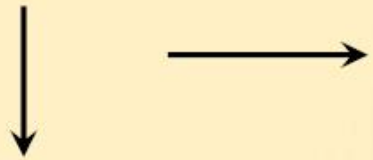



## Fixed Support:



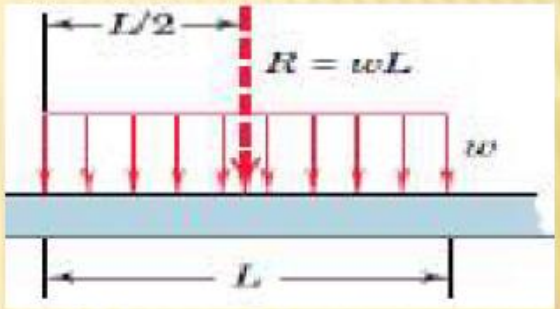
# Types of Loads:

## Concentrated Load:



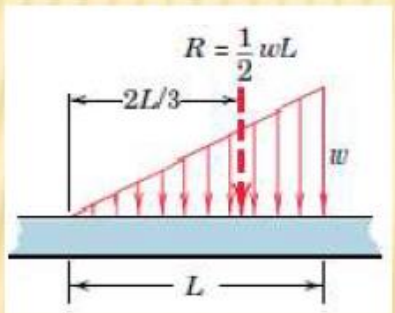
## Distributed Loads:

### 1. Uniformly Distributed Load:

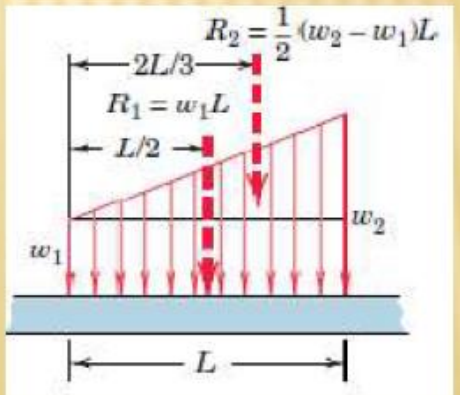



### 2. Uniformly Varying Distributed Load:

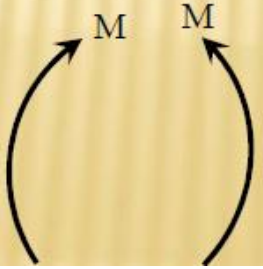
#### Triangular Load:




#### Trapezoidal Load:




## Concentrated Moment



**Example**: For the simply supported beam as shown in figure. Determine the reactions at A and B.

$\sum F_x = 0, \sum F_y = 0, \sum M = 0$

$\sum F_x = 0 \rightarrow Rh_B = 0$

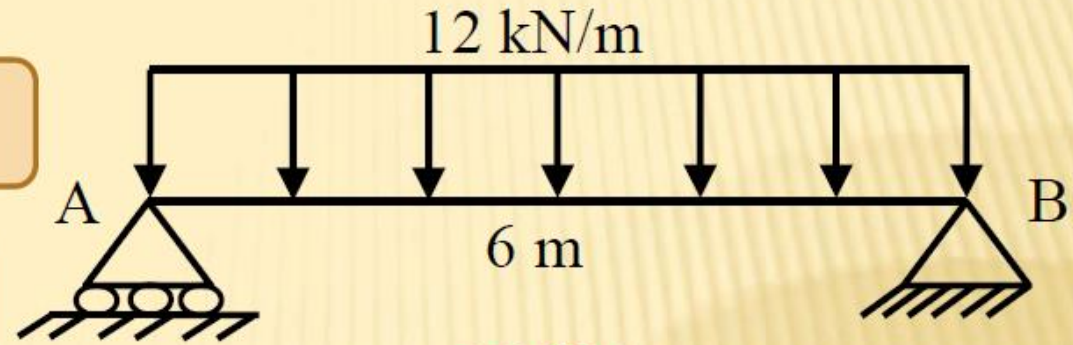
$\sum M_A = 0$

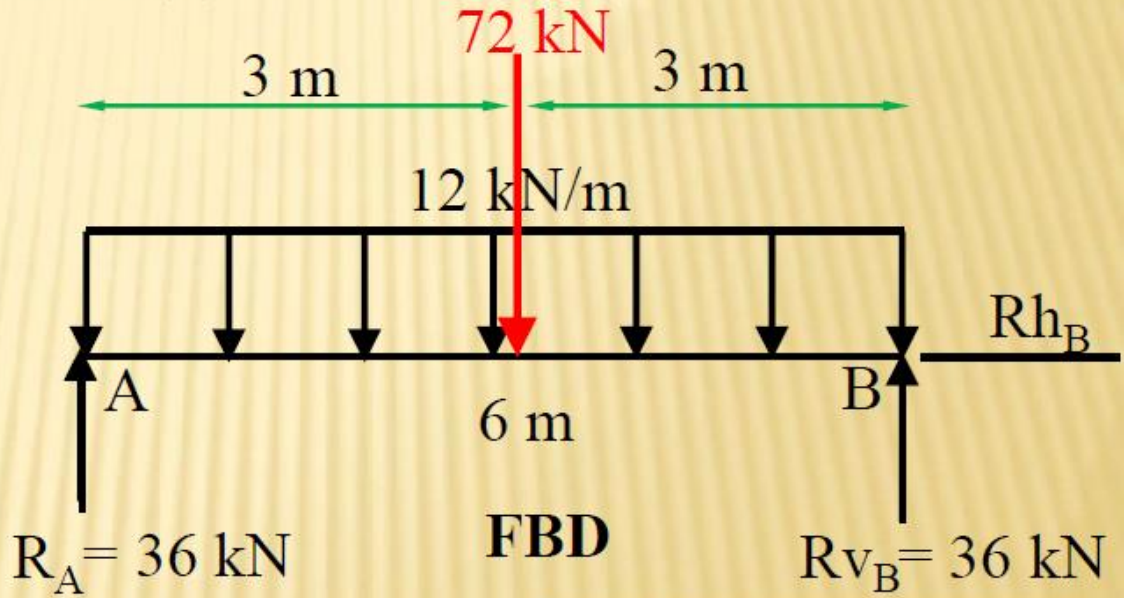
$72(3) - Rv_B\ (6) = 0$

$Rv_B = 36\ kN$

$\sum M_B = 0$

$-72(3) + R_A\ (6) = 0 \rightarrow R_A = 36\ kN$



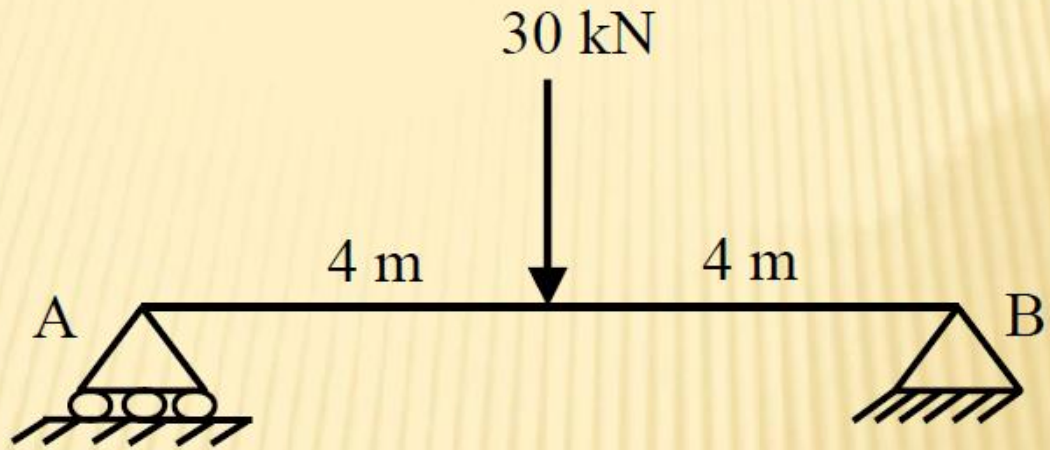

***For Check:***

$\sum F_Y = 0$

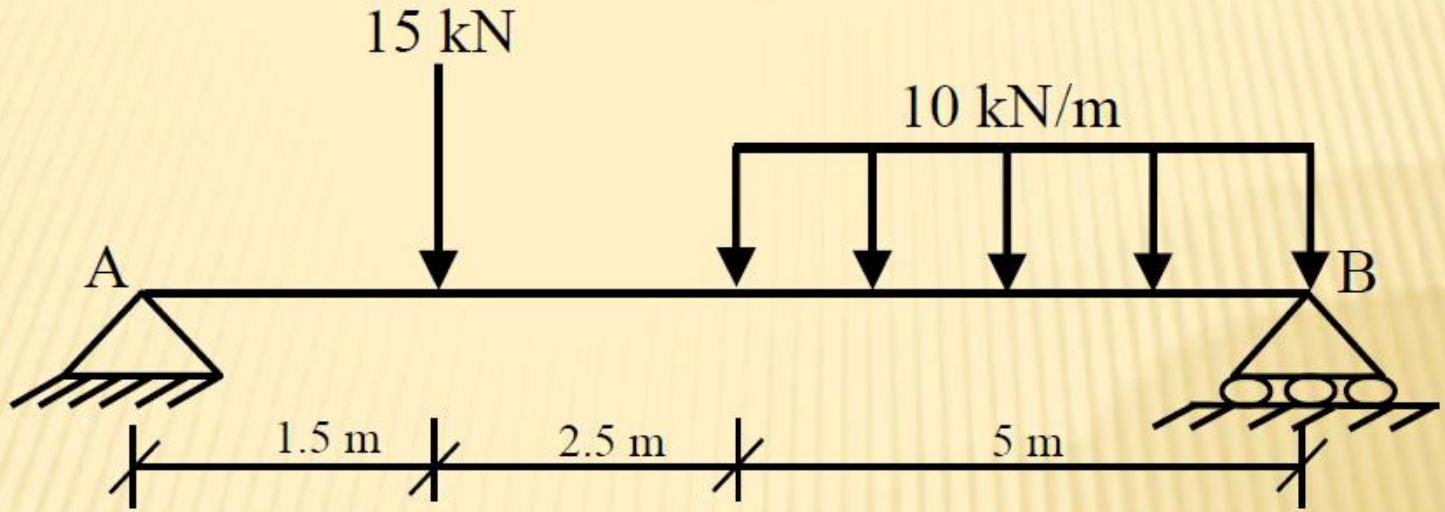
$36 + 36 - 72 = 0$

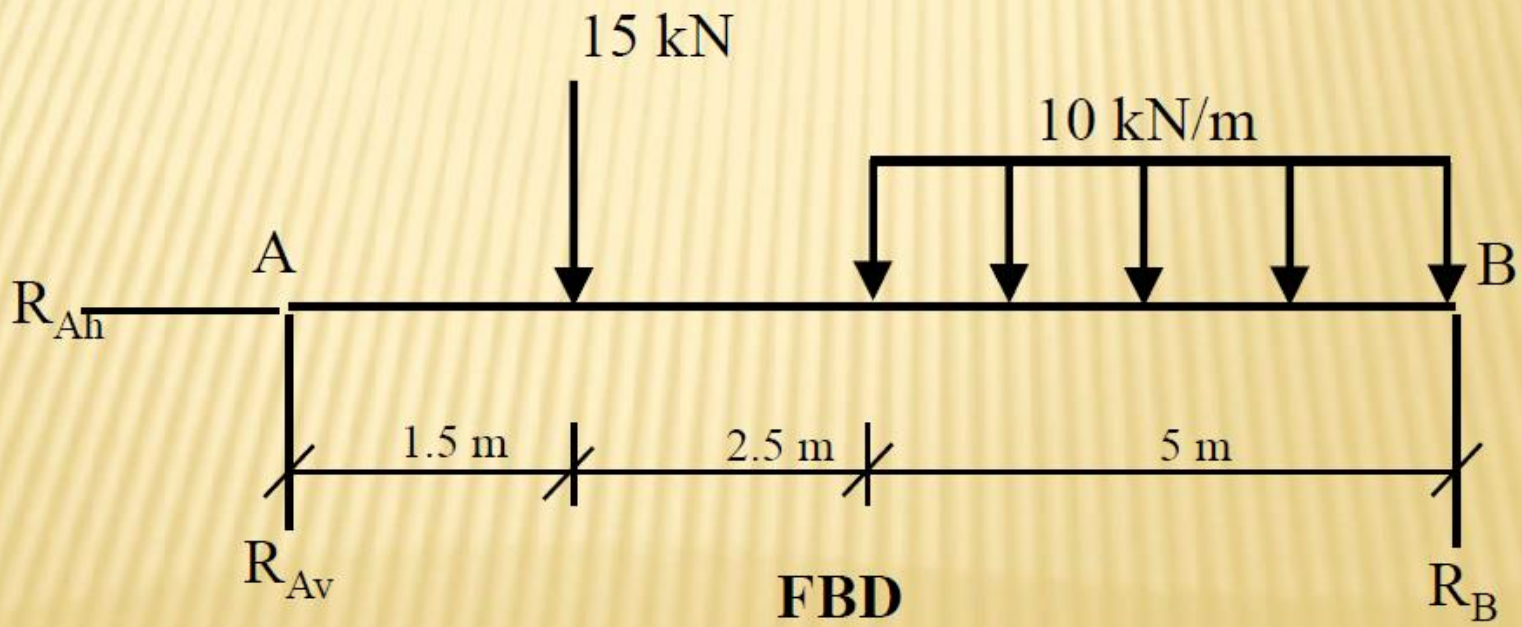
$\therefore$ ***oK.***

**H.W.**: For the simply supported beam as shown in figure. Determine the reactions at A and B.

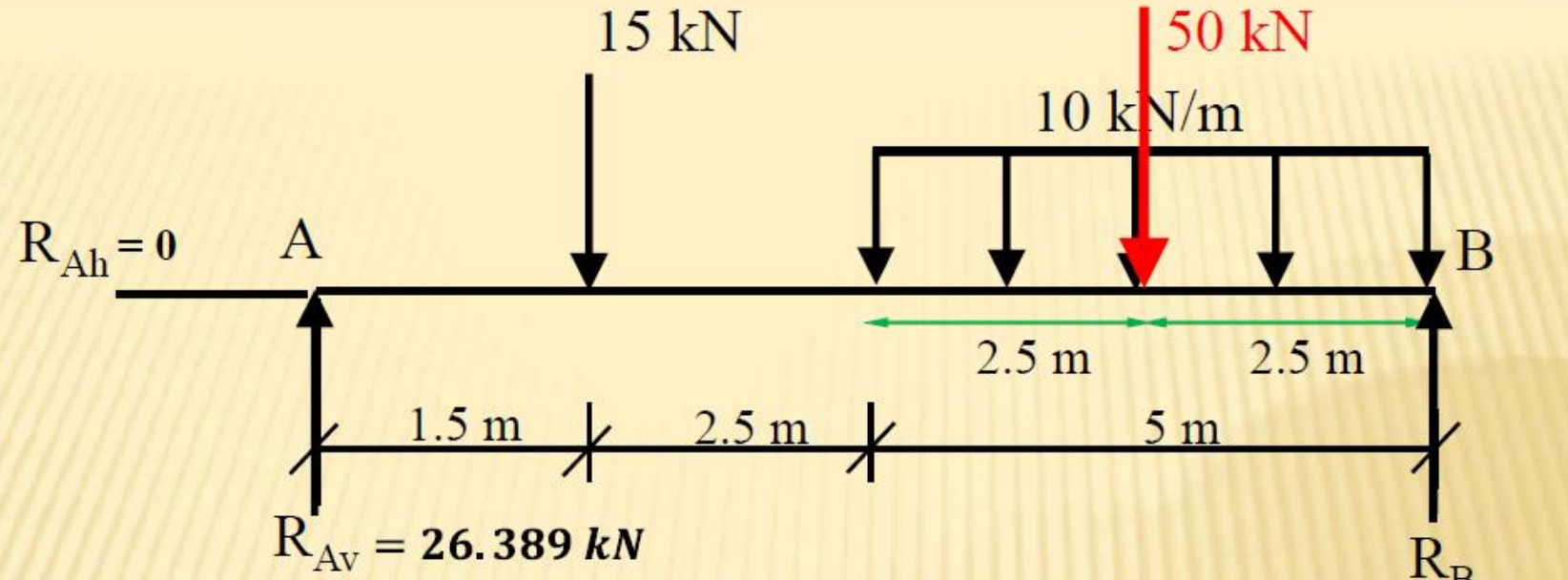


**Example:** For the beam as shown in figure. Calculate the reactions at A and B.




**FBD**




$$\sum F_x = 0 \Rightarrow R_{Ah} = 0$$

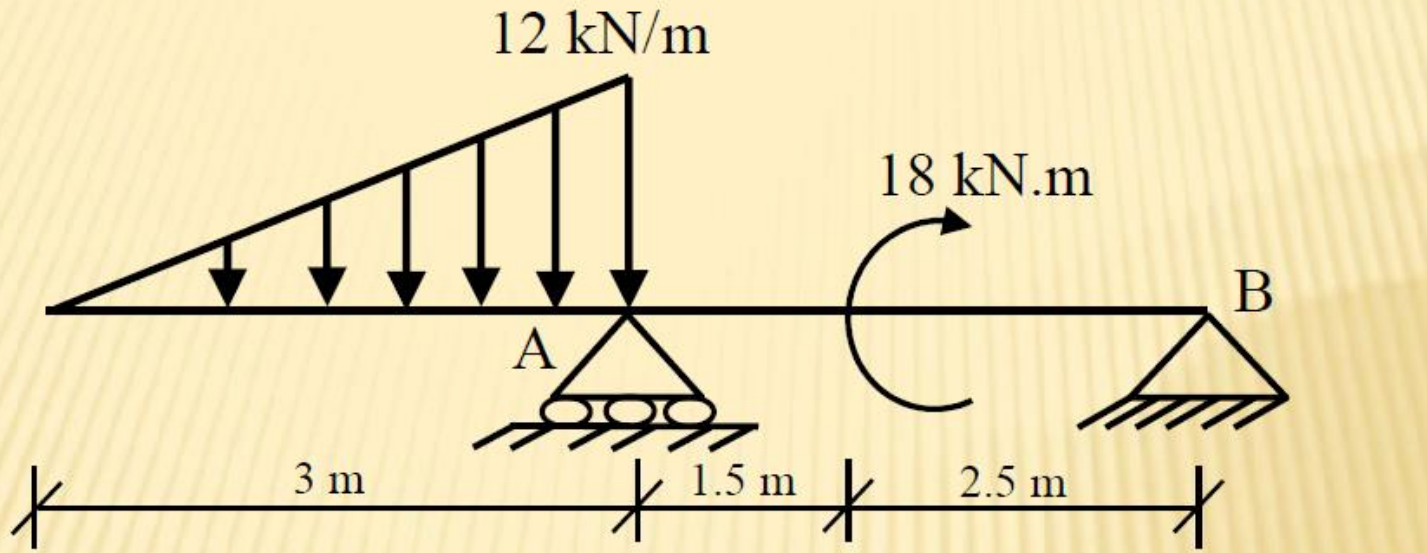
$$\sum M_A = 0$$

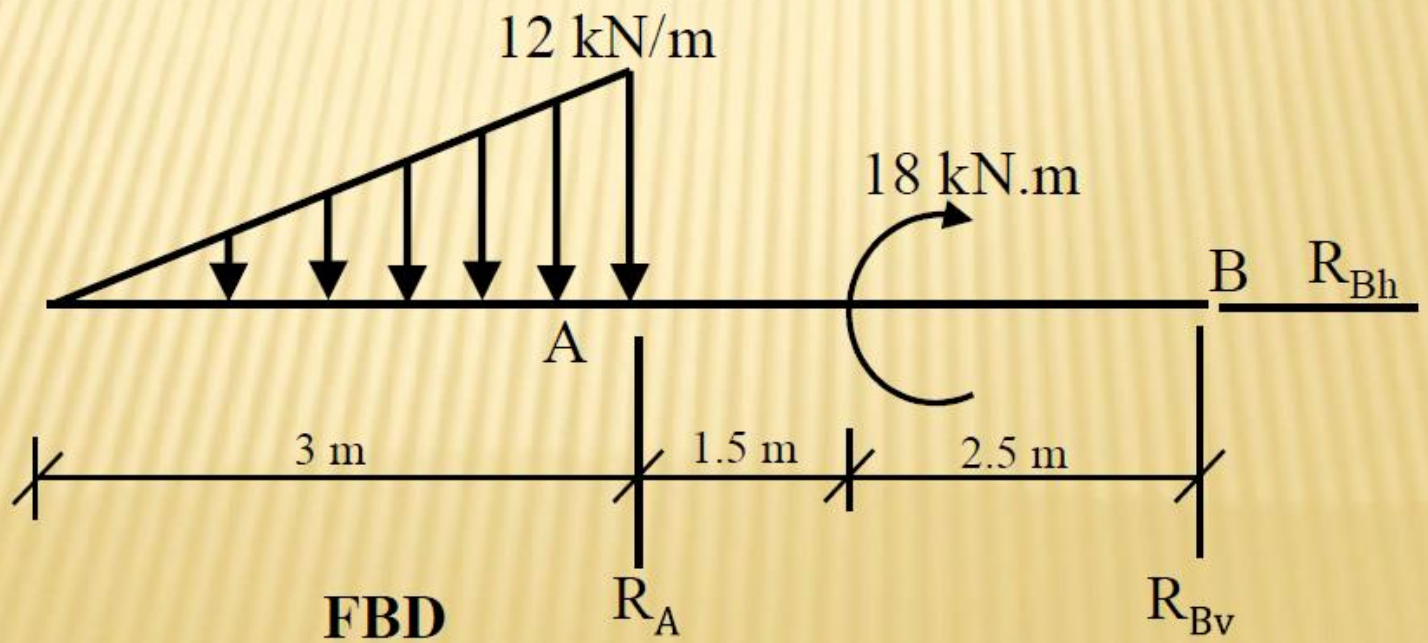
$$15(1.5) + 50\,(6.5) - R_B\,(9) = 0 \Rightarrow R_B = 38.611\ kN \uparrow$$

$$\sum M_B = 0$$

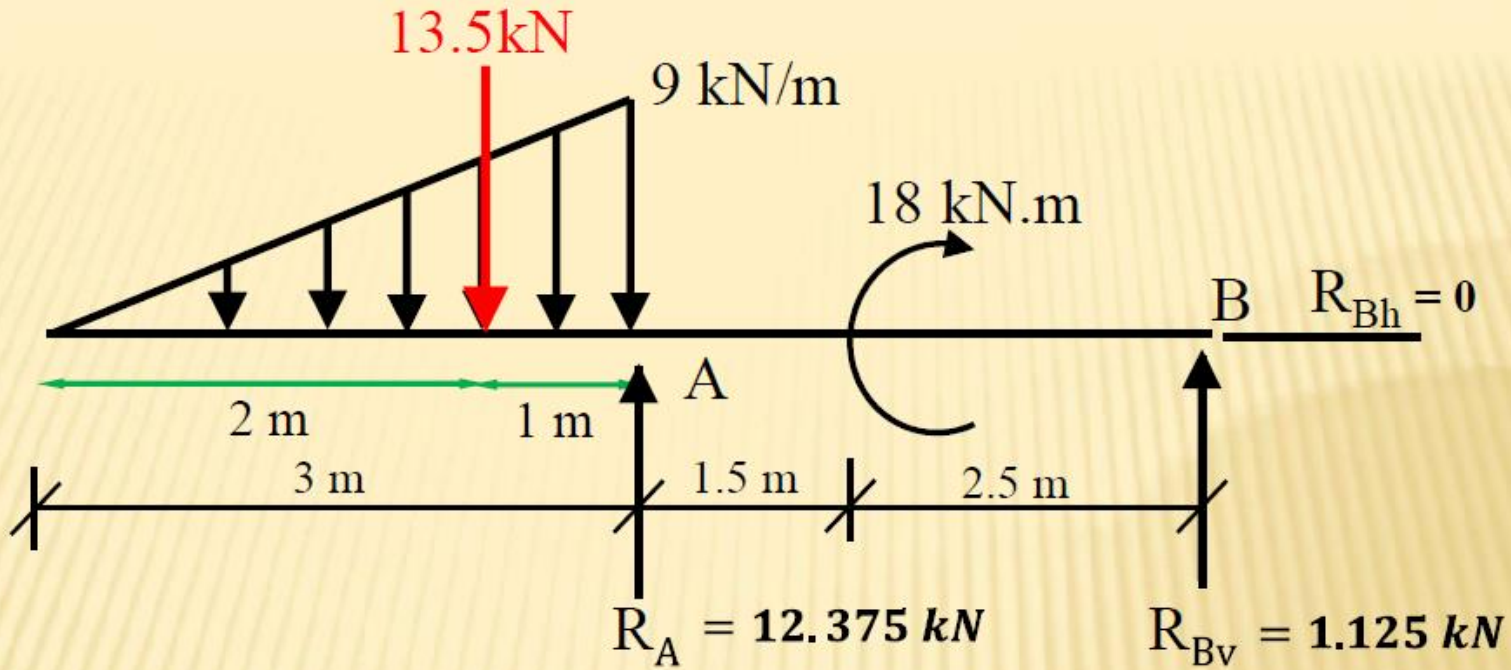
$$-15(7.5) - 50\,(2.5) + R_{Av}\,(9) = 0 \Rightarrow R_{Av} = 26.389\ kN \uparrow$$

**Example:** Calculate all reactions for the beam shown in figure.




**FBD**




$$\sum F_x = 0 \Rightarrow R_{Bh} = 0$$

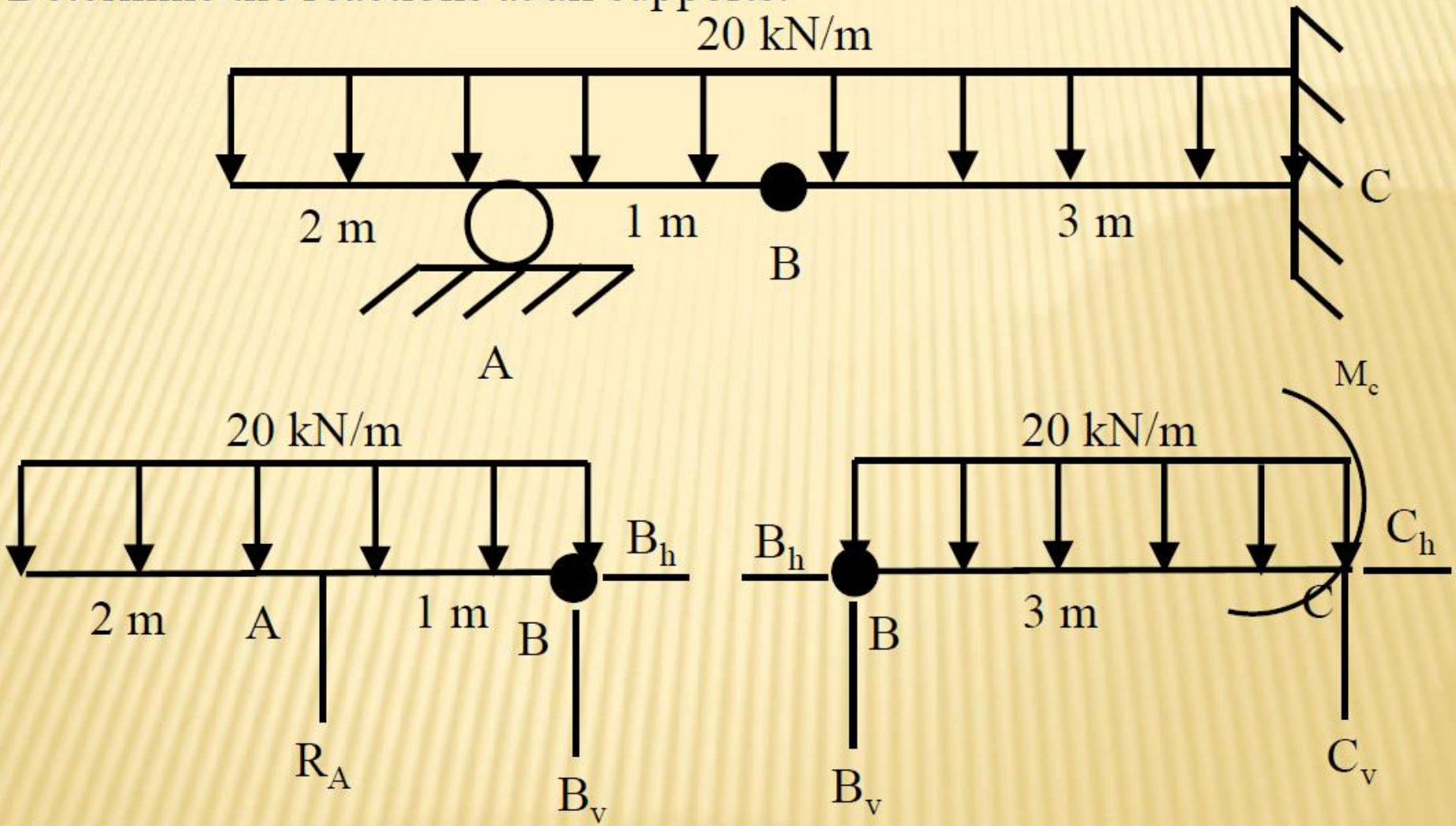
$$\sum M_A = 0$$

$$18 - 13.5\,(1) - R_{Bv}\,(4) = 0 \Rightarrow R_{Bv} = 1.125\ kN \uparrow$$

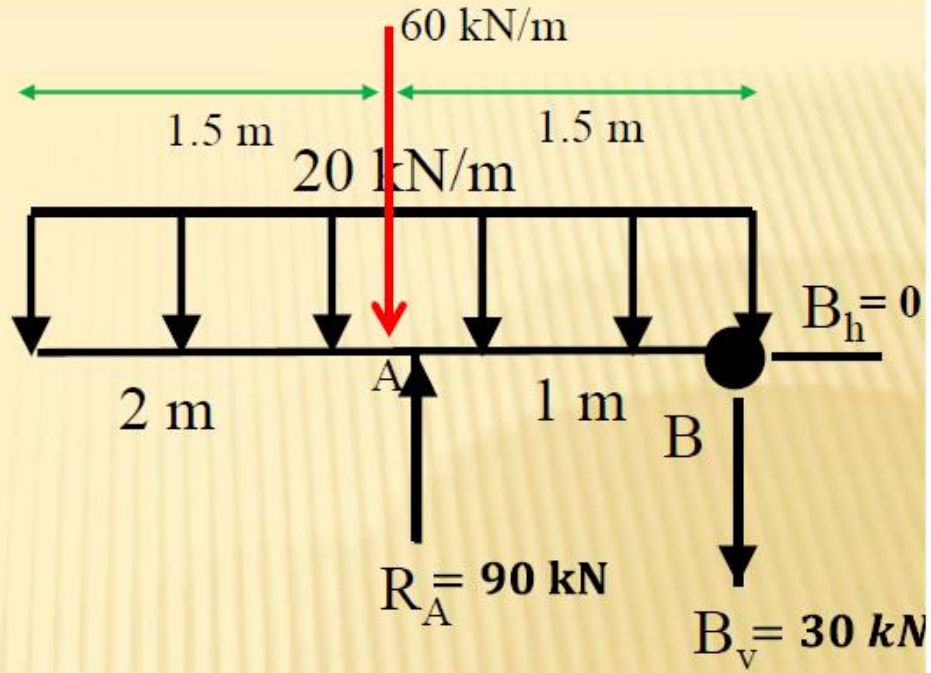
$$\sum M_B = 0$$

$$18 - 13.5\,(5) + R_A\,(4) = 0 \Rightarrow R_{Av} = 12.375\ kN \uparrow$$

**Example:** For the compound beam as shown in figure below. Determine the reactions at all supports.




**FBD**



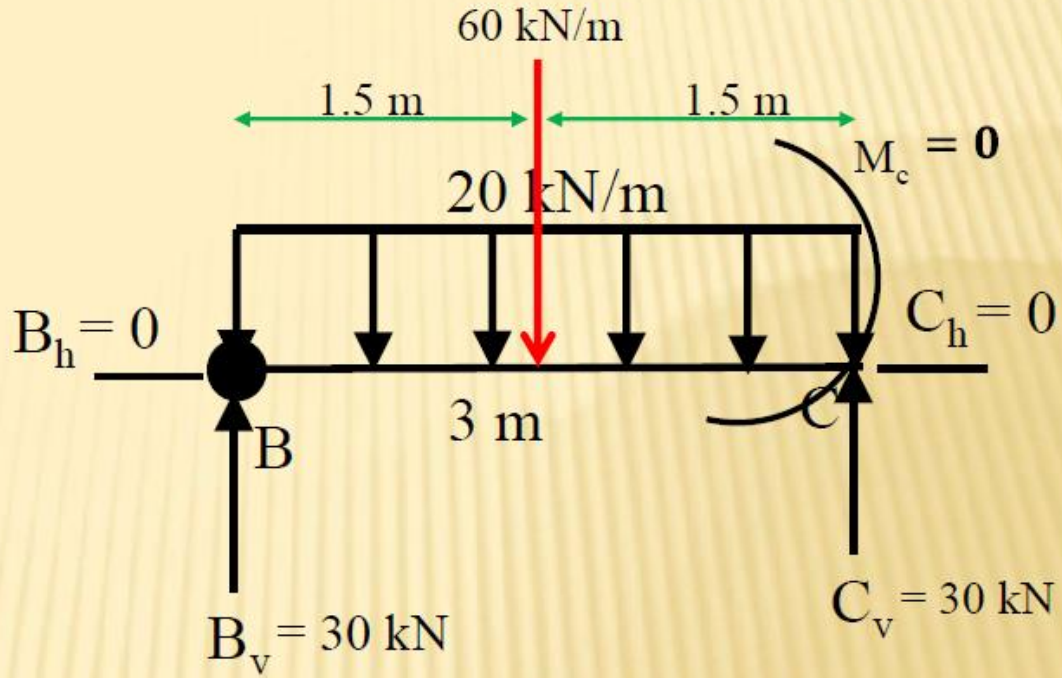

$$\sum F_x = 0 \Rightarrow B_h = 0$$

$$\sum M_B = 0$$

$$-60\,(1.5) + R_A\,(1) = 0 \Rightarrow R_A = 90\ kN \uparrow$$

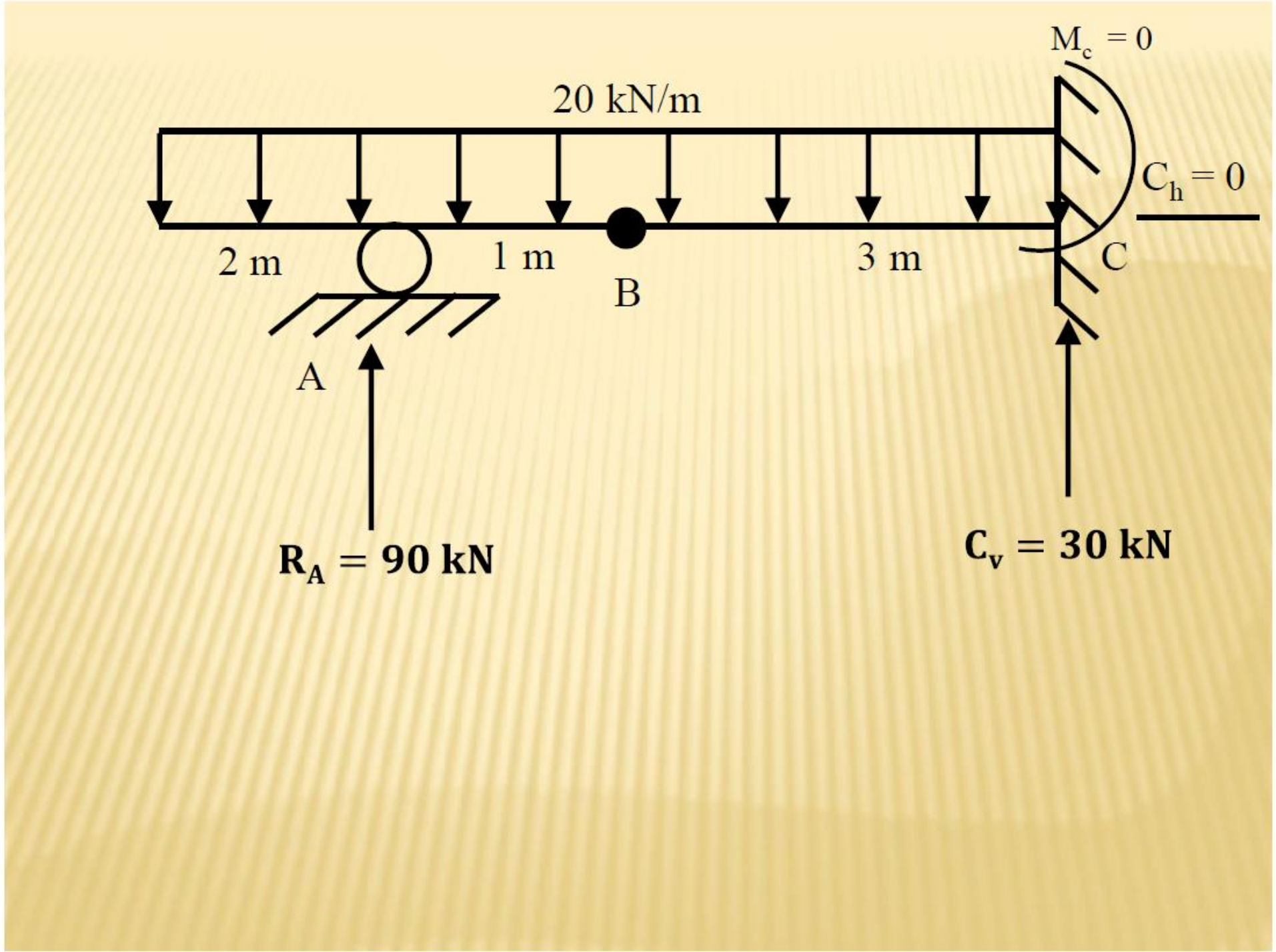
$$\sum F_Y = 0$$

$$90 - 60 + B_v = 0 \Rightarrow B_v = -30\ kN \downarrow$$

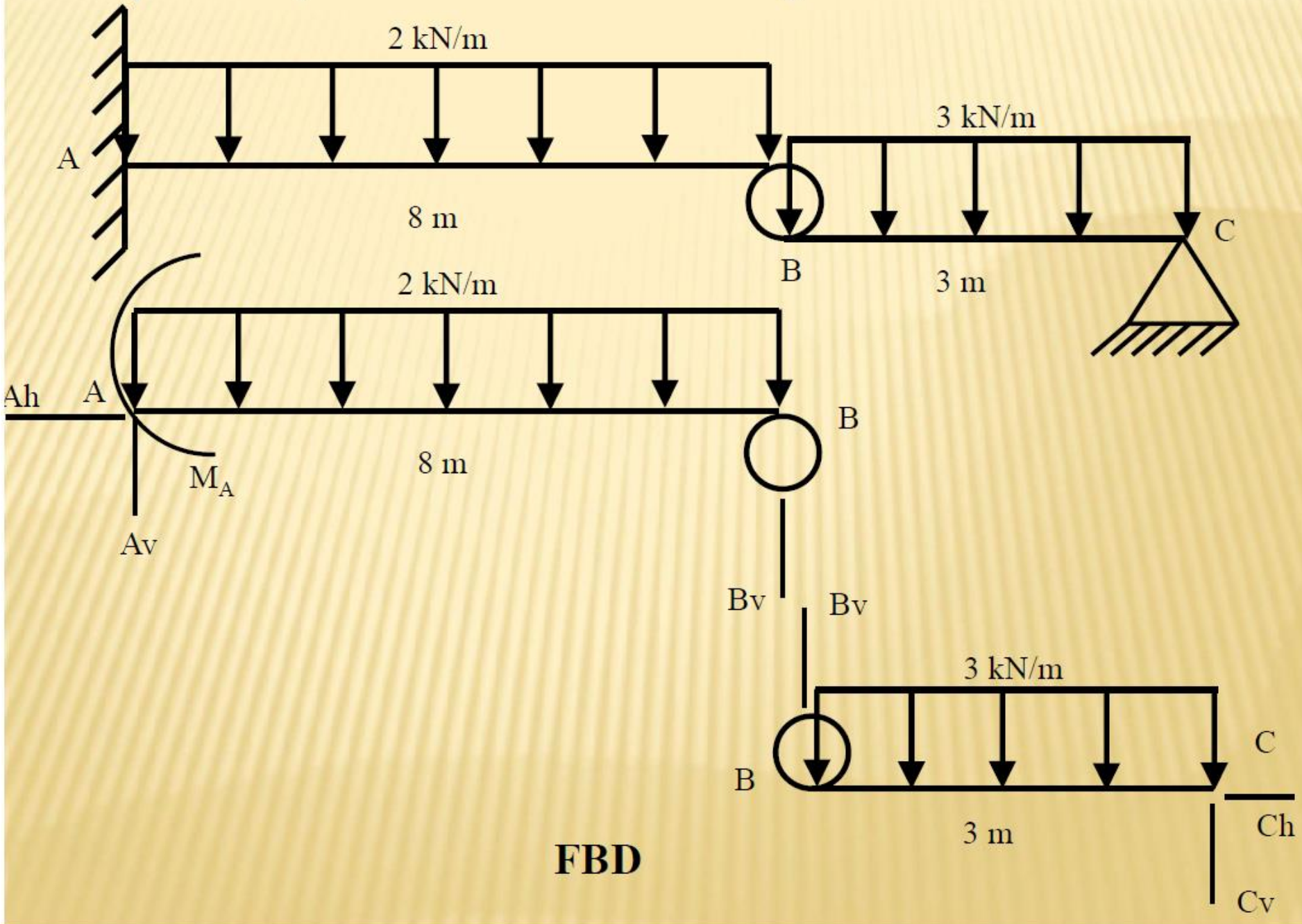


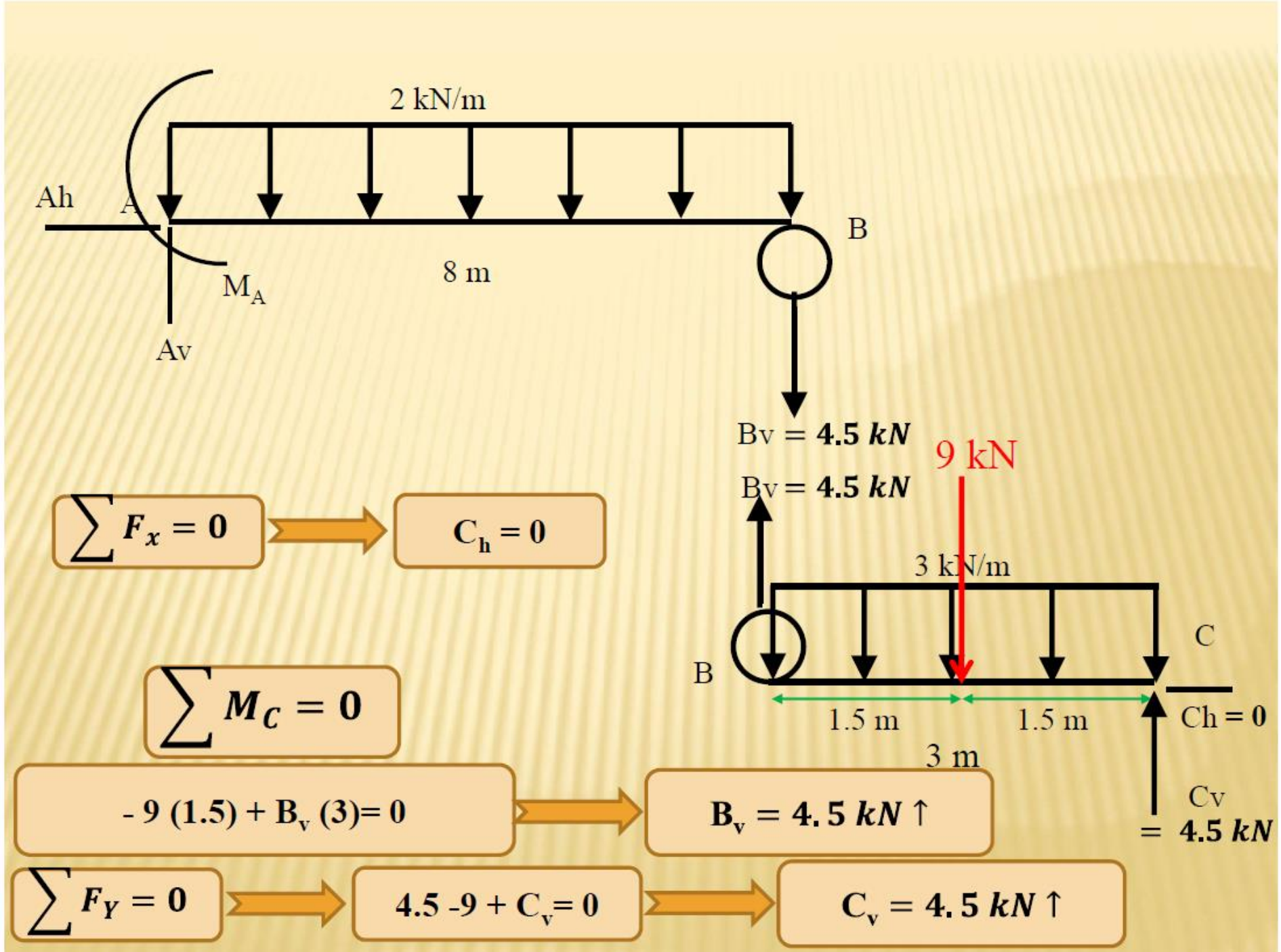

$$\sum M_B = 0$$

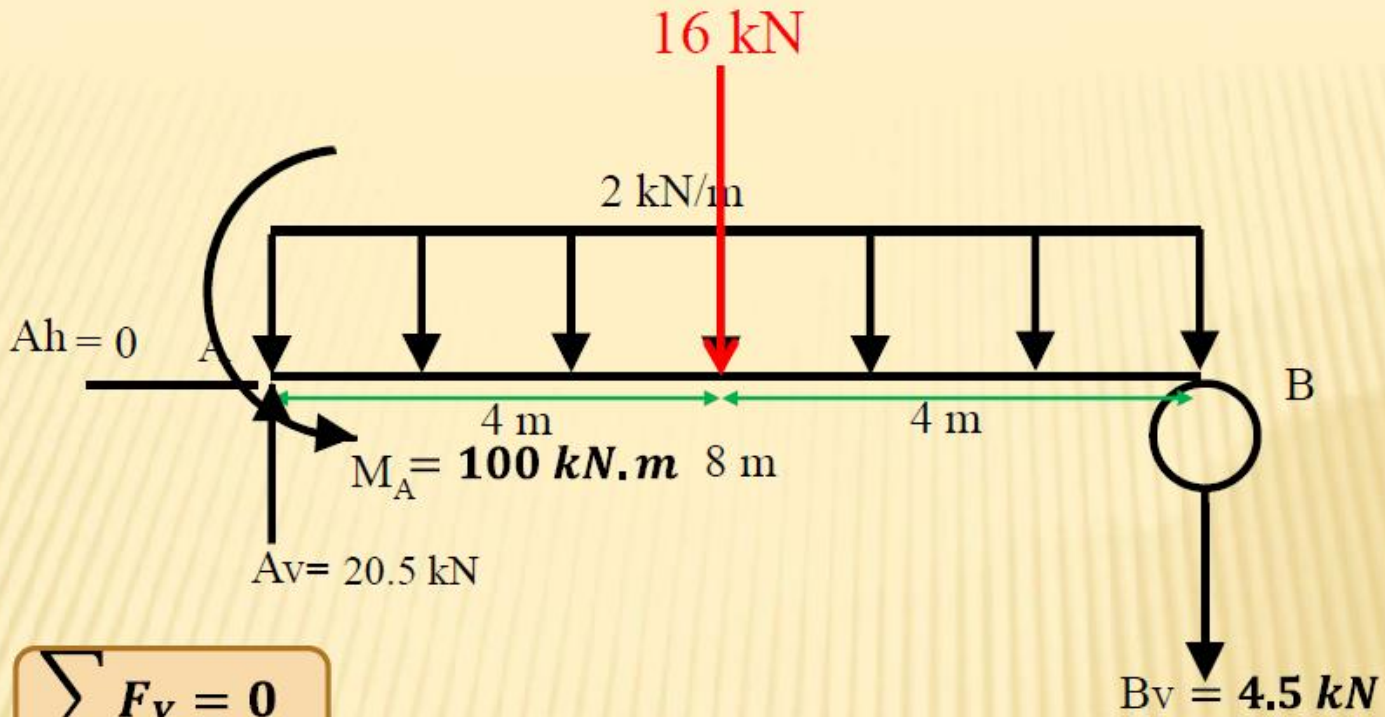
$$60\,(1.5) - 30\,(3) + M_C = 0 \quad \Rightarrow \quad M_C = 0$$


$M_c = 0$
20 kN/m
$C_h = 0$
2 m
1 m
3 m
B
C
A
$R_A = 90$ kN
$C_v = 30$ kN

**Example:** Analyze the beam as shown in figure below.




2 kN/m
Ah
A
B
8 m
$M_A$
Av
Bv = 4.5 kN
9 kN
Bv = 4.5 kN
3 kN/m
C
B
1.5 m
1.5 m
Ch = 0
3 m
Cv
= 4.5 kN
$\sum F_x = 0$
$C_h = 0$
$\sum M_C = 0$
- 9 (1.5) + $B_v$ (3)= 0
$B_v = 4.5\ kN \uparrow$
$\sum F_Y = 0$
4.5 -9 + $C_v$= 0
$C_v = 4.5\ kN \uparrow$

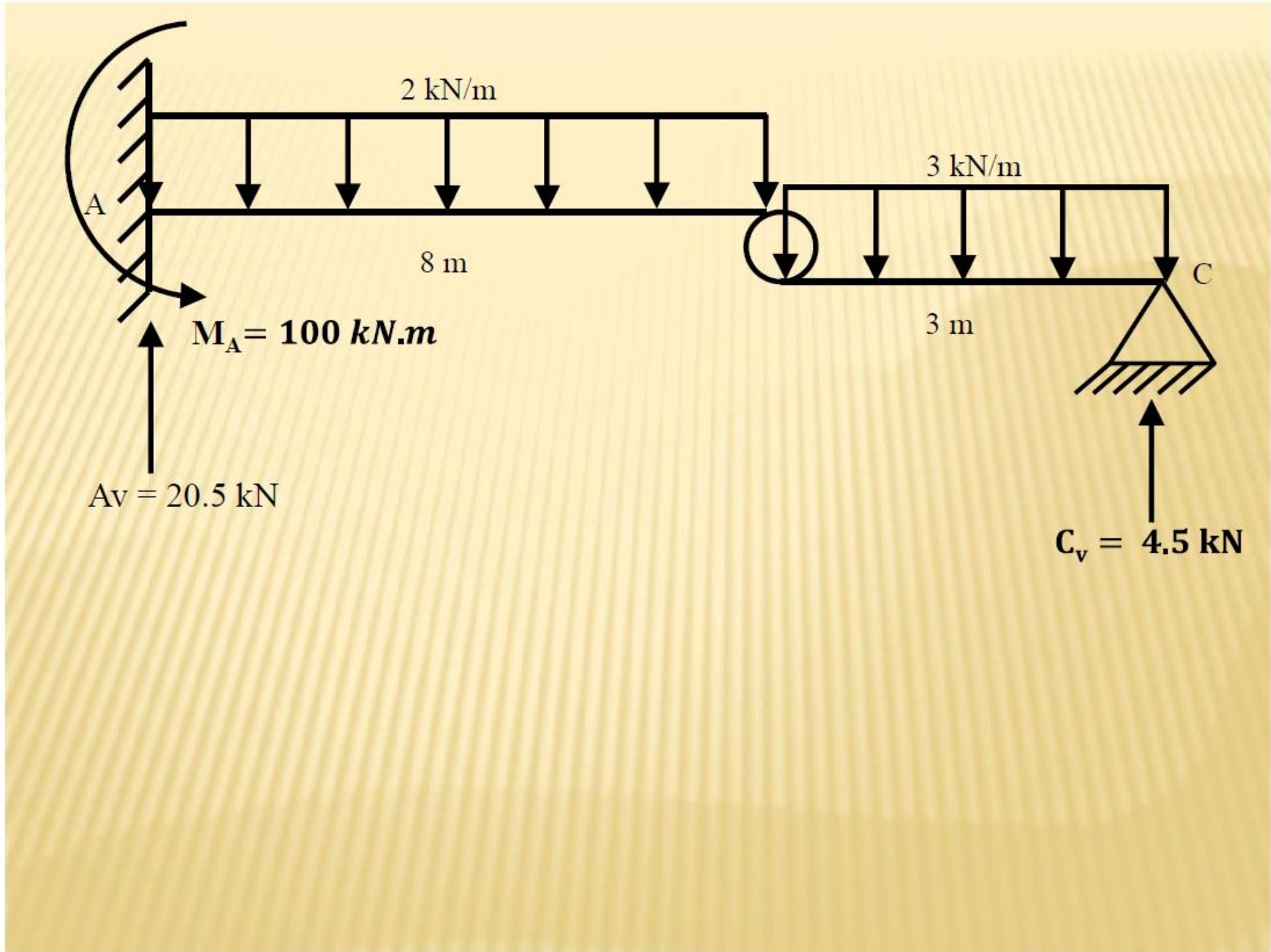



$$\sum F_Y = 0$$

$$-4.5 - 16 + A_v = 0 \Rightarrow A_v = 20.5\ kN \uparrow$$

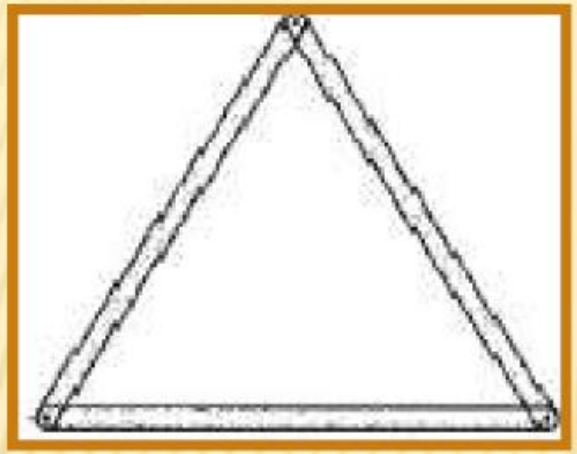
$$\sum M_A = 0$$

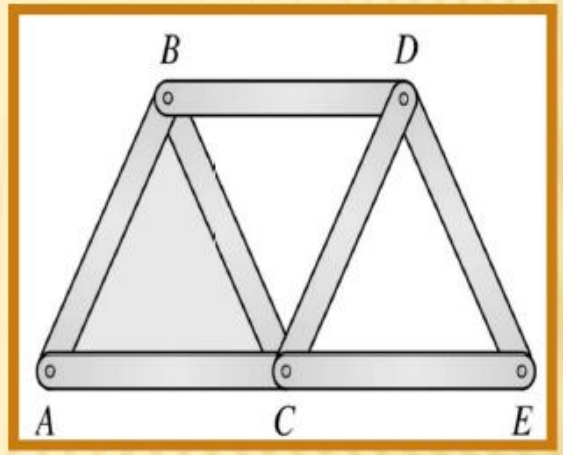
$$16\,(4) + 4.5\,(8) + M_A = 0 \Rightarrow M_A = -100\ kN.m$$


2 kN/m
3 kN/m
A
8 m
3 m
C
$M_A = 100\ kN.m$
Av = 20.5 kN
$C_v = 4.5\ kN$

# Analysis of Trusses

A truss is a structure that is made of straight, slender bars that are joined together to form a pattern of triangles. Trusses are usually designed to transmit forces over relatively long spans; common examples are **bridge trusses** and **roof trusses**.








**Bridge truss**



**Roof trusses**

**The analysis of trusses is based on the following three assumptions:**

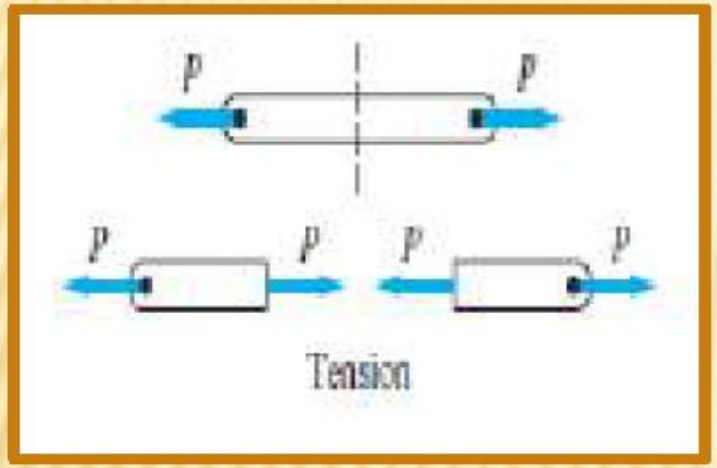
**1. The weights of the members are negligible.** A truss can be classified as a lightweight structure, meaning that the weights of its members are generally much smaller than the loads that it is designed to carry.

**2. All joints are pins.** In practice, the members at each joint are usually riveted or welded to a plate, called a gusset plate. However, if the members at a joint are aligned so that their centroidal axes (axes that pass through the centroids of the cross-sectional areas of the members) intersect at a common point, advanced methods of analysis indicate that the assumption of pins is justified.

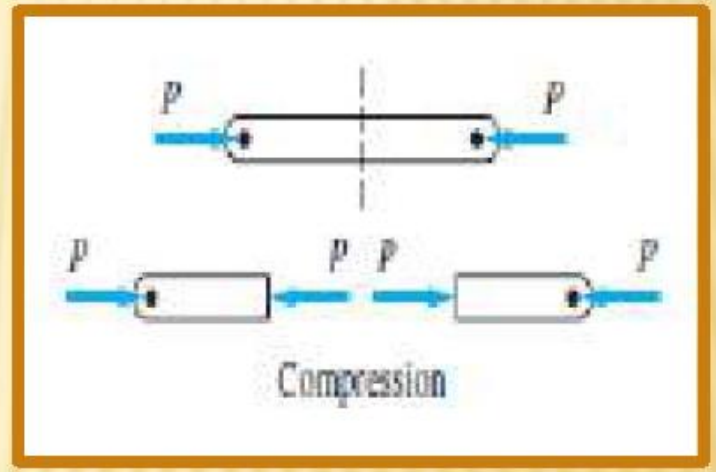
**3. The applied forces act at the joints.** Because the members of a truss are slender, they may fail in bending when subjected to loads applied at locations other than the joints. Therefore, trusses are designed so that the major applied loads act at the joints.

## Forces in Truss

The forces act along the longitudinal axis of the member, they are often called ***axial forces***. The figure below shows the external and internal forces in **tension** and **compression**. **Tensile** forces elongate (stretch) the member, whereas **compressive** forces compress (shorten) it.




**Tension Force**

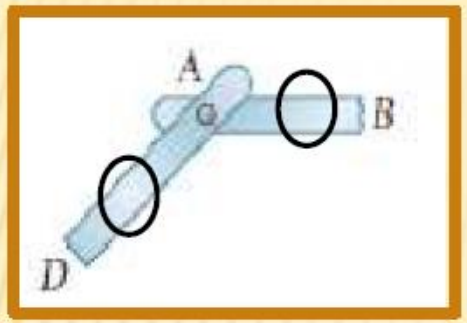



**Compressive Force**

# Zero-force members

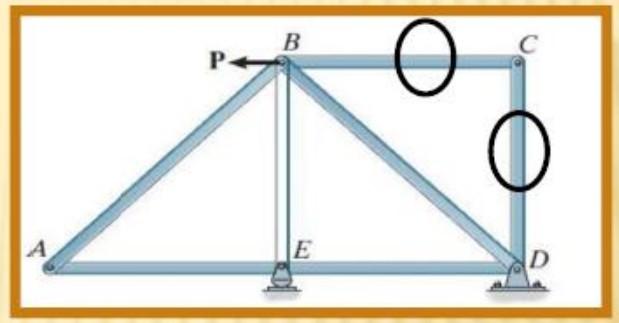
**1**. If only two non – collinear members are connected to a joint that has no external loads applied to it, then the force in both members is **zero**.

اذا التقى ضلعان في مفصل لا تؤثر عليه قوة خارجي فان القوة في كلا الضلعين تساوي صفر.



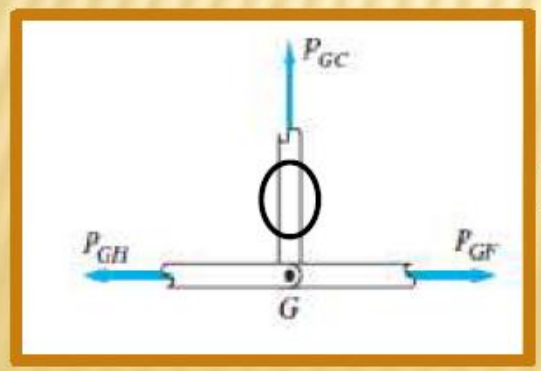

$F_{AB} = 0$

$F_{AD} = 0$

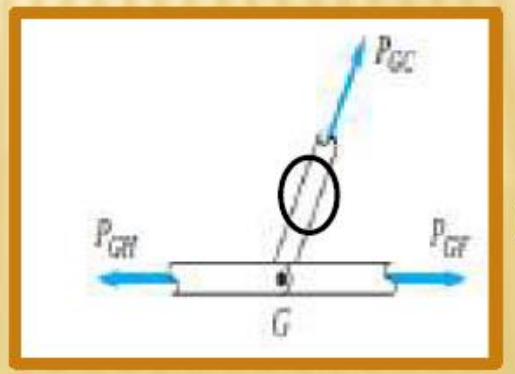



**2**. If there are three members at a joint out of which two are collinear and no external force applied in that joint, then the third non collinear member is zero.

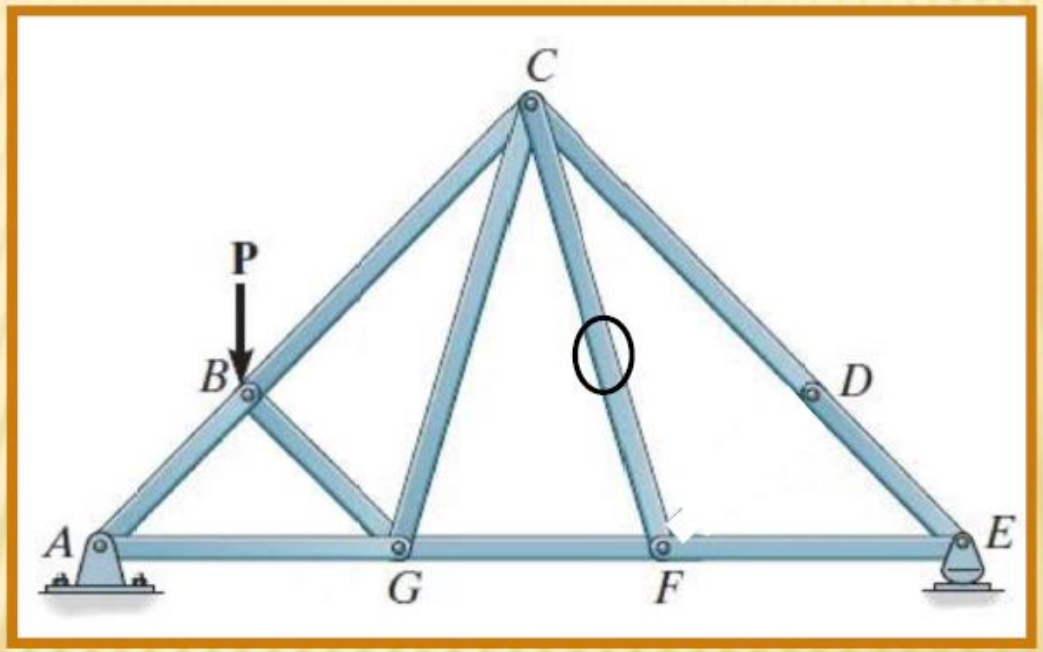
اذا كان هناك ثلاث اضلاع ملتقية في مفصل، اثنان منهما على استقامة واحدة فالقوة في الضلع الثالث تساوي صفر.



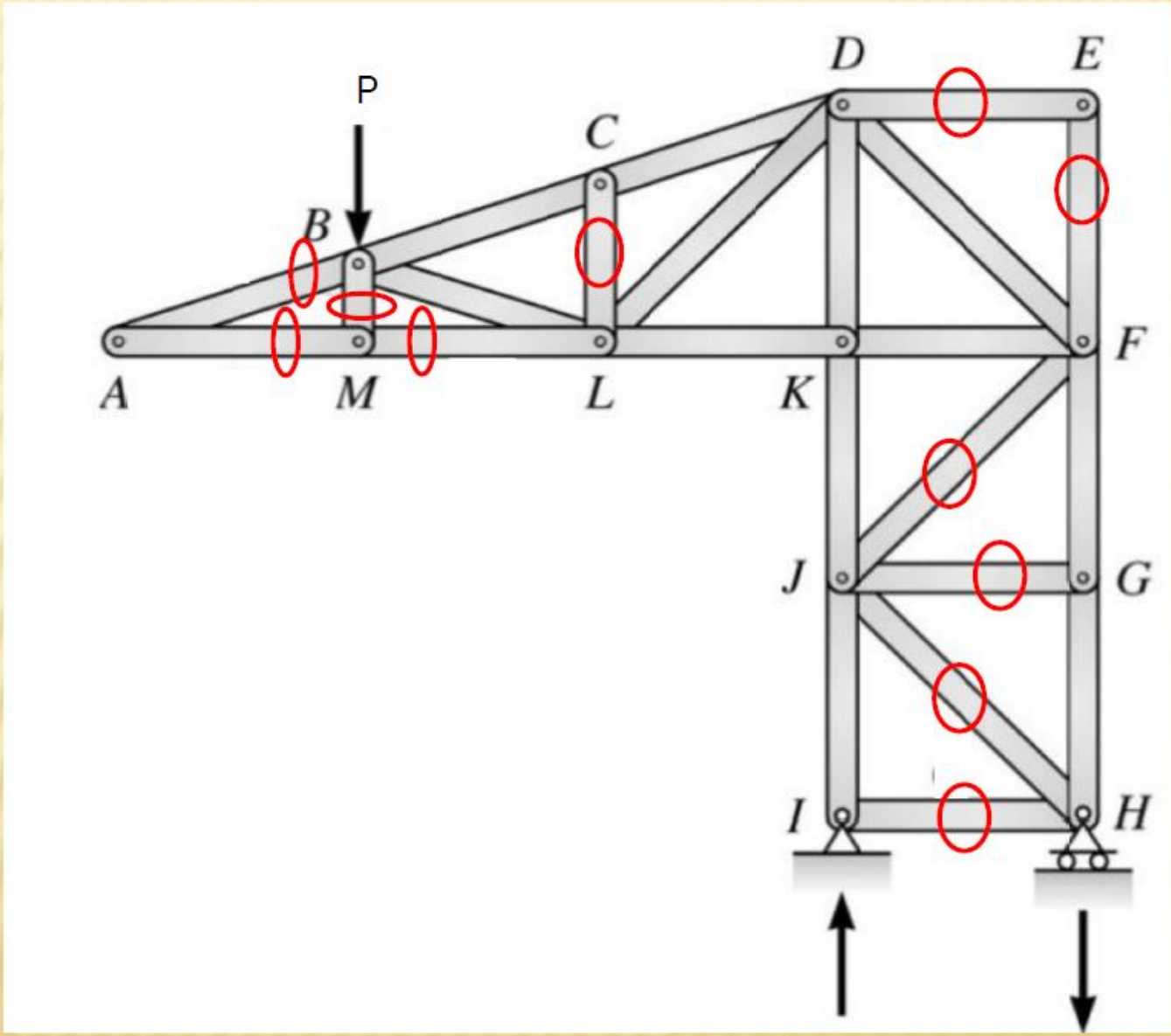

$F_{GC} = 0$




$F_{GC} = 0$


C
P
B
D
A
G
F
E

**Example:** Determine the ***zero-force members*** in the truss shown for the given loading.



# Method of trusses analysis

- Method of Joints
- Method of Sections

## Method of Joints

This method for finding the forces in the members of a truss consists of satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint. The method therefore deals with the equilibrium of concurrent forces, and **only two independent equilibrium equations are involved.**

$$\sum F_x = 0 \qquad \sum F_Y = 0$$

We begin the analysis with any joint where at least one known load exists and where **not more than two unknown forces are present**

**Example:** Determine the force in each member of the truss shown in figure below. State if the members are in tension or compression. **Using joint method.**

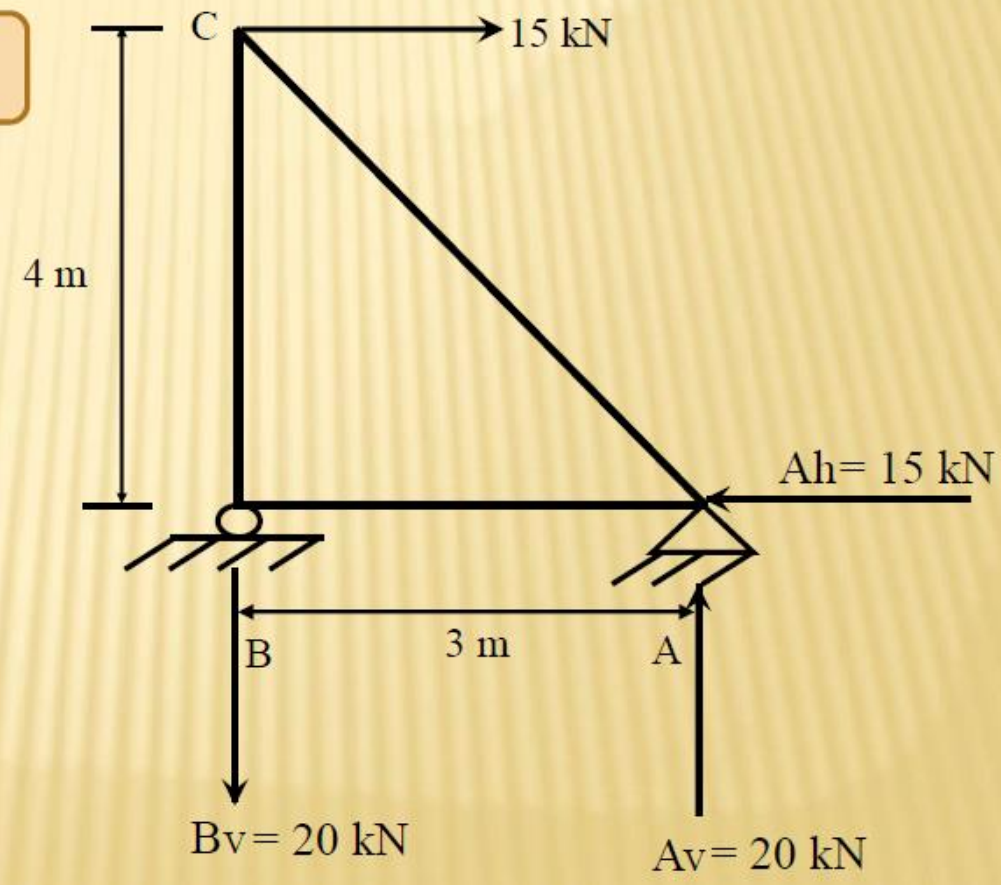
$$\sum F_x = 0$$

$$15 - Ah = 0 \Rightarrow Ah = 15 \text{ kN} \leftarrow$$

$$\sum M_B = 0$$

$$15(4) - Av\ (3) = 0$$

$$Av = 20kN$$



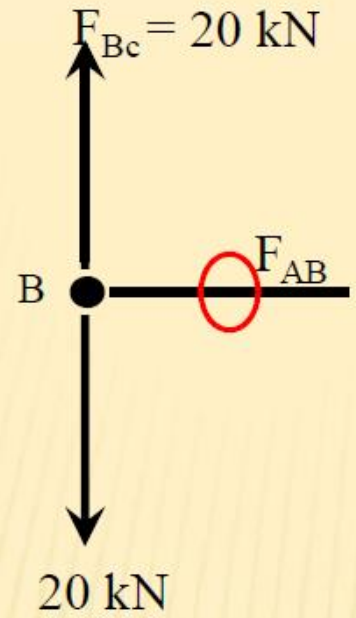
## Joint B

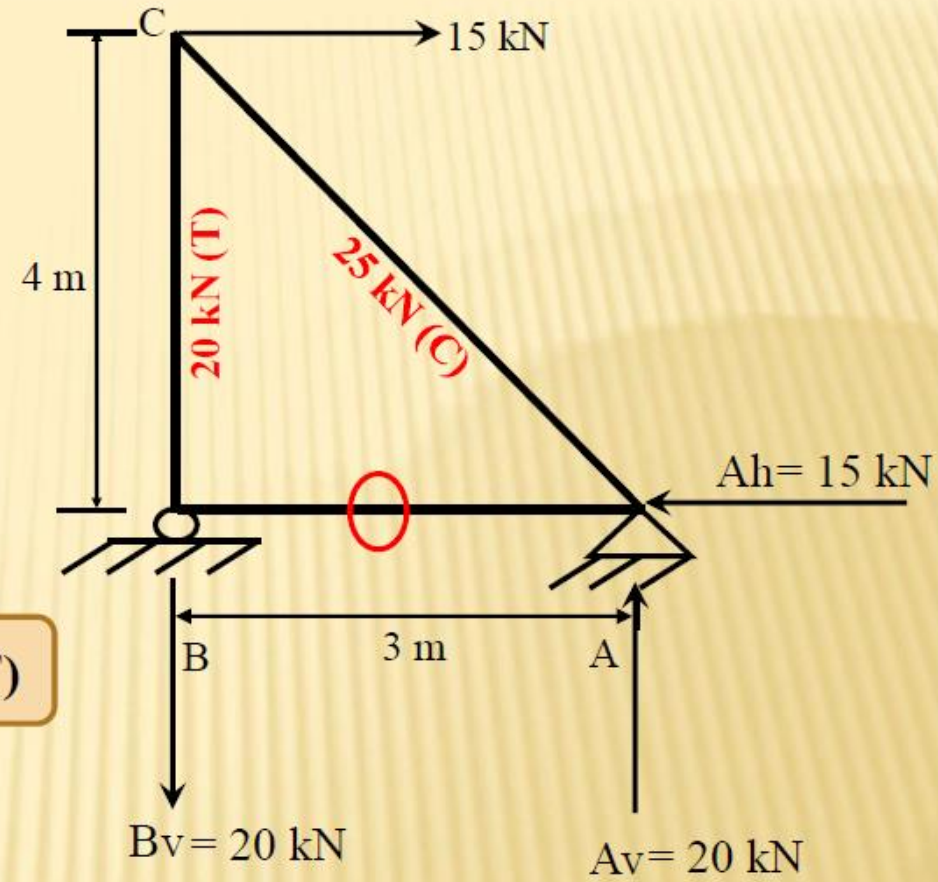
$\sum F_x = 0$

$F_{AB} = 0$

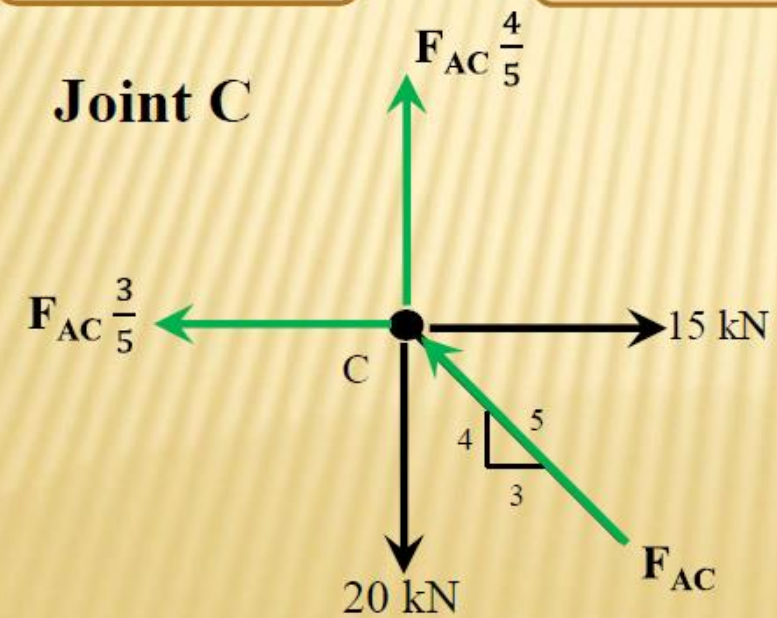
$\sum F_Y = 0$

$- 20 + F_{BC} =$ → $F_{BC} = 20$ kN (T)




## Joint C




$\sum F_x = 0$

$15 - F_{AC}\frac{3}{5} = 0$ → $F_{AC} = 25$ kN (C)

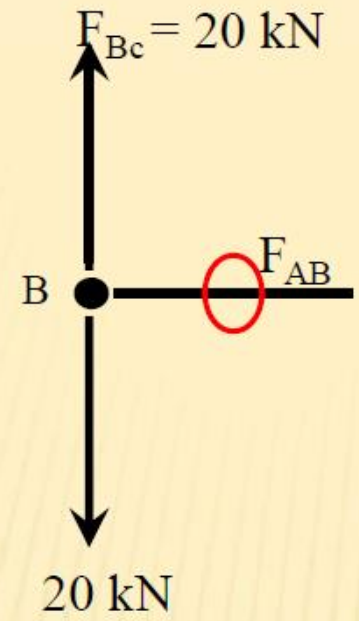
## Joint B

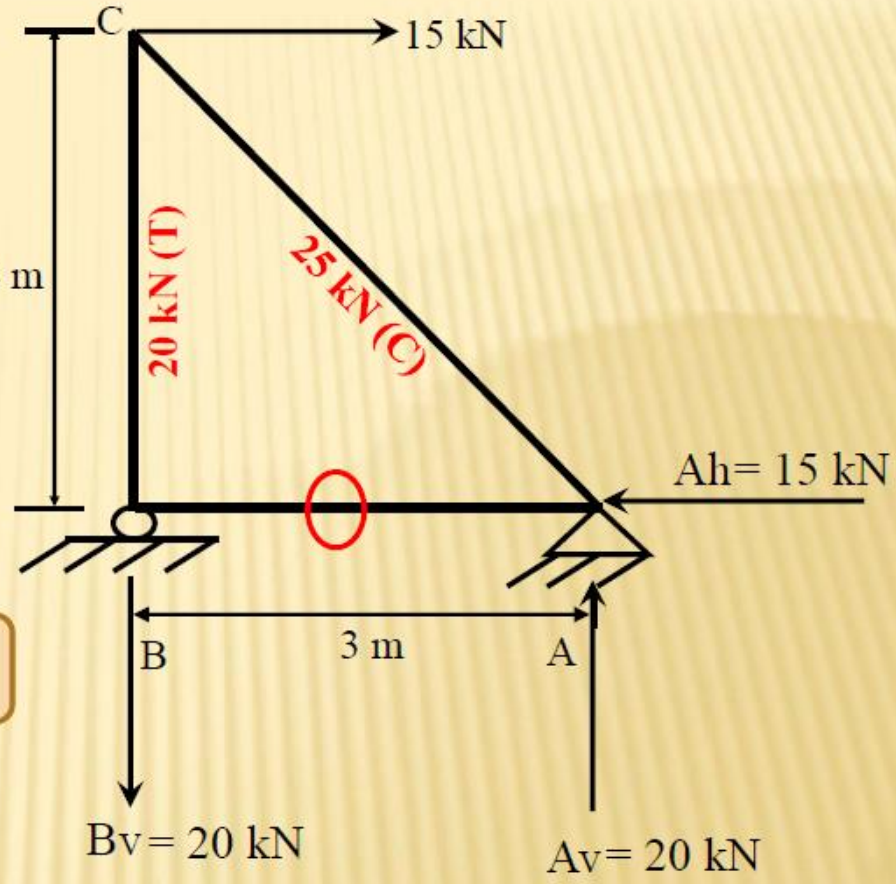
$\sum F_x = 0$

$F_{AB} = 0$

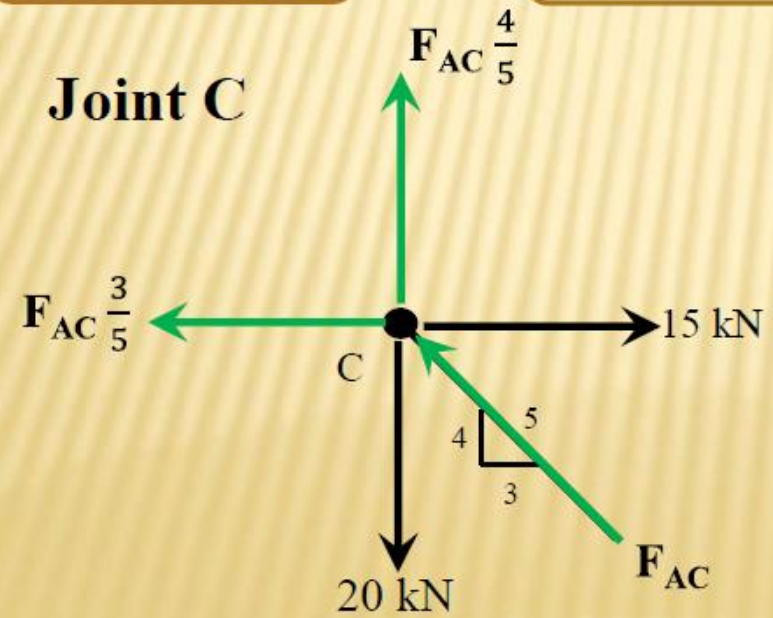
$\sum F_Y = 0$

$-20 + F_{BC} =$ → $F_{BC} = 20$ kN (T)



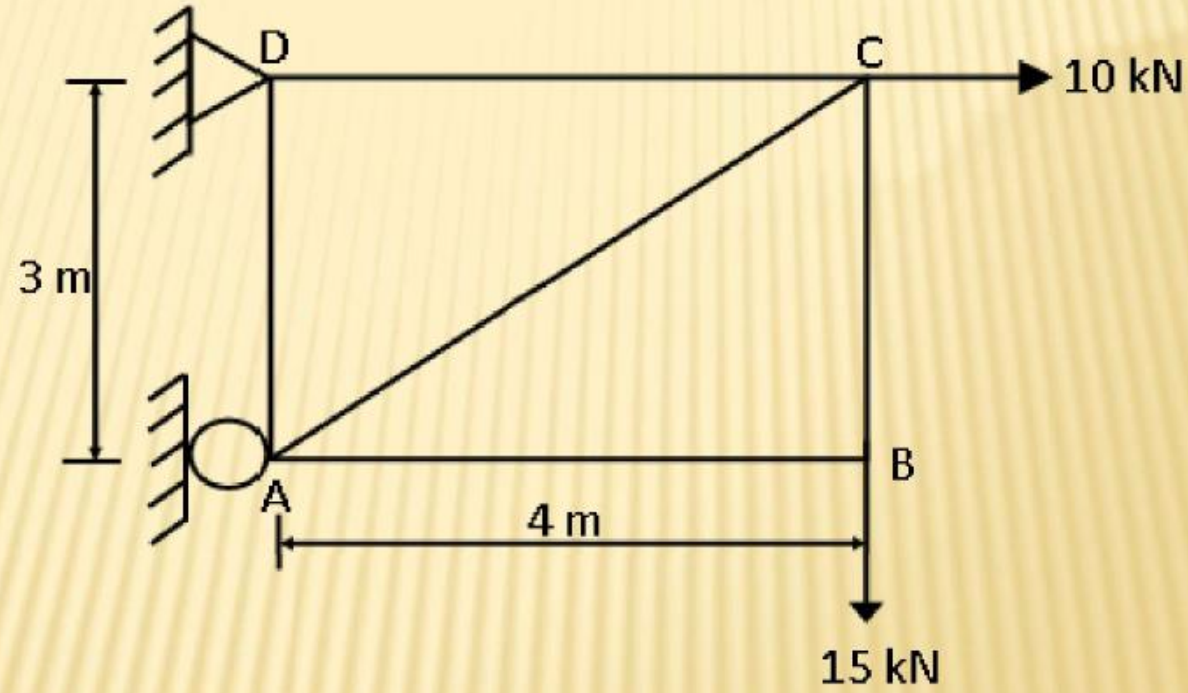

## Joint C



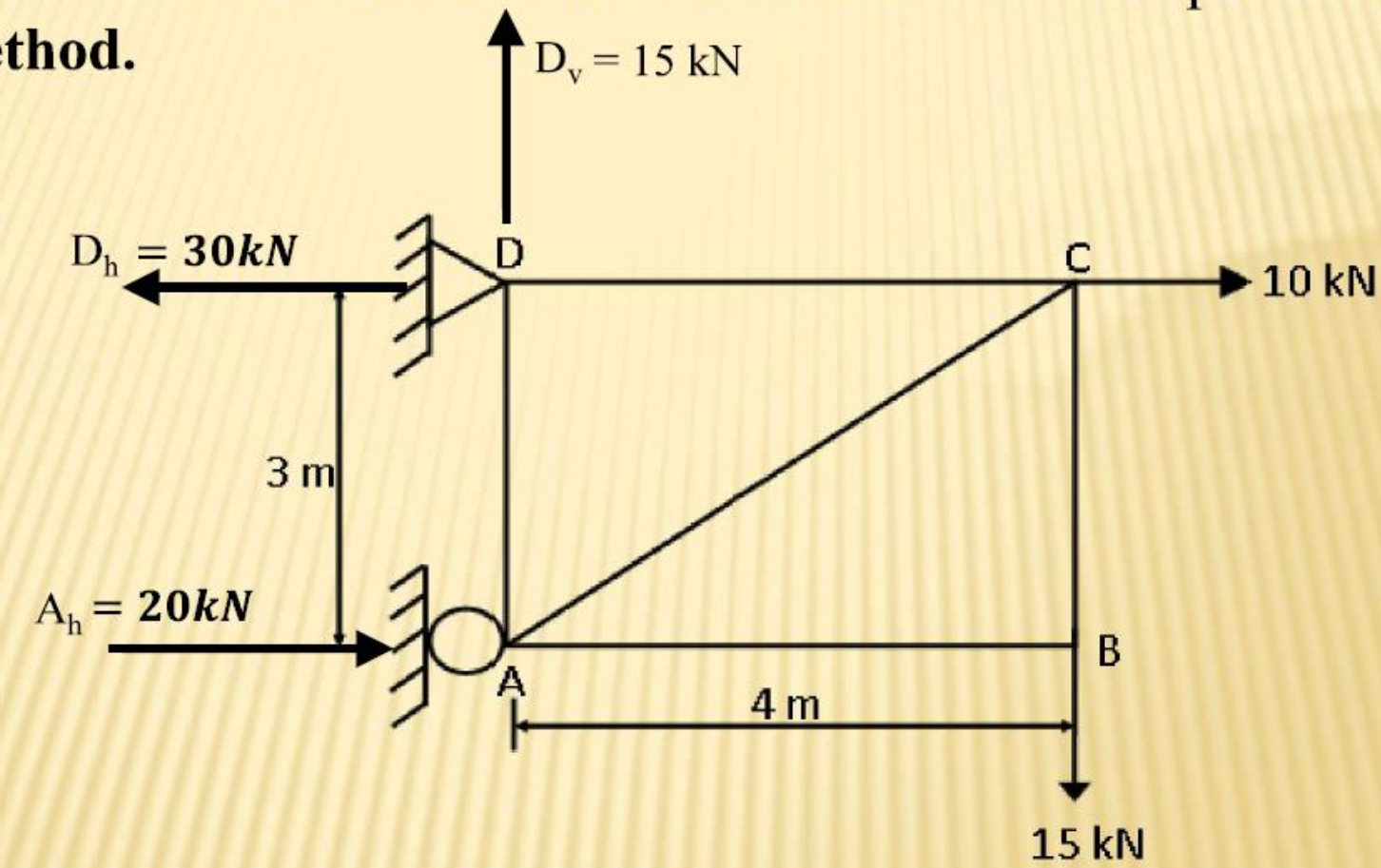

$\sum F_x = 0$

$15 - F_{AC}\frac{3}{5} = 0$ → $F_{AC} = 25$ kN (C)

**Example:** Determine the force in each member of the truss shown in figure below. State if the members are in tension or compression. **Using joint method.**



**Example:** Determine the force in each member of the truss shown in figure below. State if the members are in tension or compression. **Using joint method.**




$\sum F_Y = 0$

- 15 + Dv =

$D_v = 15kN\uparrow$

$\sum M_A = 0$

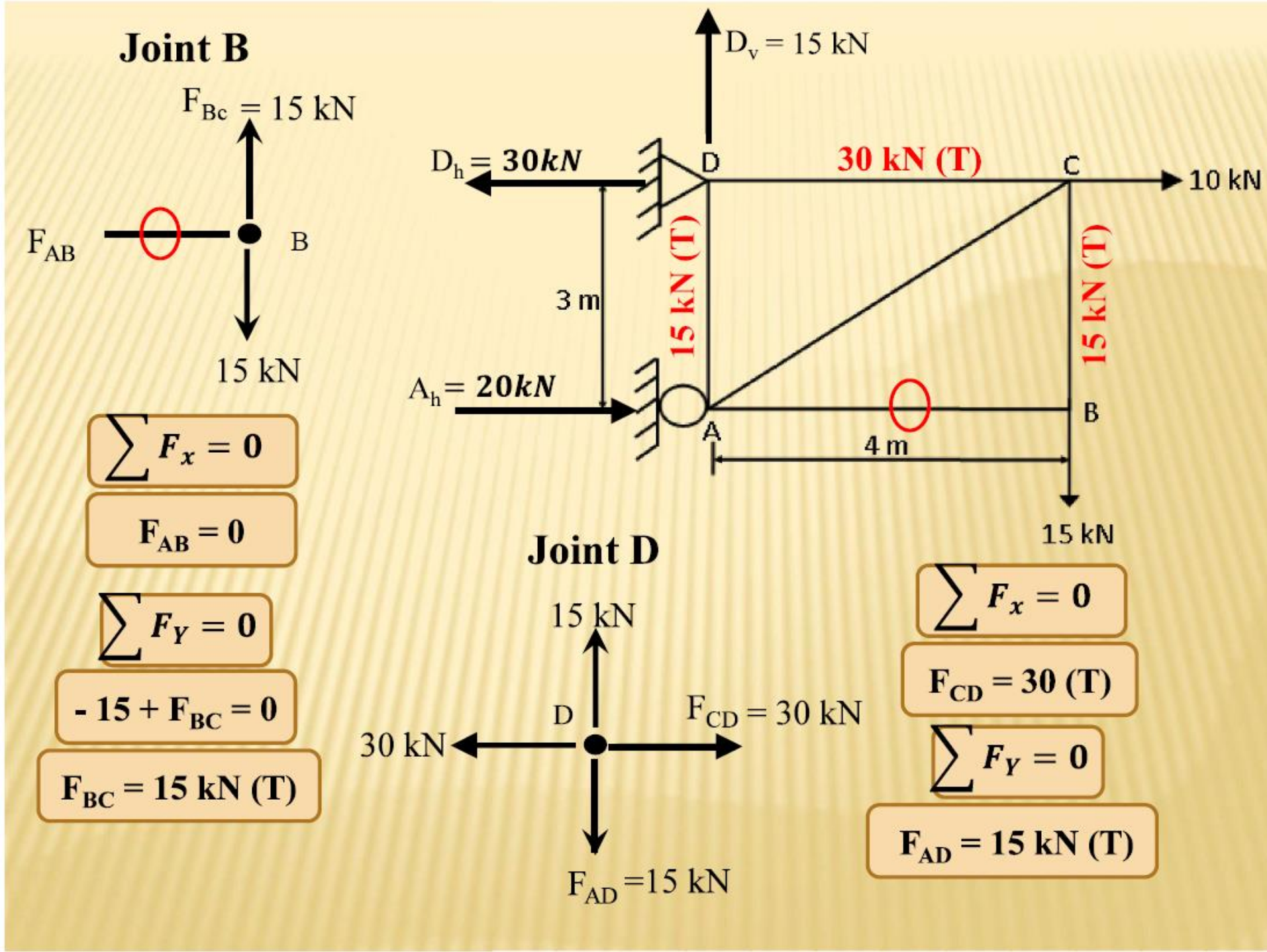
15(4) + 10(3) + $D_h$ (3)= 0

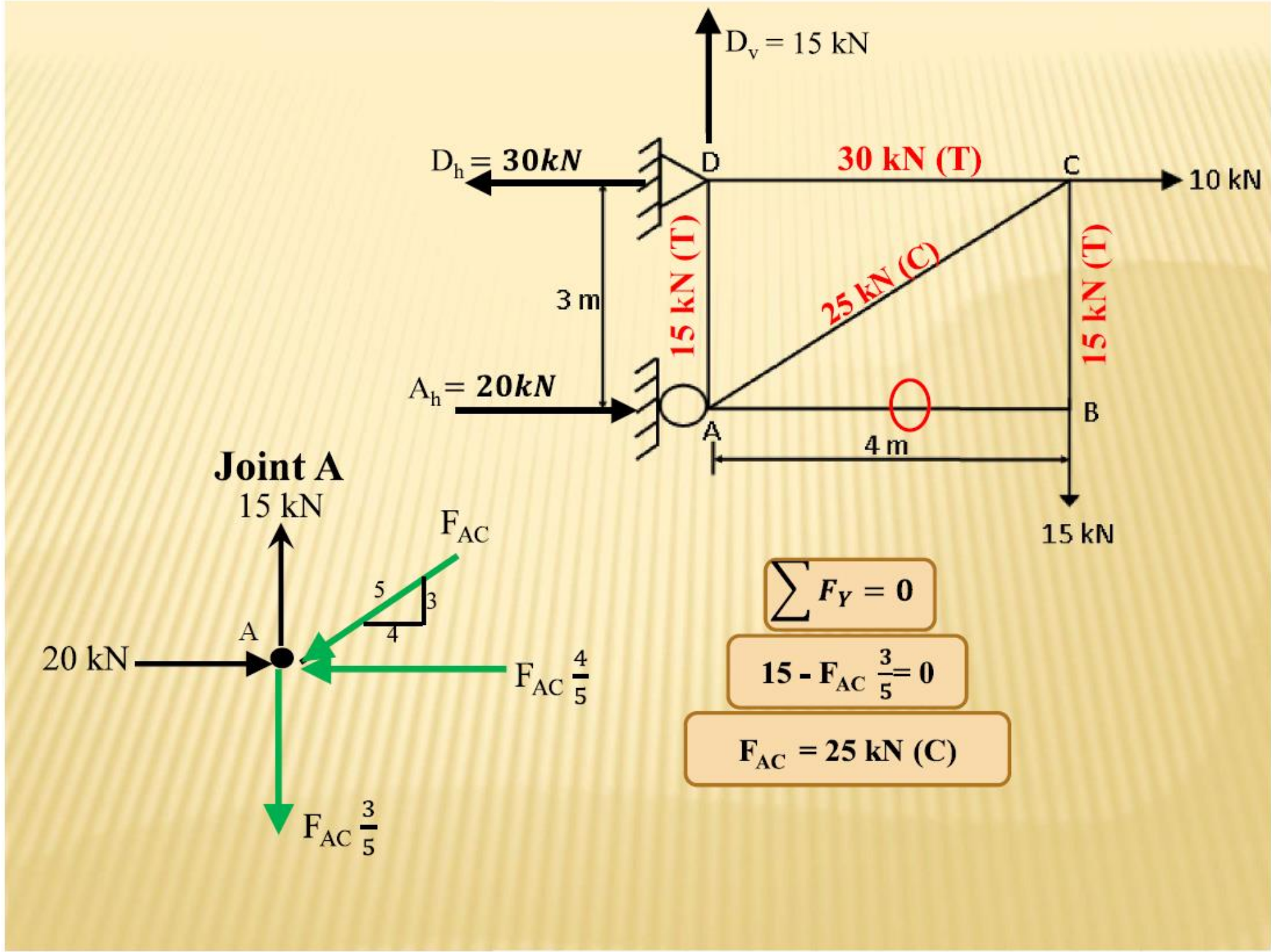
$D_h = -30kN \leftarrow$

$\sum M_D = 0$

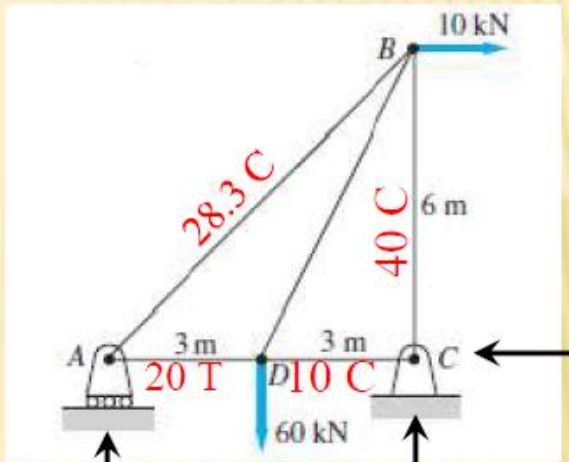
15(4) - $A_h$ (3)= 0

$A_h = 20kN \rightarrow$

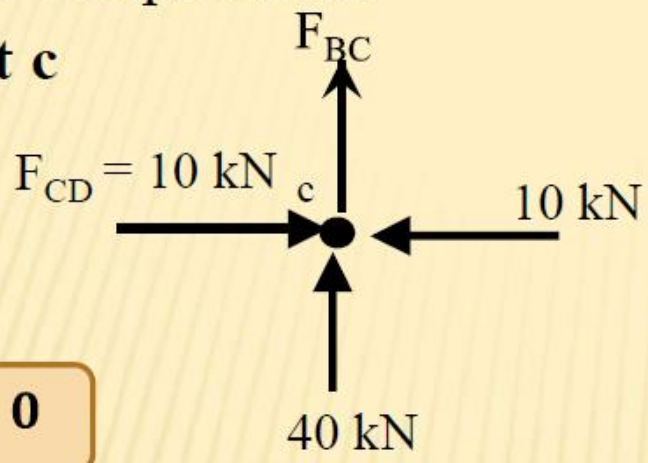

Joint B
$F_{Bc}$ = 15 kN
$F_{AB}$
B
15 kN
$D_v$ = 15 kN
$D_h$ = 30kN
D
30 kN (T)
C
10 kN
3 m
15 kN (T)
15 kN (T)
$A_h$ = 20kN
A
B
4 m
15 kN
$\sum F_x = 0$
$F_{AB} = 0$
$\sum F_Y = 0$
$-15 + F_{BC} = 0$
$F_{BC}$ = 15 kN (T)
Joint D
15 kN
D
$F_{CD}$ = 30 kN
30 kN
$F_{AD}$ =15 kN
$\sum F_x = 0$
$F_{CD}$ = 30 (T)
$\sum F_Y = 0$
$F_{AD}$ = 15 kN (T)


$D_v = 15$ kN
$D_h = 30kN$
D
30 kN (T)
C
10 kN
3 m
15 kN (T)
25 kN (C)
15 kN (T)
$A_h = 20kN$
A
B
4 m
15 kN
Joint A
15 kN
$F_{AC}$
5
3
4
A
20 kN
$F_{AC}\frac{4}{5}$
$F_{AC}\frac{3}{5}$
$\sum F_Y = 0$
$15 - F_{AC}\frac{3}{5} = 0$
$F_{AC} = 25$ kN (C)

**Example**: Using the method of joints, determine the force in each member of the truss shown in figure. Indicate whether the members are in tension or compression.




## Joint c




$$\sum F_x = 0$$

$$-10 + F_{CD} = 0 \Rightarrow F_{CD} = 10 \text{ kN (C)}$$

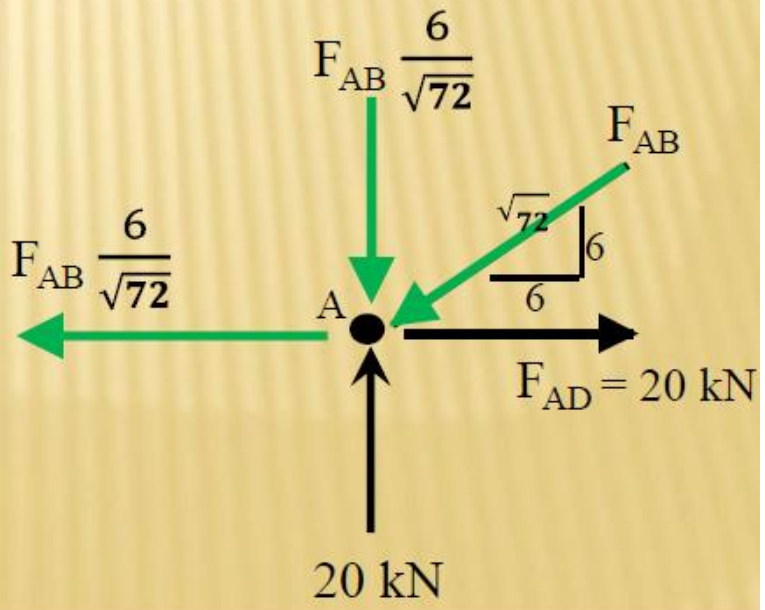
$$\sum F_Y = 0$$

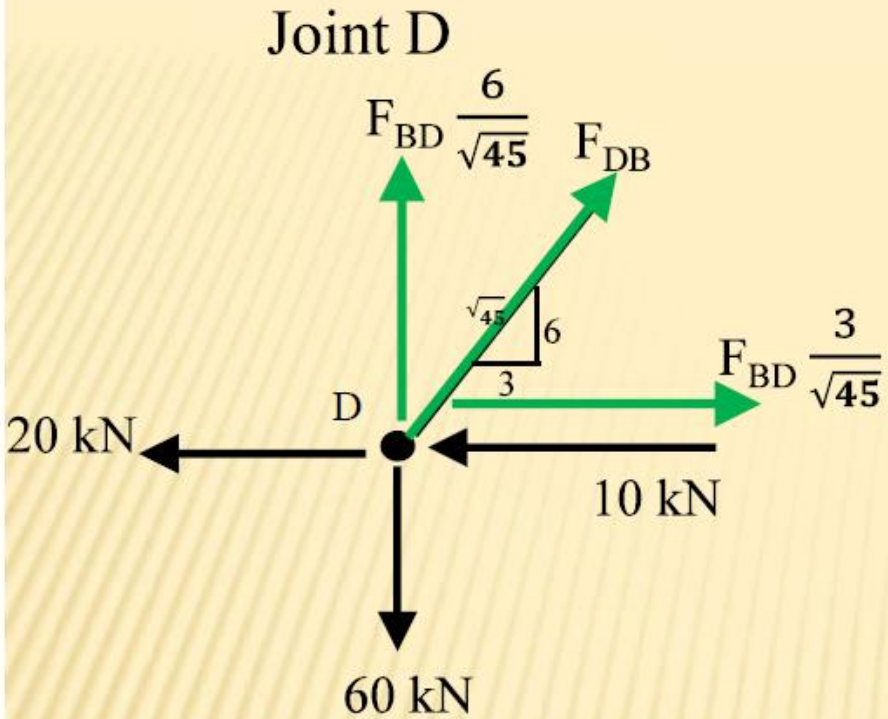
$$40 - F_{BC} = 0 \Rightarrow F_{BC} = 40 \text{ kN (C)}$$

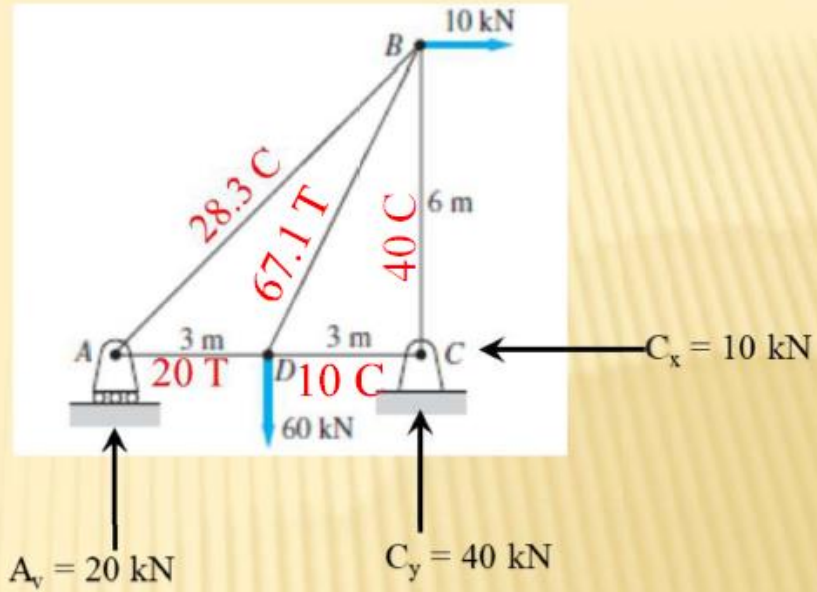
## Joint A

$$\sum F_Y = 0$$

$$20 - F_{AB}\frac{6}{\sqrt{72}} = 0 \Rightarrow F_{AB} = 28.3 \text{ kN (C)}$$



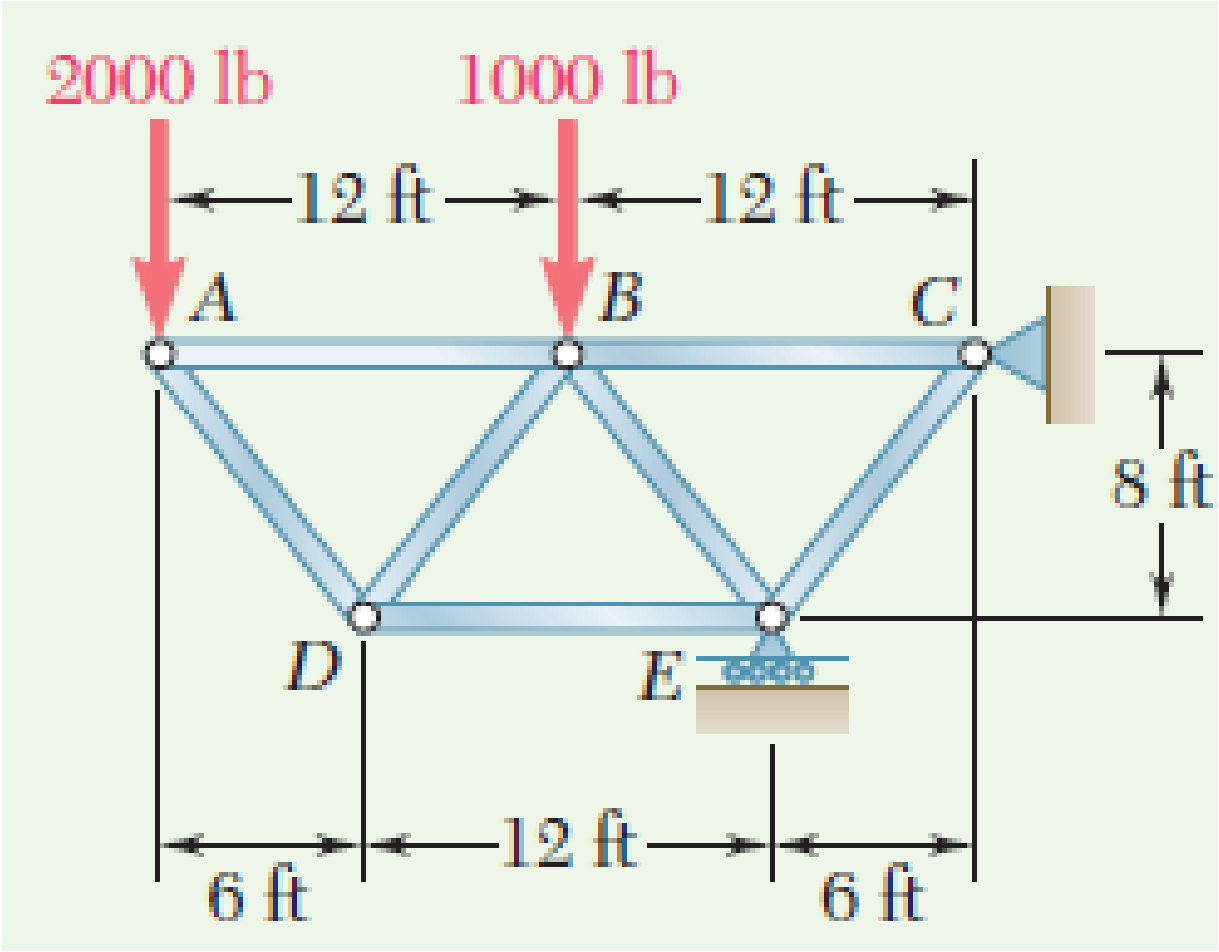



$$\sum F_Y = 0$$

$$-60 + F_{BD}\frac{6}{\sqrt{45}} = 0 \Rightarrow F_{BD} = 67.1 \text{ kN (T)}$$

Using the method of joints, determine the force in each member of the truss shown.



$+\circlearrowleft \Sigma M_C = 0$: $(2000\ \text{lb})(24\ \text{ft}) + (1000\ \text{lb})(12\ \text{ft}) - E(6\ \text{ft}) = 0$

$E = +10{,}000\ \text{lb}$ $\qquad \mathbf{E = 10{,}000\ lb\uparrow}$

$\overset{+}{\rightarrow} \Sigma F_x = 0$: $\qquad \mathbf{C_x = 0}$

$+\uparrow \Sigma F_y = 0$: $-2000\ \text{lb} - 1000\ \text{lb} + 10{,}000\ \text{lb} + C_y = 0$

$C_y = -7000\ \text{lb}$ $\qquad \mathbf{C_y = 7000\ lb\downarrow}$

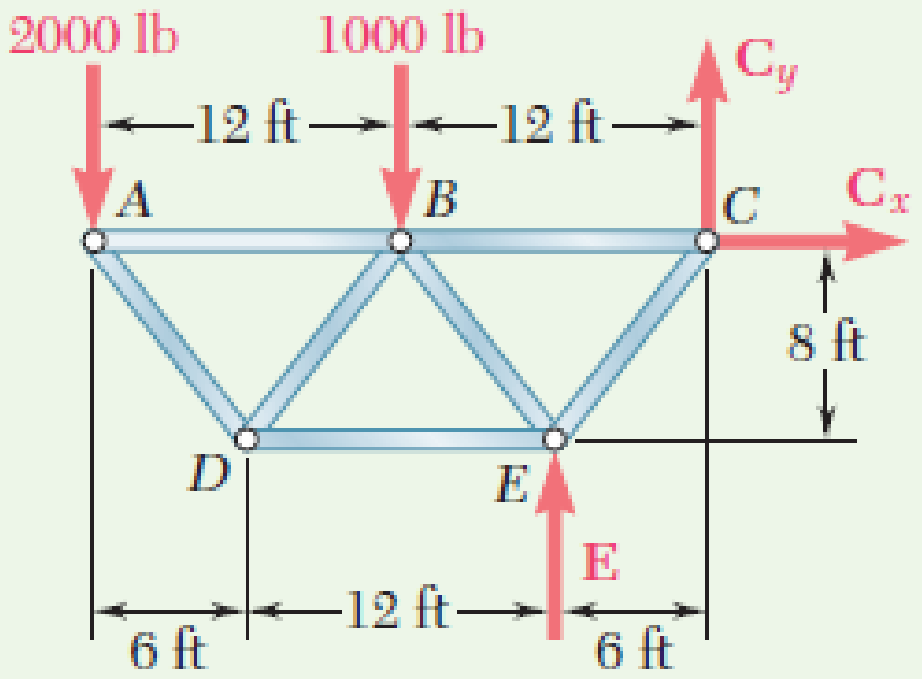



**Fig. 1** Free-body diagram of entire truss.

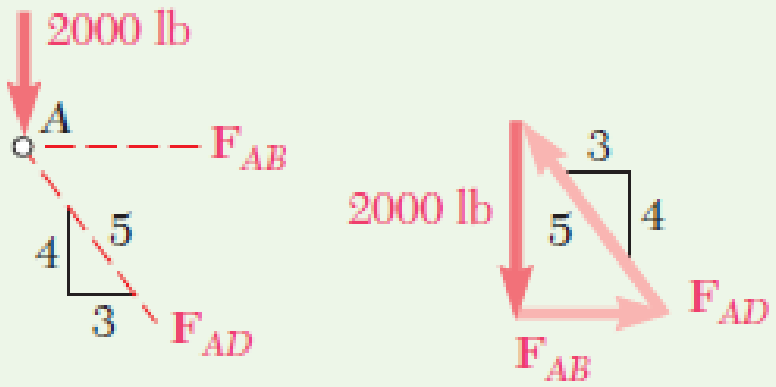



Fig. 2 Free-body diagram of joint *A*.

$$\frac{2000 \text{ lb}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$F_{AB} = 1500 \text{ lb } T$$
$$F_{AD} = 2500 \text{ lb } C$$

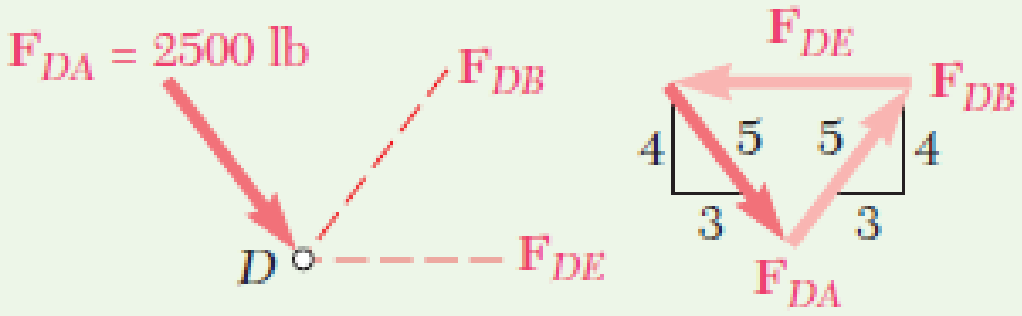



Fig. 3 Free-body diagram of joint *D*.

$$F_{DB} = F_{DA}$$
$$F_{DE} = 2\left(\tfrac{3}{5}\right)F_{DA}$$

$$F_{DB} = 2500 \text{ lb } T$$
$$F_{DE} = 3000 \text{ lb } C$$

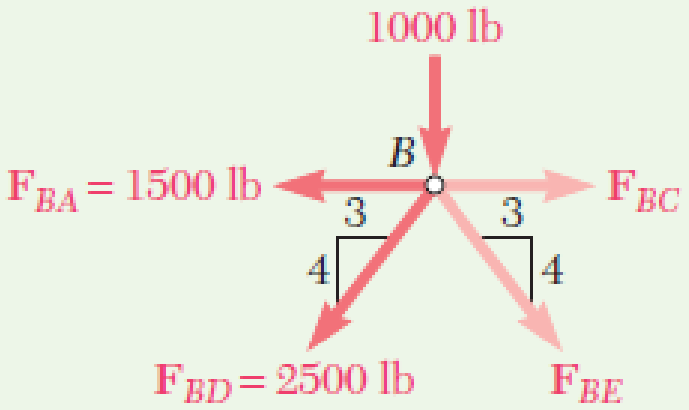



**Fig. 4** Free-body diagram of joint *B*.

$$+\uparrow \Sigma F_y = 0: \quad -1000 - \tfrac{4}{5}(2500) - \tfrac{4}{5}F_{BE} = 0$$

$$F_{BE} = -3750 \text{ lb} \qquad F_{BE} = 3750 \text{ lb } C$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad F_{BC} - 1500 - \tfrac{3}{5}(2500) - \tfrac{3}{5}(3750) = 0$$

$$F_{BC} = +5250 \text{ lb} \qquad F_{BC} = 5250 \text{ lb } T$$

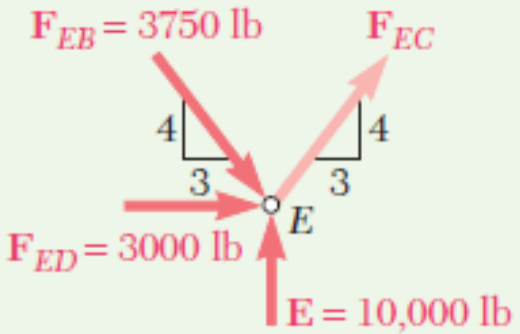



**Fig. 5** Free-body diagram of joint *E*.

**Joint *E*.** Assume the unknown force $\mathbf{F}_{EC}$ acts away from the joint (Fig. 5). Summing $x$ components, you obtain

$$\overset{+}{\rightarrow}\Sigma F_x = 0: \qquad \tfrac{3}{5}F_{EC} + 3000 + \tfrac{3}{5}(3750) = 0$$

$$F_{EC} = -8750 \text{ lb} \qquad F_{EC} = 8750 \text{ lb } C$$

Summing $y$ components, you obtain a check of your computations:

$$+\uparrow\Sigma F_y = 10{,}000 - \tfrac{4}{5}(3750) - \tfrac{4}{5}(8750)$$

$$= 10{,}000 - 3000 - 7000 = 0 \qquad \text{(checks)}$$

$$\overset{+}{\rightarrow}\Sigma F_x = -5250 + \tfrac{3}{5}(8750) = -5250 + 5250 = 0 \qquad \text{(checks)}$$

$$+\uparrow\Sigma F_y = -7000 + \tfrac{4}{5}(8750) = -7000 + 7000 = 0 \qquad \text{(checks)}$$

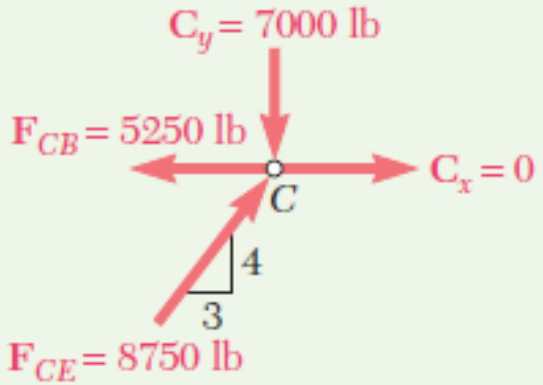



**Fig. 6** Free-body diagram of joint *C*.

1. Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

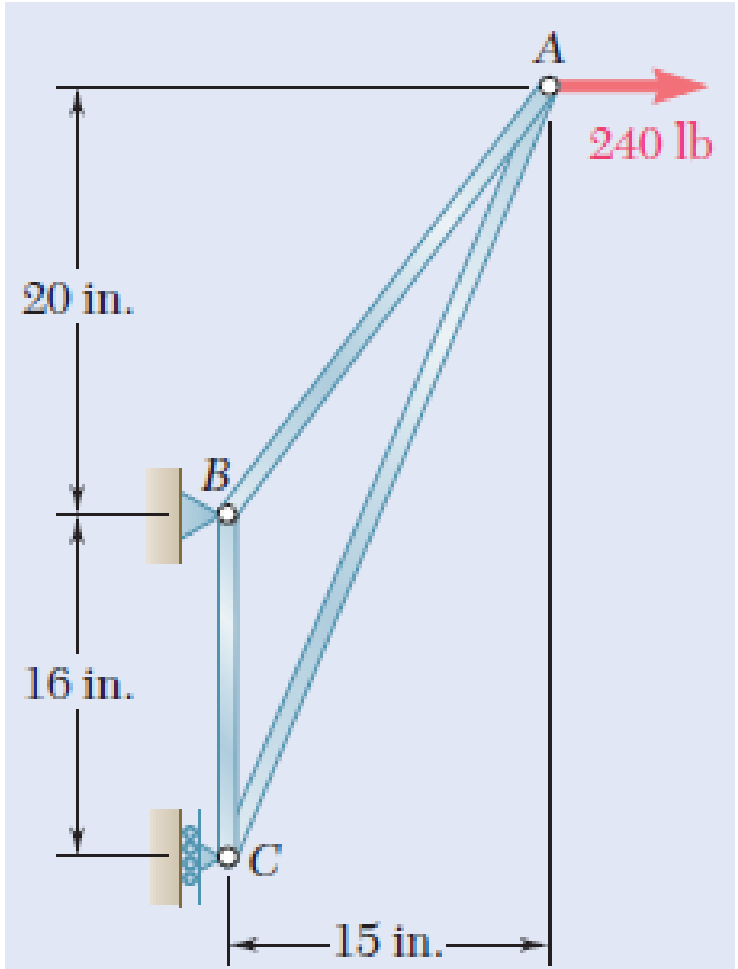



Fig. P6.1

6.1 $F_{AB} = 900$ lb $T$; $F_{AC} = 780$ lb $C$; $F_{BC} = 720$ lb $T$.

2. Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

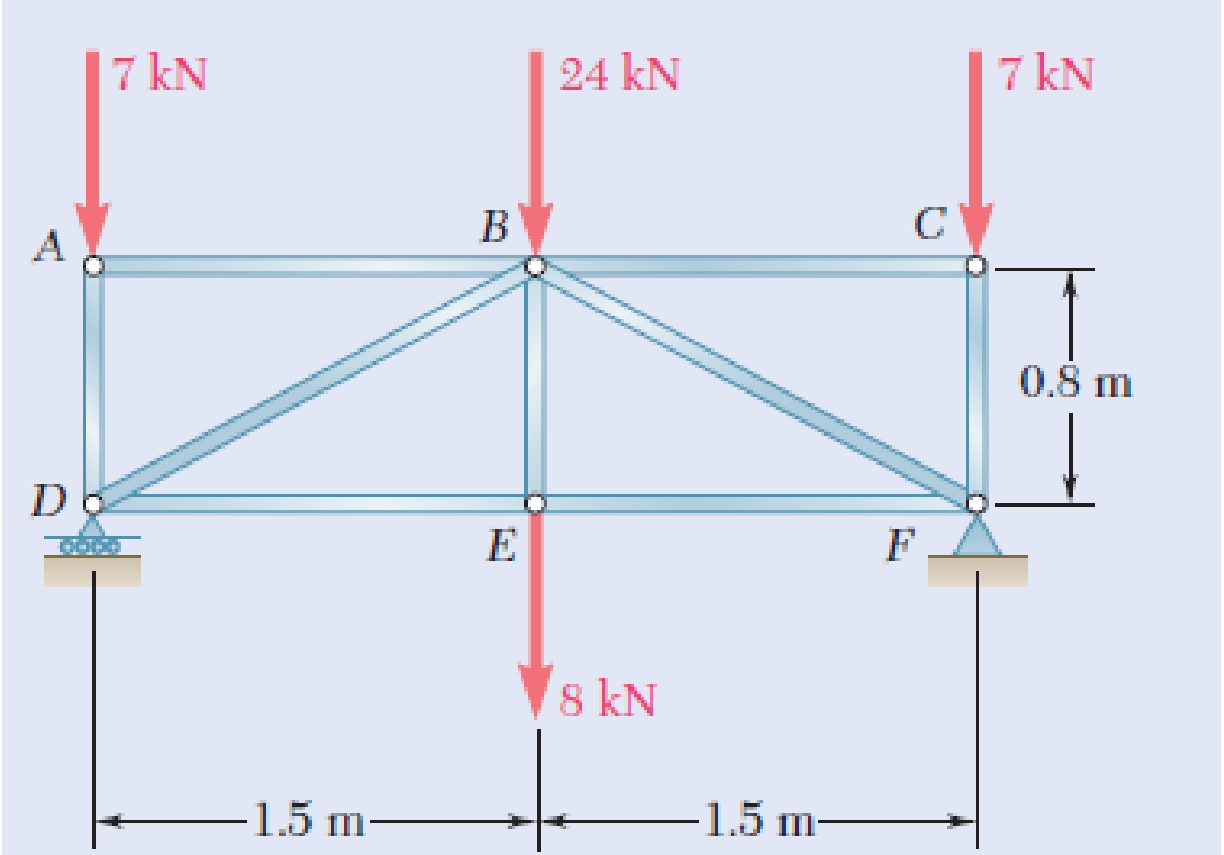



Fig. P6.4

**6.4** $F_{AB} = F_{BC} = 0$; $F_{AD} = F_{CF} = 7.00$ kN $C$; $F_{BD} = F_{BF} = 34.0$ kN $C$; $F_{BE} = 8.00$ kN $T$; $F_{DE} = F_{EF} = 30.0$ kN $T$.

3. Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

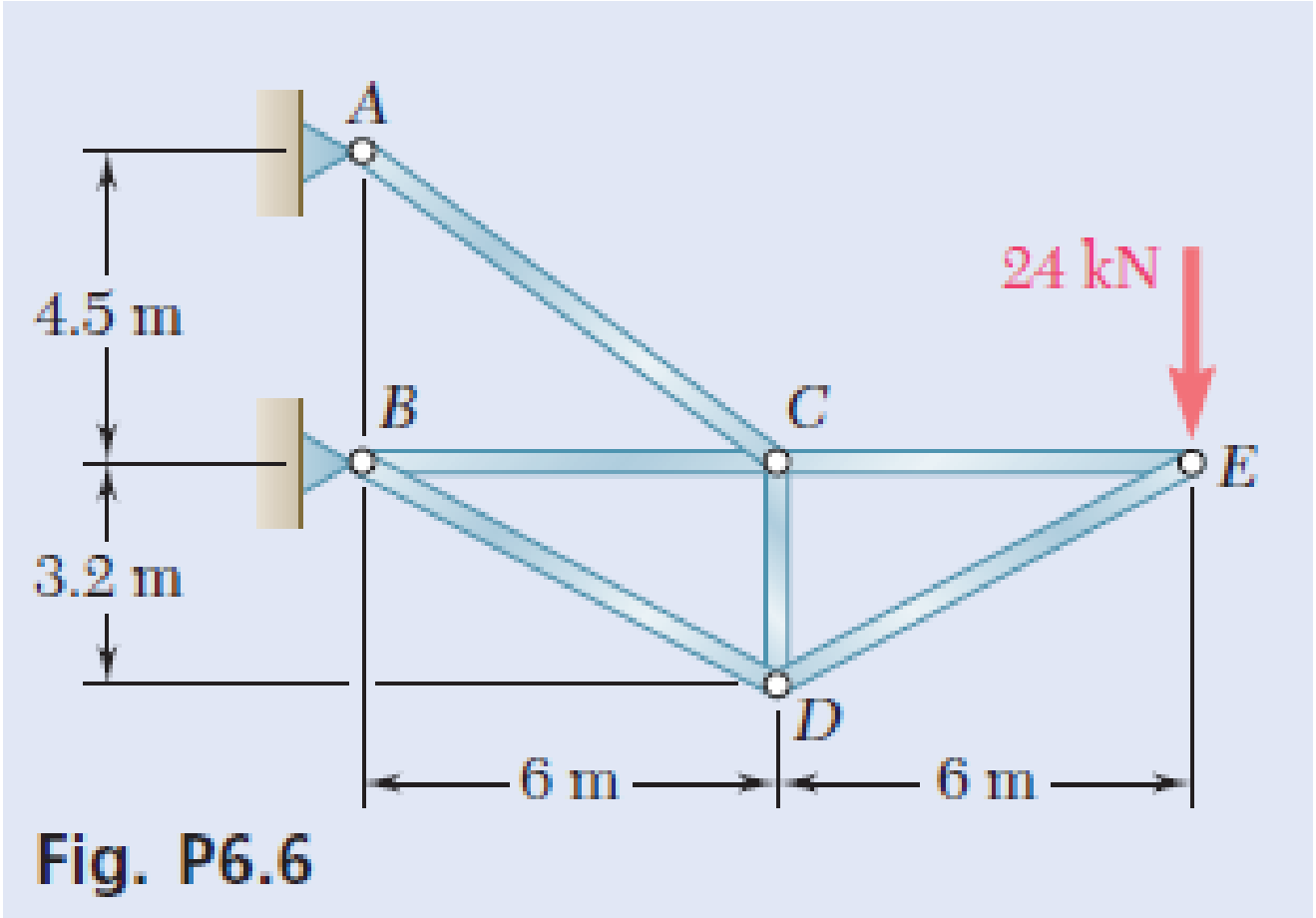



Fig. P6.6

**6.6** $F_{AC} = 80.0$ kN $T$; $F_{CE} = 45.0$ kN $T$; $F_{DE} = 51.0$ kN $C$;
$F_{BD} = 51.0$ kN $C$; $F_{CD} = 48.0$ kN $T$; $F_{BC}$ 19.00 kN $C$.

# Method of Sections

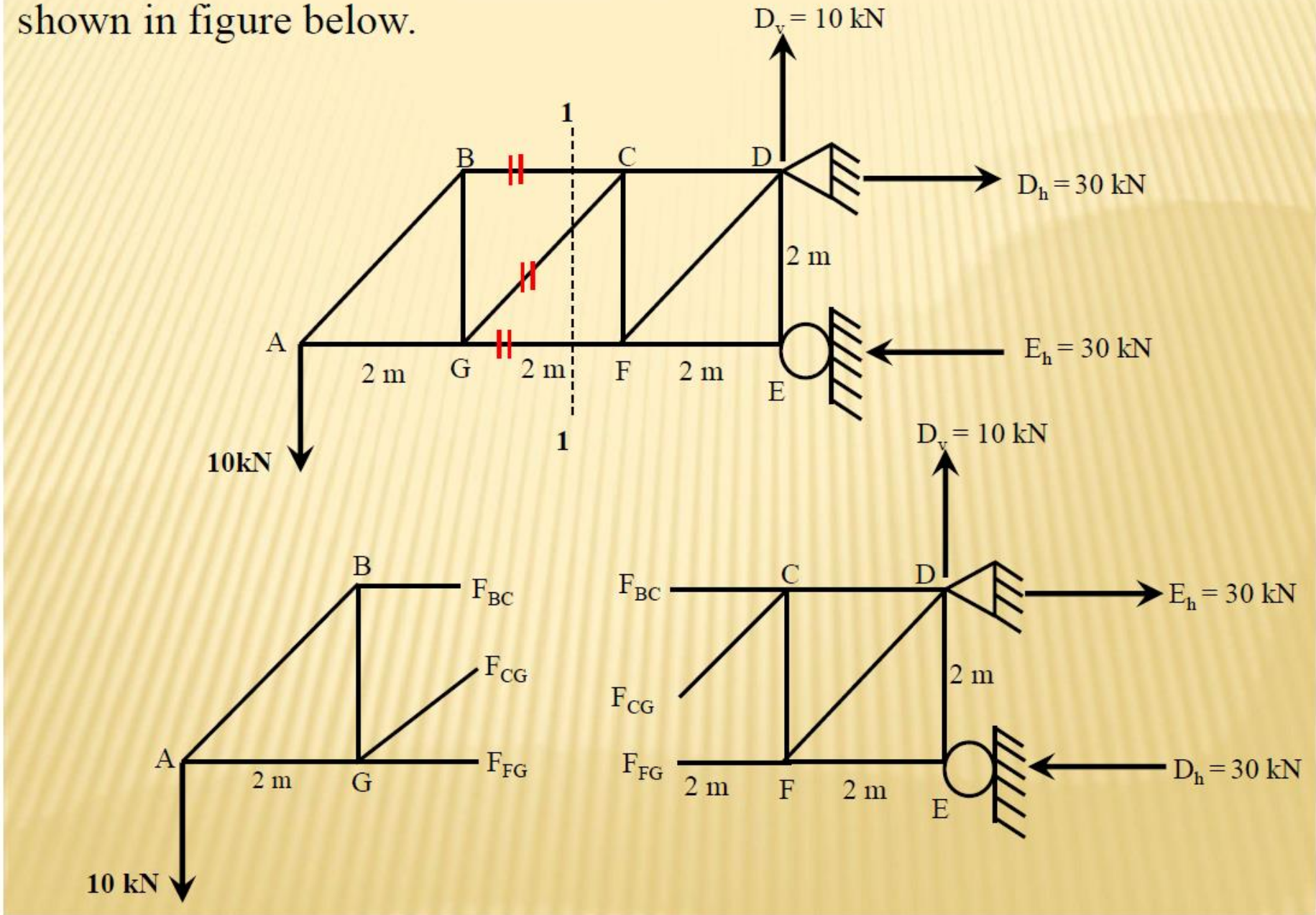
The method involves dividing the truss into sections by cutting through the selected members and analyzing the section as a rigid body. The advantage of the Method of Sections is that the only internal member forces exposed are those which you have cut through, the remaining internal forces are not exposed and thus ignored.



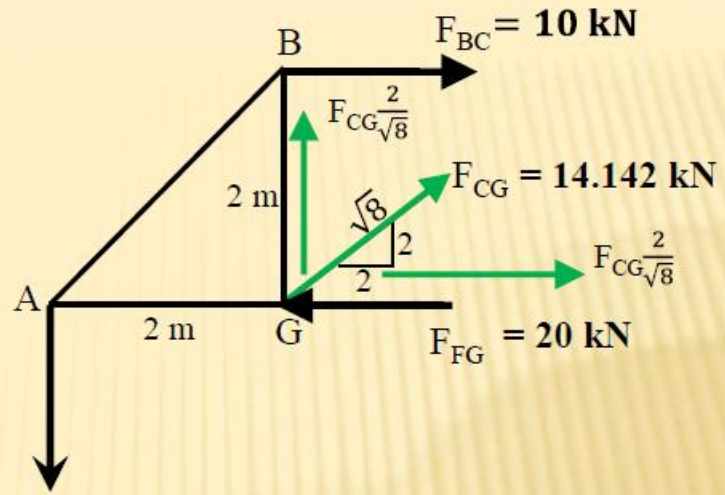
Method of Sections can be used to determine **three unknowns** member forces per FBD since all **three equations** can be used.

$$\sum F_x = 0 \qquad \sum F_Y = 0 \qquad \sum M_o = 0$$

**Example:** Determine the force in members **BC**, **CG**, and **FG** of the truss shown in figure below.



## Section 1 – 1 left




$$\sum F_Y = 0$$

$$-10 + F_{CG}\frac{2}{\sqrt{8}} = 0 \Rightarrow F_{CG} = 14.142 \text{ kN (T)}$$

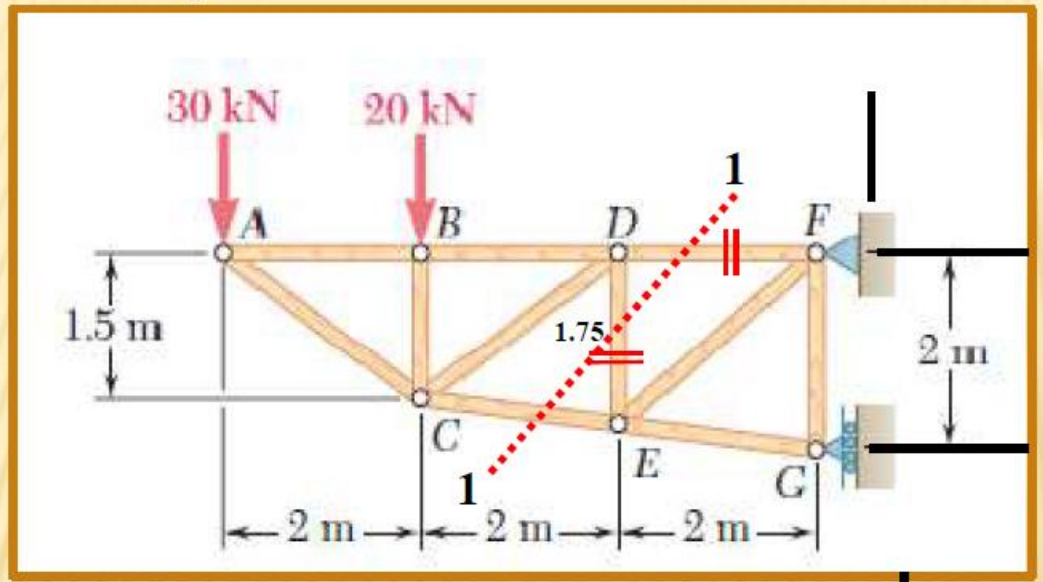
$$\sum M_G = 0$$

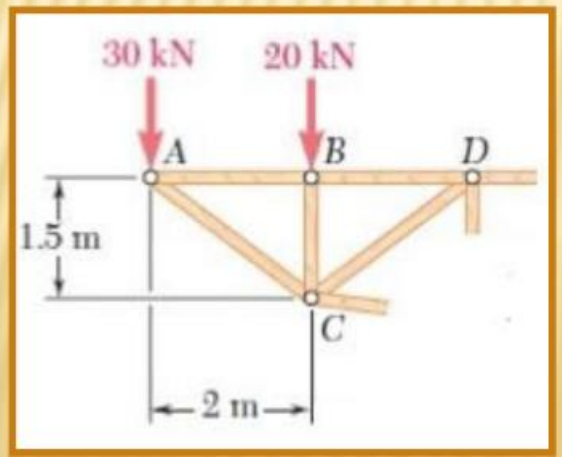
$$-10\,(2) + F_{BC}\,(2) = 0 \Rightarrow F_{BC} = 10 \text{ kN (T)}$$

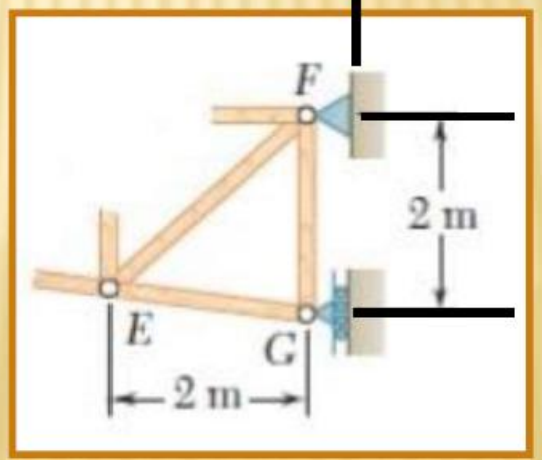
$$\sum F_X = 0$$

$$10 + 14.142\frac{2}{\sqrt{8}} + F_{FG} = 0 \Rightarrow F_{FG} = -20 \text{ kN (C)}$$

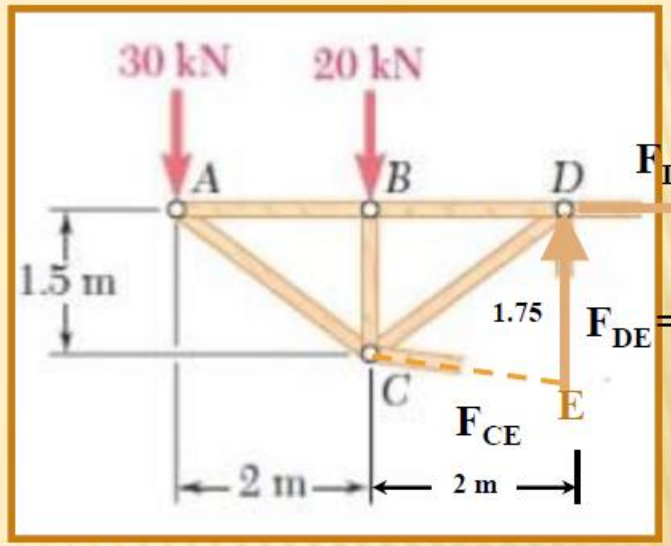
**Example:** Determine the force in members ***DF*** and ***DE*** of the truss shown in figure. **Using section method**. State whether each member is in tension or compression.



## Section 1 – 1 left



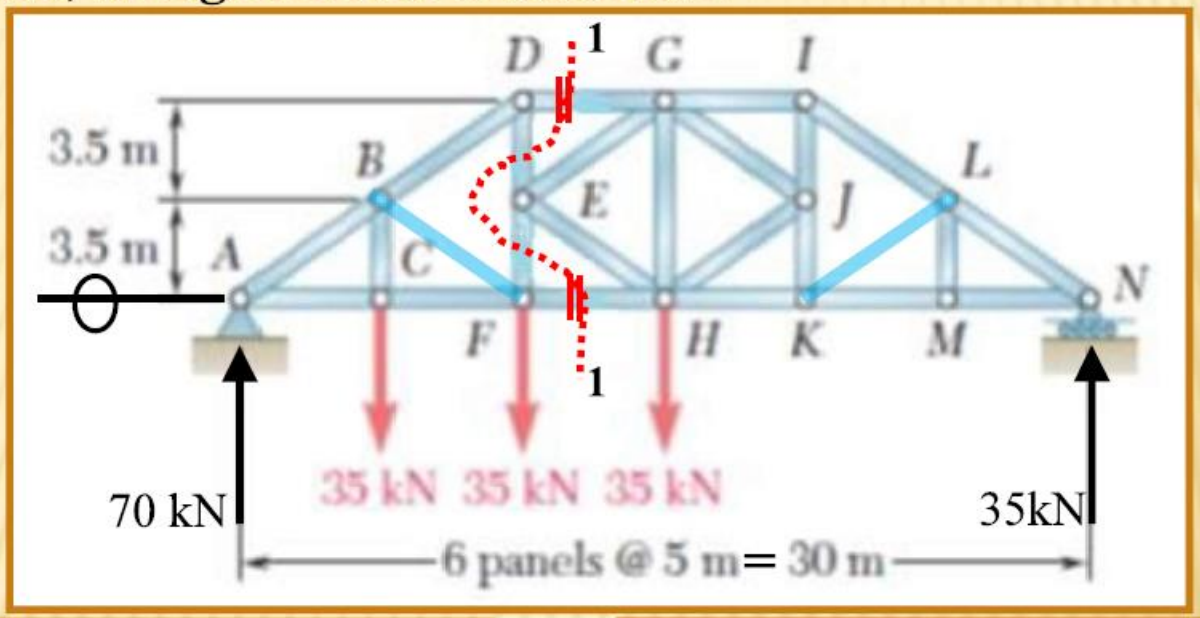

$$\sum M_E = 0$$

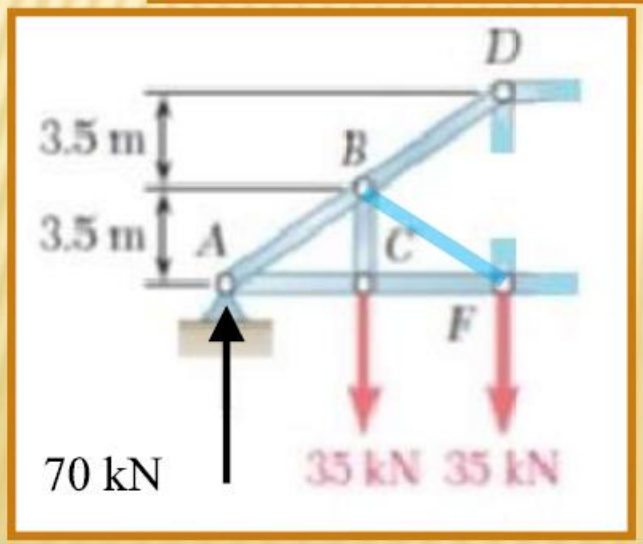
$$-20(2) - 30(4) + F_{DF}(1.75) = 0 \Rightarrow F_{DF} = 91.429 \text{ kN (T)}$$

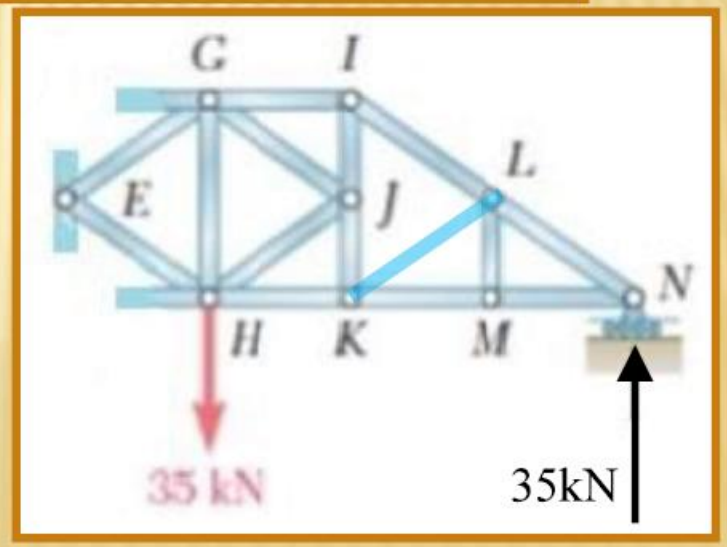
$$\sum M_C = 0$$

$$91.429(1.5) - 30(2) + F_{DE}(2) = 0 \Rightarrow F_{DE} = -38.572 \text{ kN (C)}$$

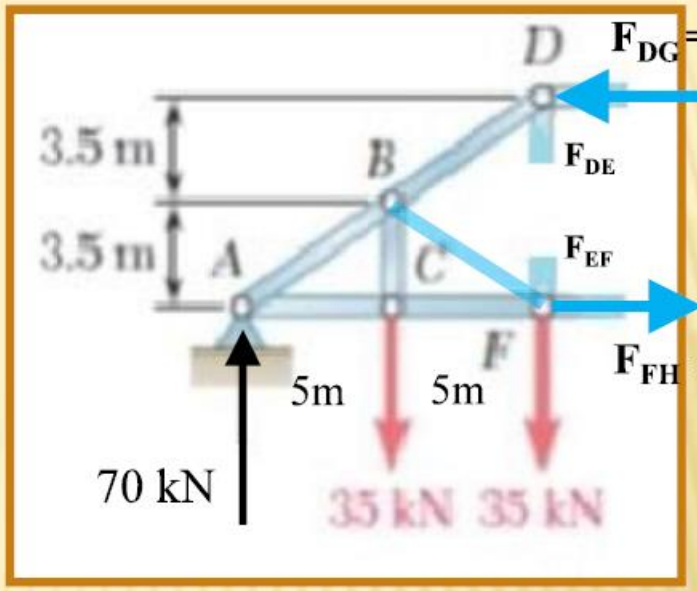
**Example:** Determine the force in members ***DG*** and ***FH*** of the truss shown in figure, **using method of sections** .



## Section 1 – 1 Left



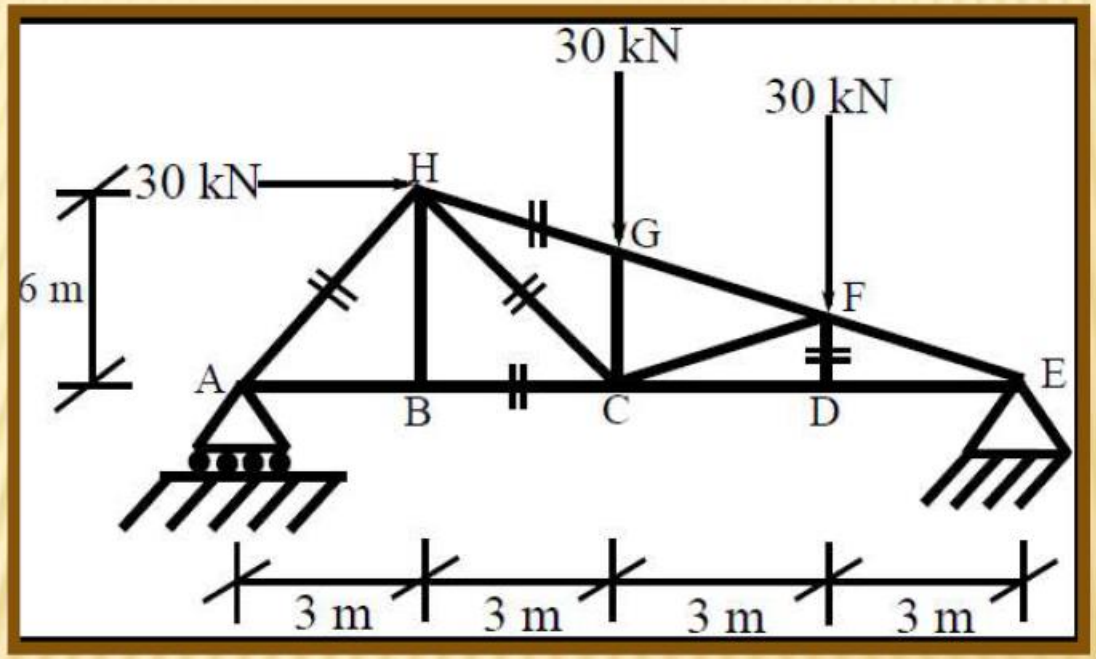

$$\sum M_F = 0$$

$$70\,(10) - 35\,(5) + F_{DG}\,(7) = 0 \Rightarrow F_{DG} = -75 \text{ kN (C)}$$

$$\sum F_X = 0$$

$$-75 + F_{FH} = 0 \Rightarrow F_{FH} = 75 \text{ kN (T)}$$

**Example:** Compute the force in members (AH, BC, CH, GH, and DF) of the truss shown in figure below. State if the members are in tension or compression.



# Frames

## الهياكل

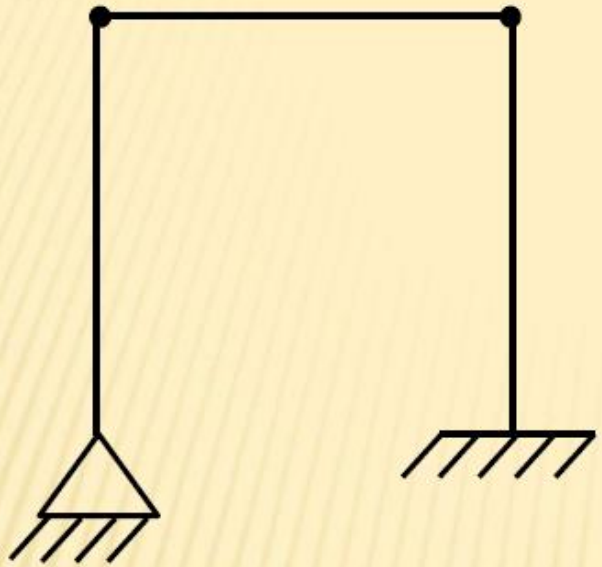
**Frame:** Is composed of several connected members that are either fixed (rigid) or pin (smooth) connected at their ends.

fixed (rigid) → Three Forces

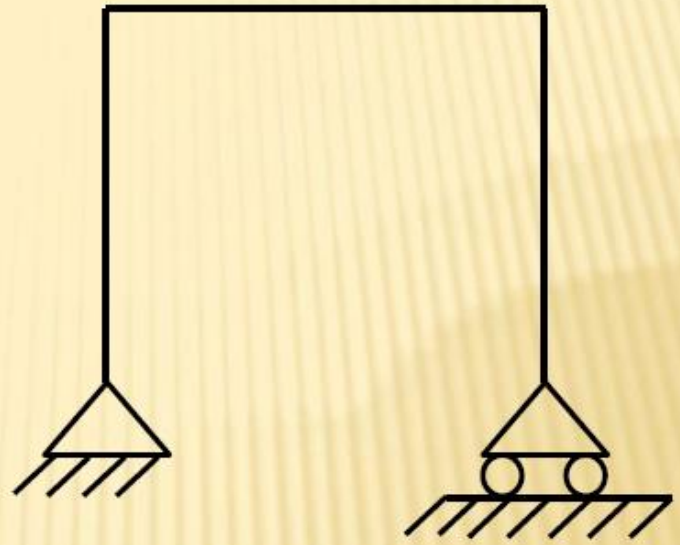
pin (smooth) → Two Forces

# difference between trusses and frames

| Truss | Frame |
| --- | --- |
| • In truss forces act only along the axis of the axis the members. Members are having transverse tension or compression. | • In frames forces are acting along the axis the members. . In addition to transverse forces. |
| • Each member is acted upon by two equal and opposite forces frame having line of action forces along the Centre of members. i.e. every member of truss is a two force member. | • One or more than one member of is subjected to more than two forces member. (Multiple force members). |
| • Member does not bend. | • Members may bend/may not bend. |
| • Forces are applied at the joints only. | • Forces may act anywhere on the member. |



Smooth Frame



Rigid Frame

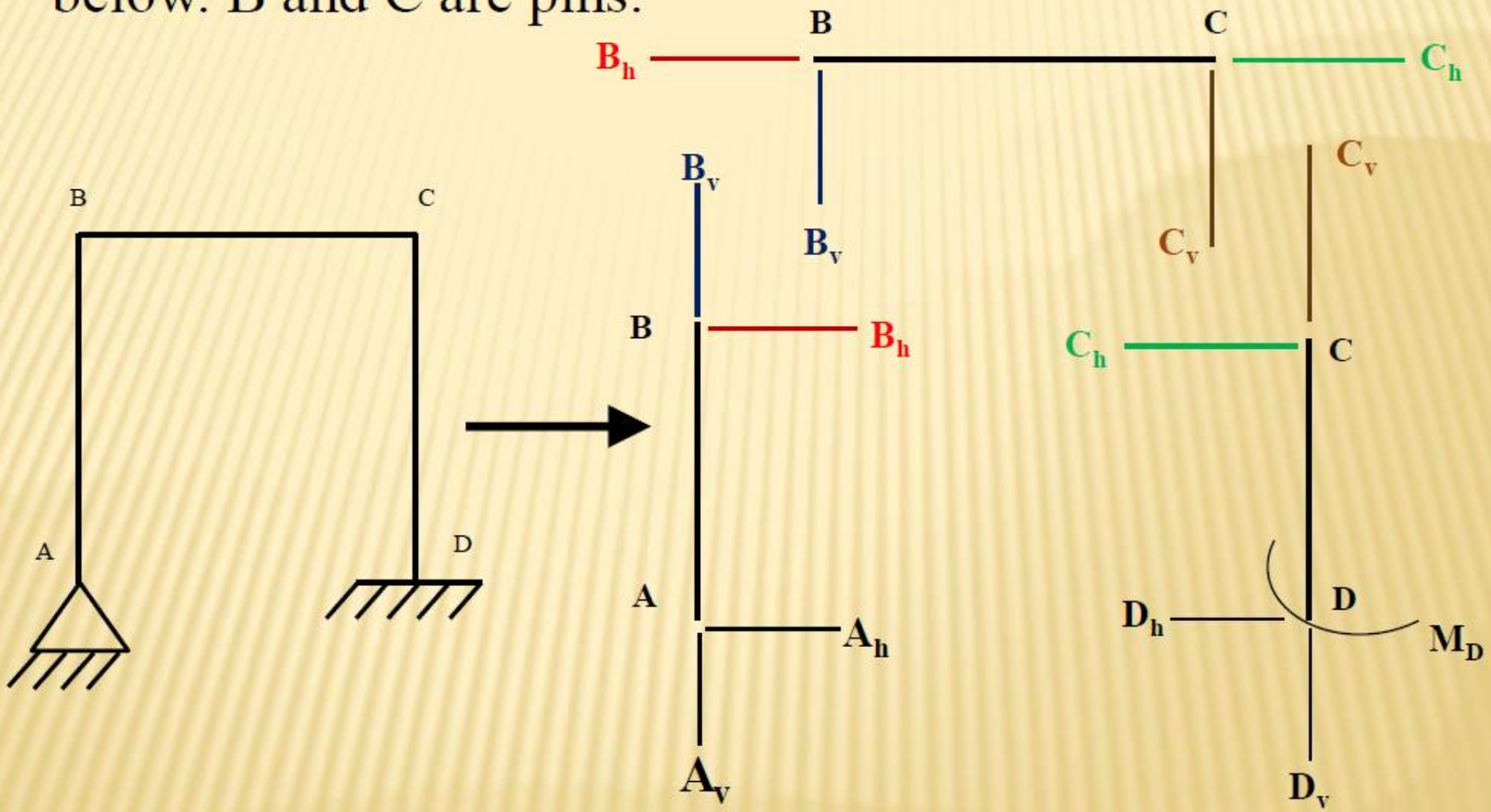
**Analysis of frames: Using Method of member**

**1**. Compute the reactions from the all frame (if possible).

**2**. Draw the Free Body Diagram (FBD).

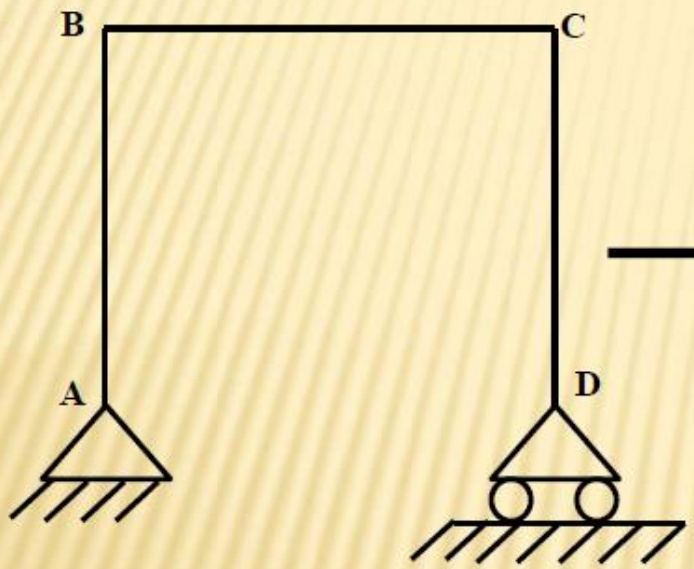
**3**. Apply the equilibrium equations on each member. ($\sum F_x=0$ , ($\sum F_y=0$ , ($\sum M=0$).

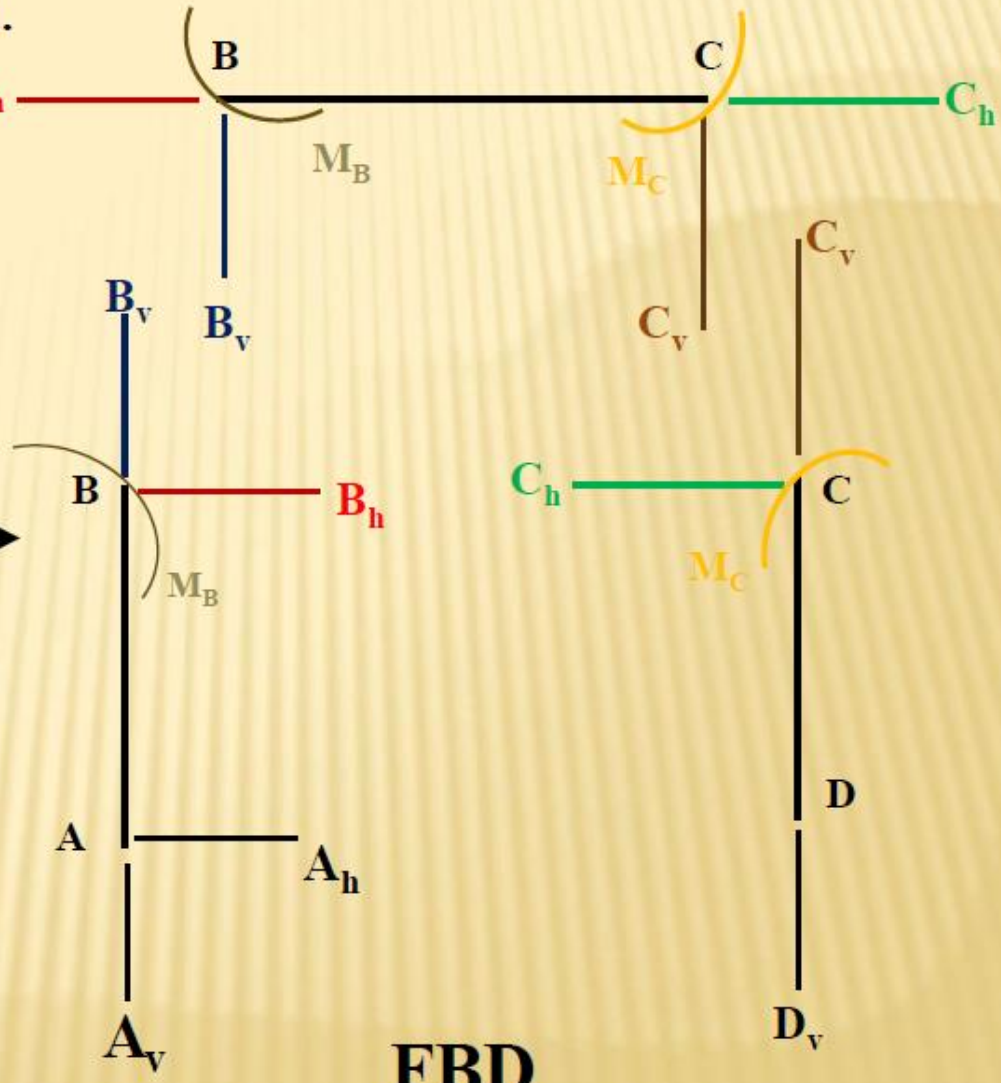
**Example**: Draw the FBD of the frame shown in figure below. B and C are pins.




**FBD**

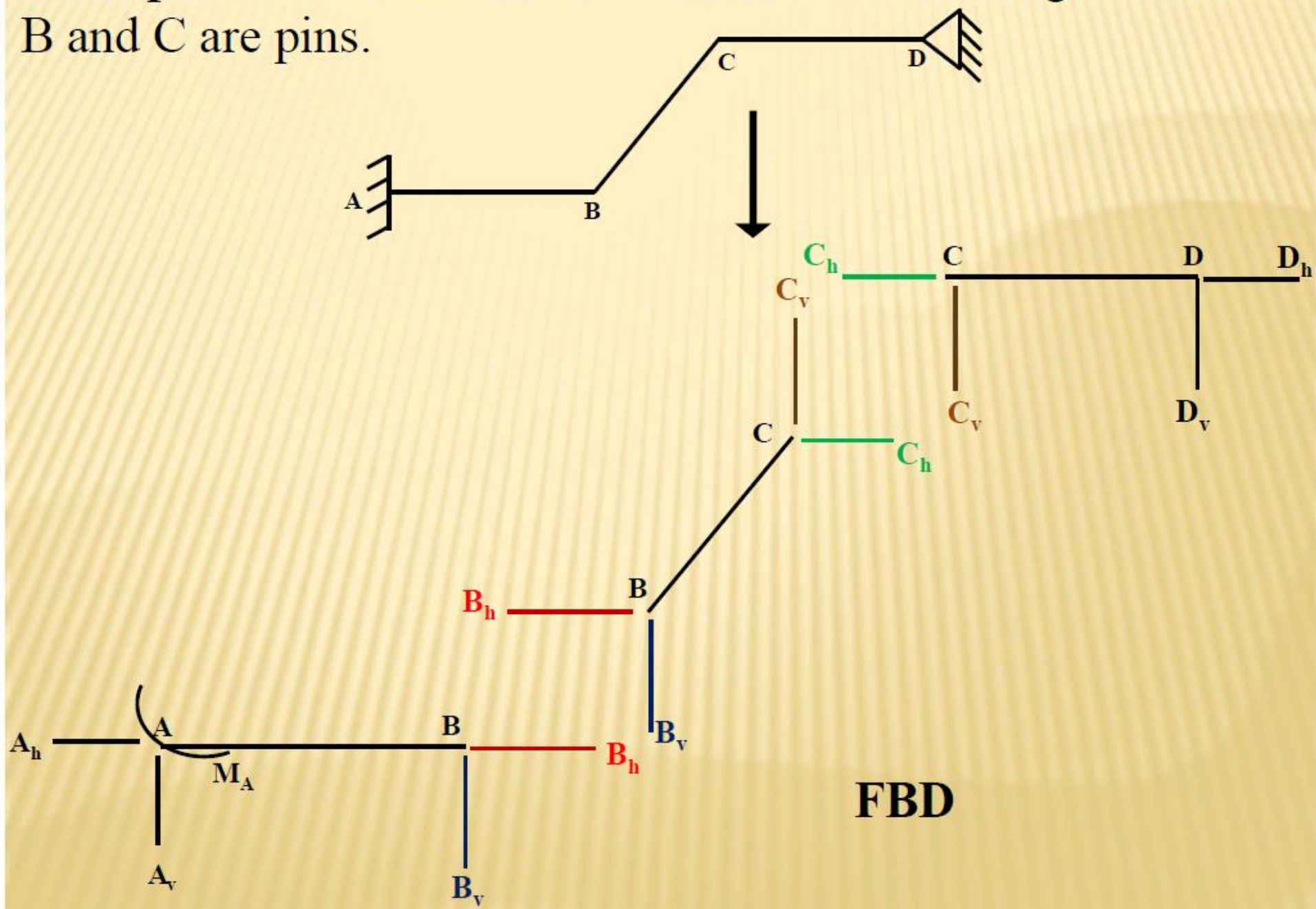
**Example**: Draw the FBD of the frame shown in figure below. B and C are rigid.



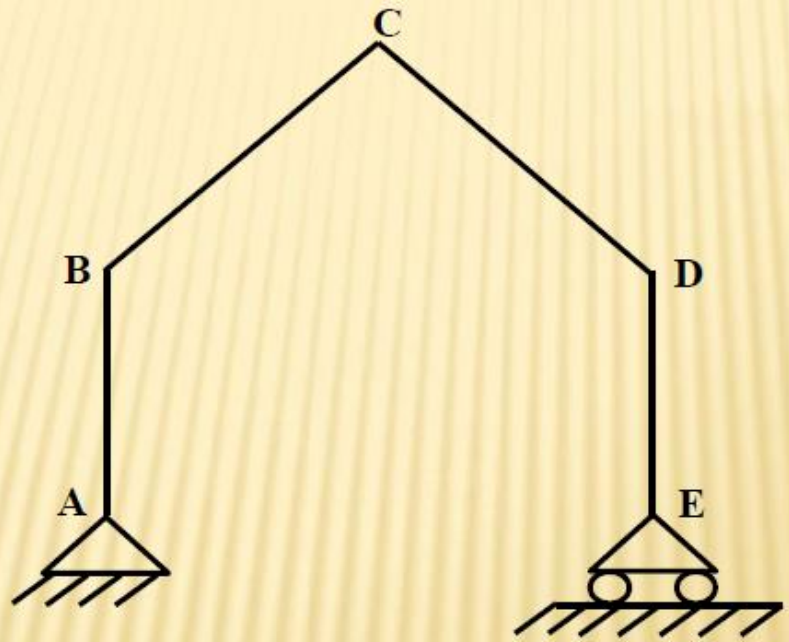

FBD

**Example**: Draw the FBD of the frame shown in figure below. B and C are pins.



# H.W.

Draw the FBD of the frame shown in figure below. B, C and D are rigid.



# Analysis of Frames
# Method of Member

- Compute the reactions from the all frame (if possible).
- Draw the Free Body Diagram (FBD).
- Apply the equilibrium equations on each member. ($\sum F_x=0$, ($\sum F_y=0$, ($\sum M=0$).

**Example:** For the frame shown in figure below, calculate the forces in all members. Joints B and C are rigid connected.

Calculate the reactions

$$\sum M@A = 0$$

$$40(2) + 20(3) - D_v(4) = 0$$

$$\rightarrow D_v = 35\ kN \uparrow$$

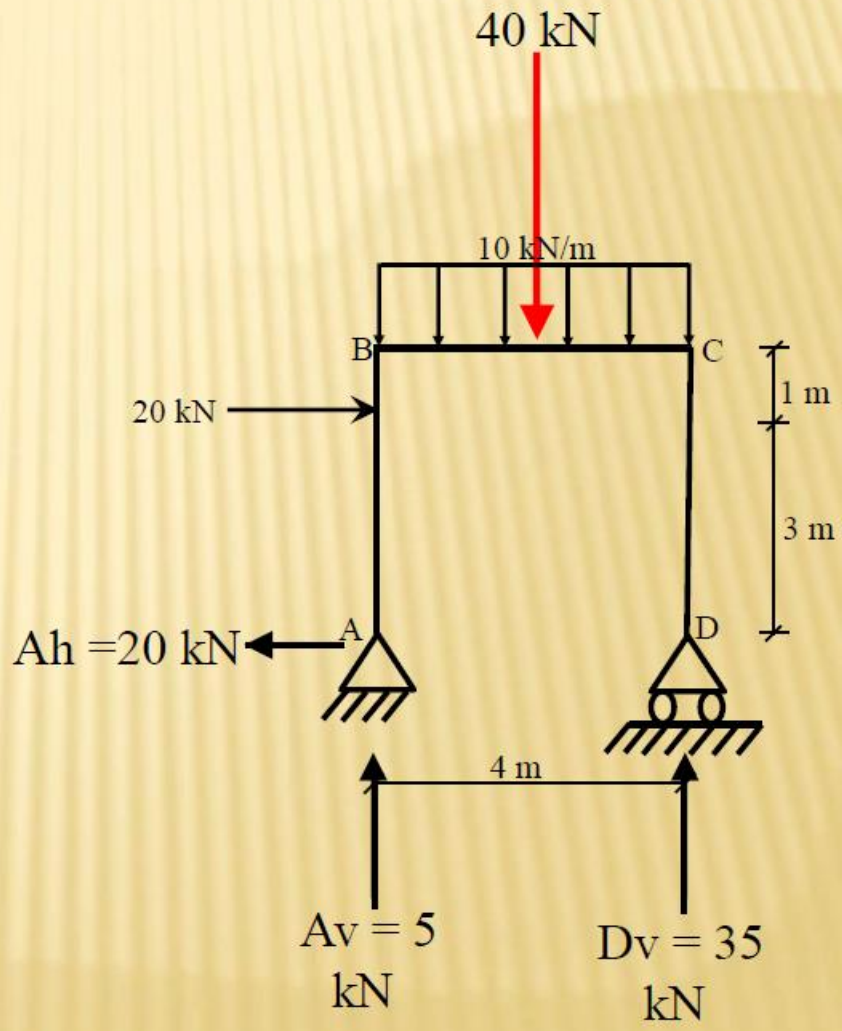
$$\sum M@D = 0$$

$$-40(2) + 20(3) + A_v(4) = 0$$

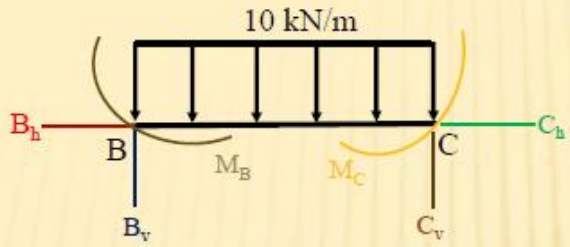
$$\rightarrow A_v = 5\ kN \uparrow$$

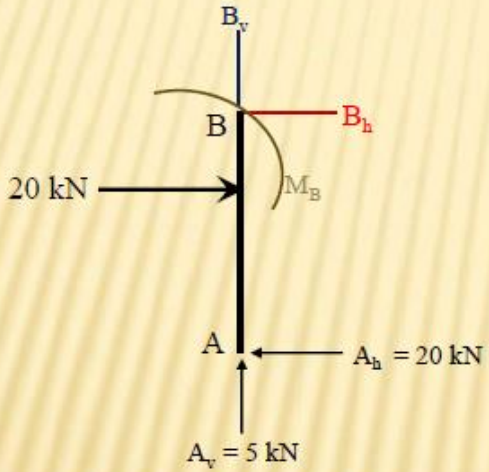
$$\sum f_x = 0$$

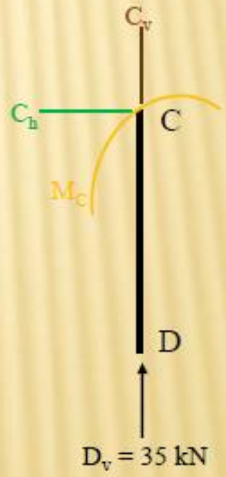
$$20 - A_h = 0 \rightarrow A_h = 20\ kN \leftarrow$$



- Draw FBD of each member:




- Apply the equations of equilibrium on each member:

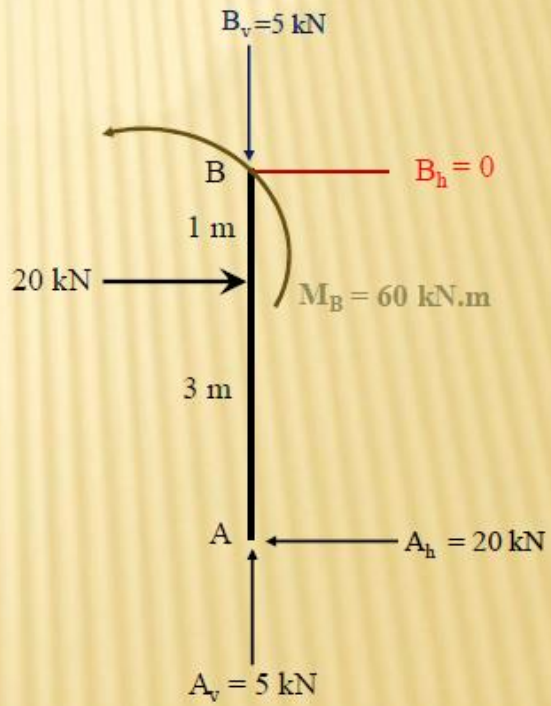
- Apply the equations of equilibrium on each member:

Member AB:

$$\sum f_x = 0$$
$$20 - 20 + B_h = 0 \rightarrow B_h = 0$$

$$\sum f_y = 0$$
$$5 - B_v = 0 \rightarrow B_v = 5 \downarrow$$

$$\sum M@A = 0$$
$$20(3) - M_B = 0 \rightarrow M_B$$
$$= 60\ kN.m \text{ Counter clock wise}$$

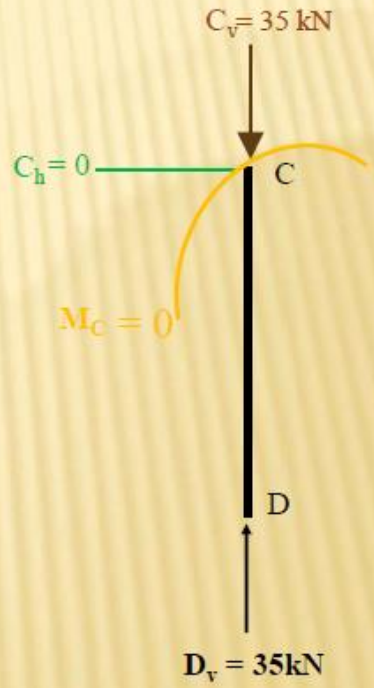


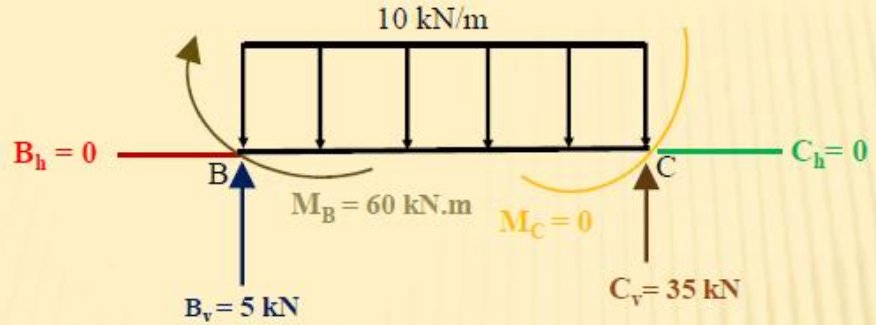
# Member CD:

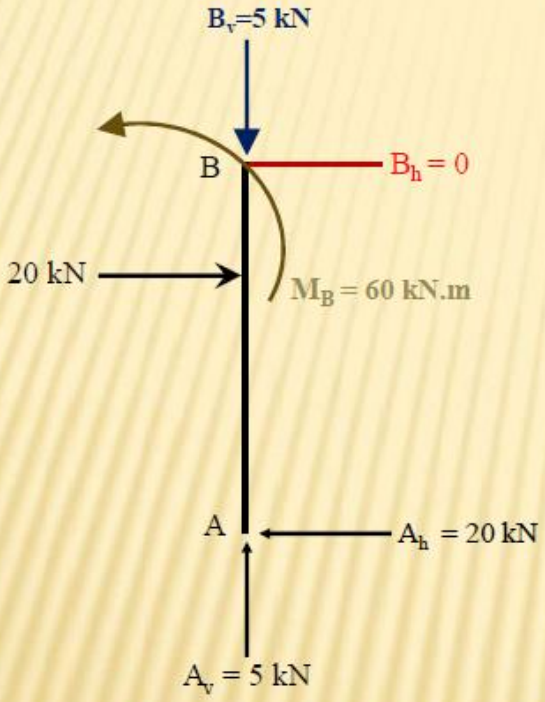
$$\sum f_x = 0$$
$$C_h = 0$$

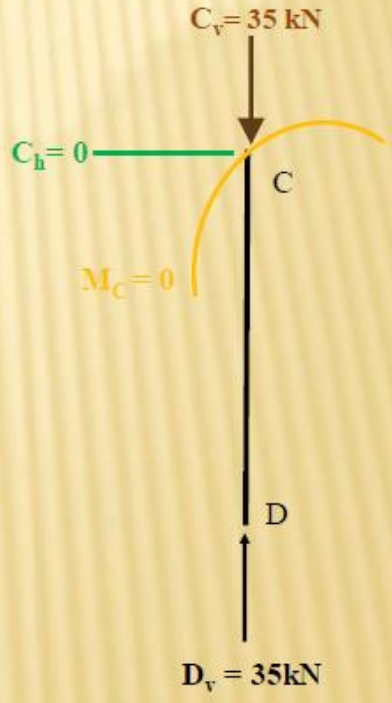
$$\sum f_y = 0$$
$$35 - c_v = \rightarrow c_v$$
$$= 35kN \downarrow$$

$$\sum M@D = 0 \rightarrow M_c = 0$$



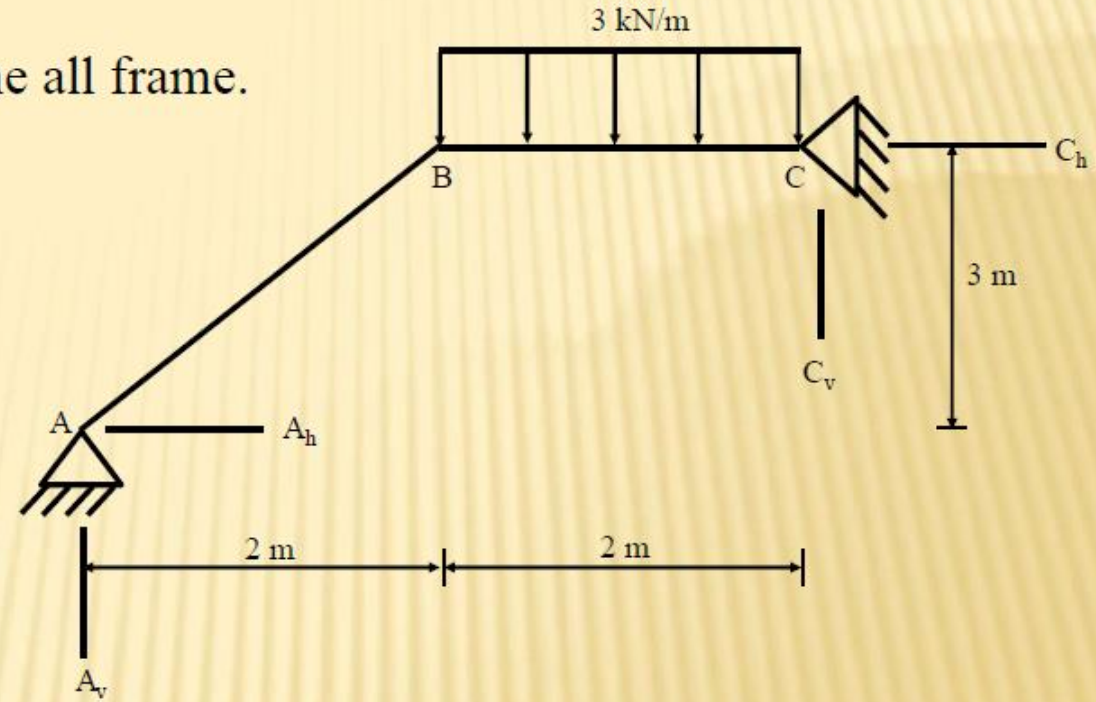

10 kN/m
$B_h = 0$
B
C
$C_h = 0$
$M_B = 60$ kN.m
$M_C = 0$
$B_v = 5$ kN
$C_v = 35$ kN


$B_v = 5$ kN
B
$B_h = 0$
20 kN
$M_B = 60$ kN.m
A
$A_h = 20$ kN
$A_v = 5$ kN

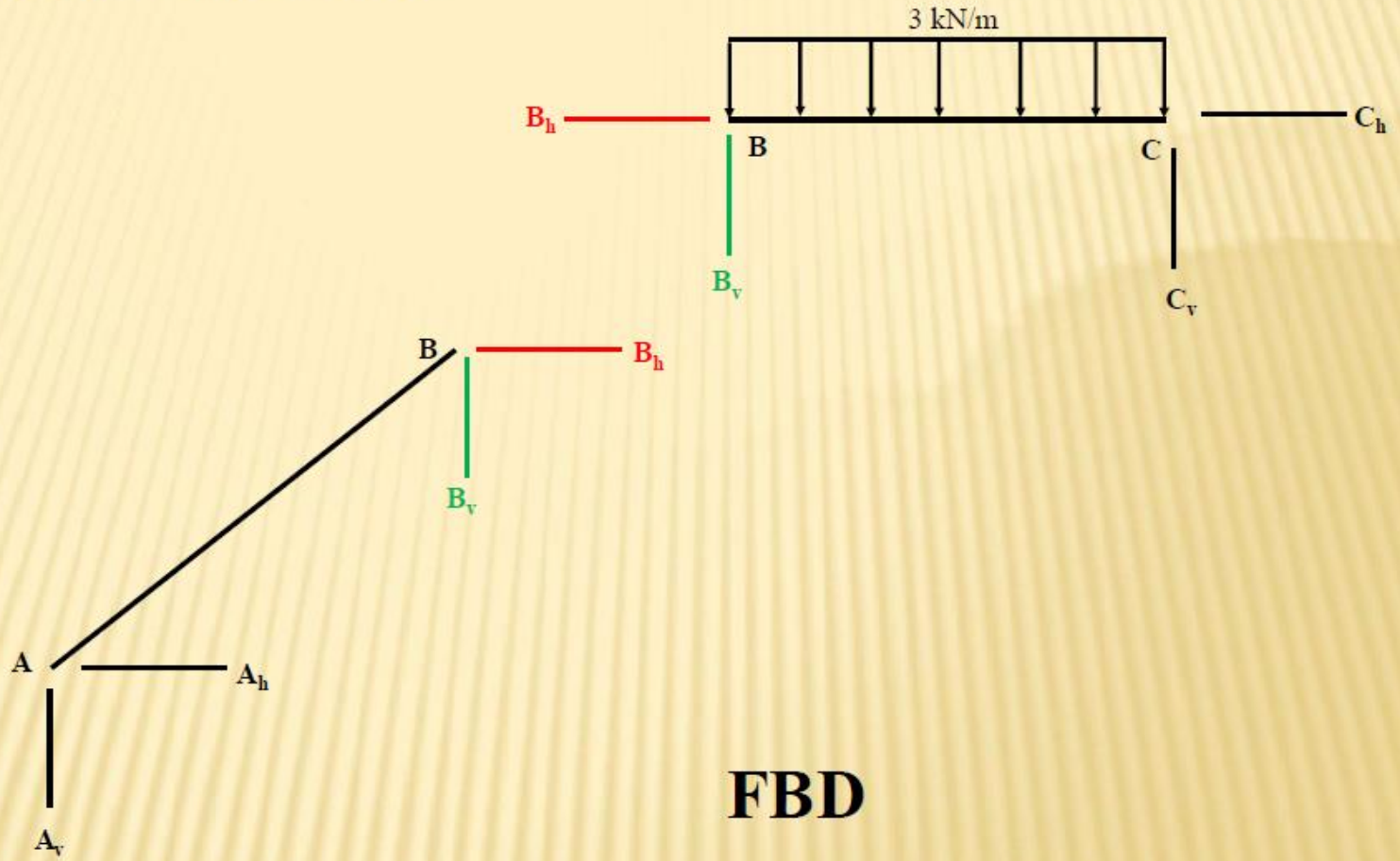

$C_v = 35$ kN
$C_h = 0$
C
$M_C = 0$
D
$D_v = 35$kN

**Example:** Determine the horizontal and vertical components of reaction at the pins *A*, *B*, and *C* of the two-member frame shown in the figure

- Calculate reactions from the all frame.



- Draw the FBD of the frame:



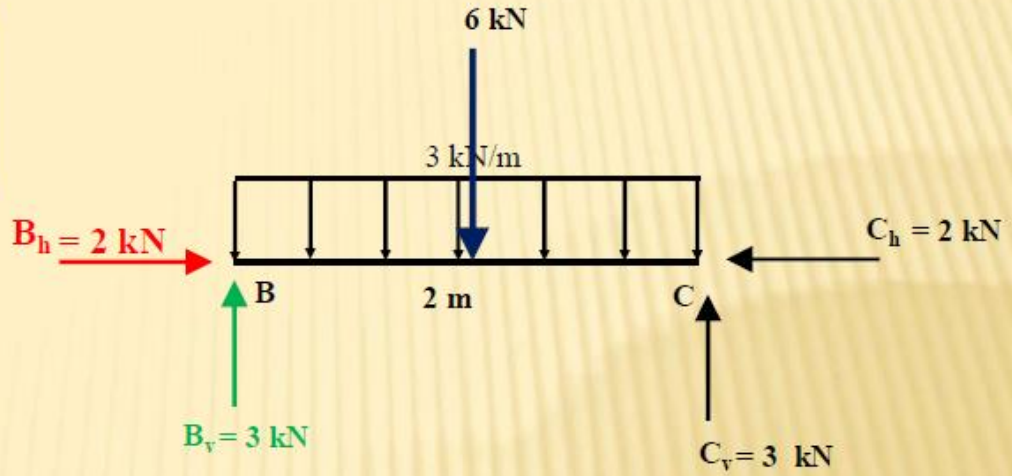

**FBD**

**- Apply the equations of equilibrium on each member:**

# Member BC

$$\sum M@B = 0$$
$$6(1) - C_v(2) = 0 \rightarrow C_v$$
$$= 3\ kN\ \uparrow$$

$$\sum M@C = 0$$
$$-6(1) + B_v(2) = 0 \rightarrow B_v$$
$$= 3\ kN\ \uparrow$$

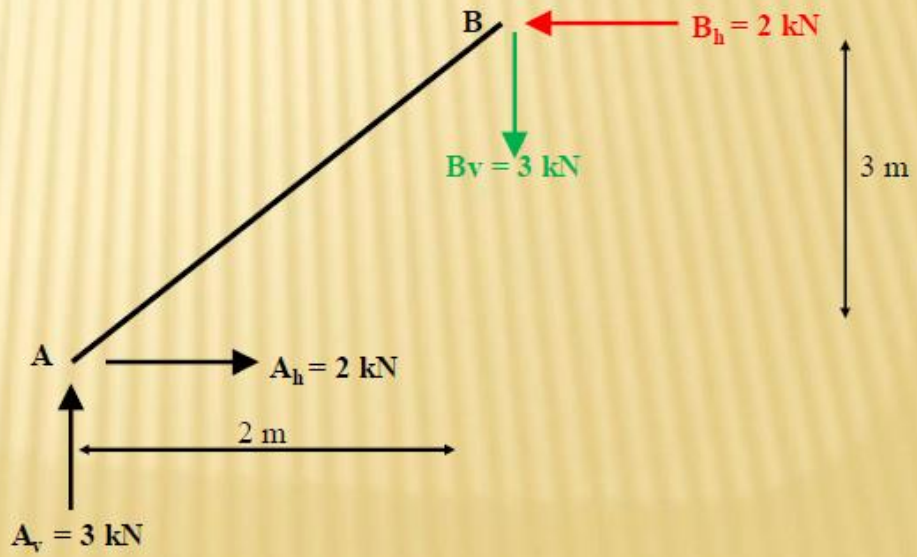



# Member AB

$$\sum F_y = 0$$
$$A_v - 3 = 0 \rightarrow A_v = 3\ kN\ \uparrow$$

$$\sum M@A = 0$$
$$3(2) - B_h(3) = 0 \rightarrow B_h$$
$$= 2\ kN\ \leftarrow$$

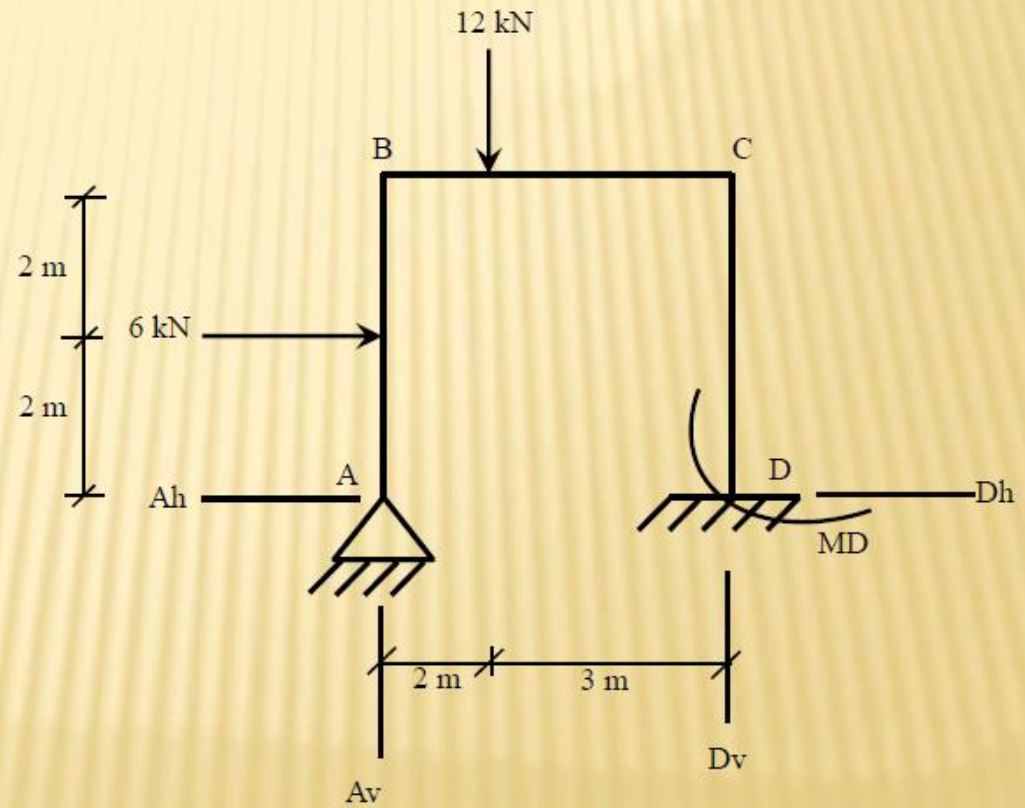
$$\sum F_x = 0$$
$$-2 + A_h = 0 \rightarrow A_h = 2\ kN\ \rightarrow$$



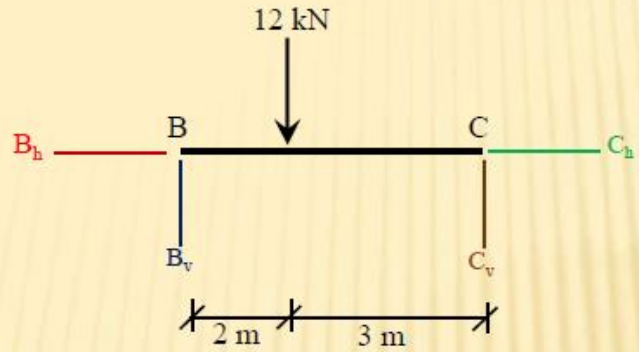
# Analysis of Frames
# Method of Member

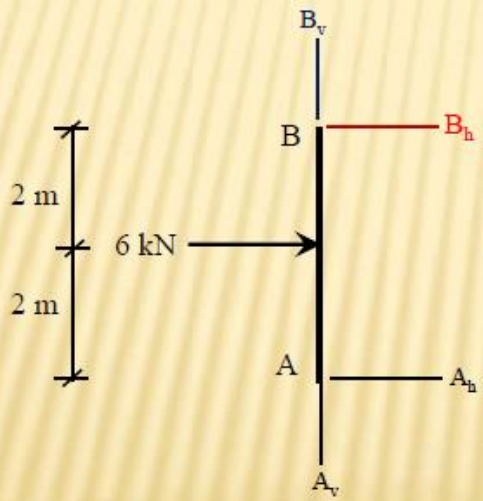
**Example:** For the frame shown in figure below, calculate the forces in all members. Joints B and C are pin connected.

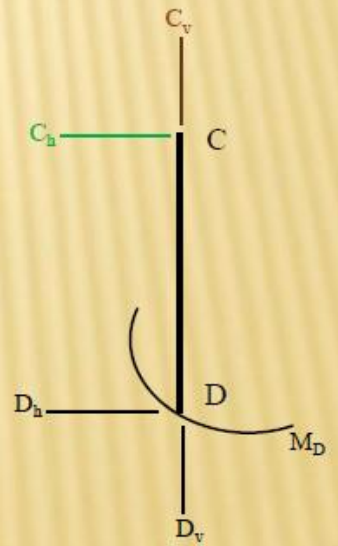
Calculate the reactions



# Draw FBD



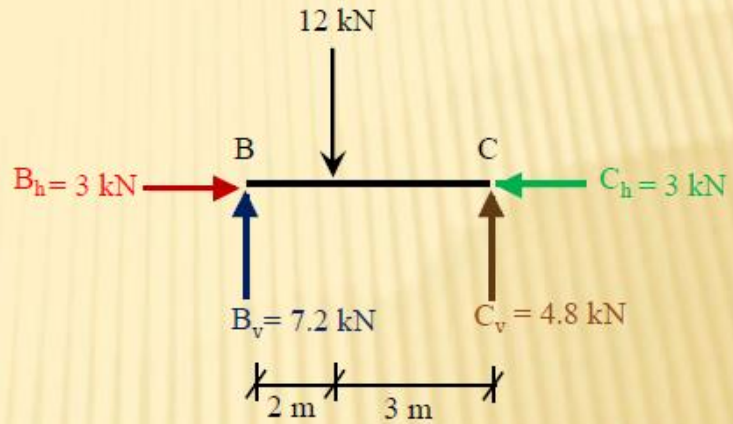

**Apply the equations of equilibrium on each member:**

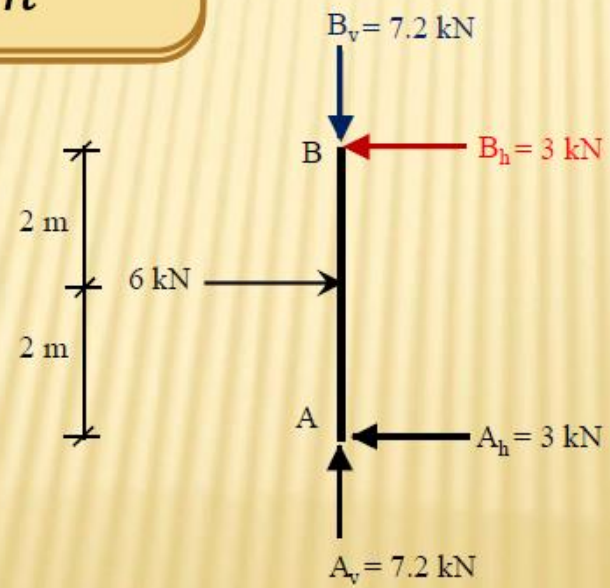
## Member BC

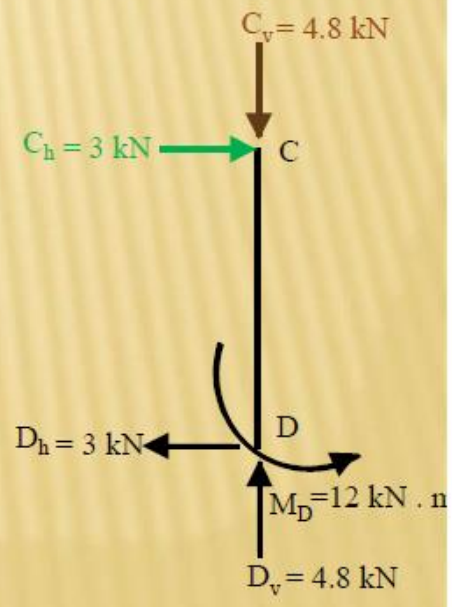
$$\sum F_y = 0$$
$$-4.8 + D_V = 0$$
$$D_V = 4.8\ kN\ \uparrow$$

$$\sum M@_D = 0$$
$$3(4) - M_D = 0$$
$$M_D = 12\ kN.m$$

$$\sum F_x = 0$$
$$3 - D_h = 0$$
$$D_h = 3\ kN\ \leftarrow$$

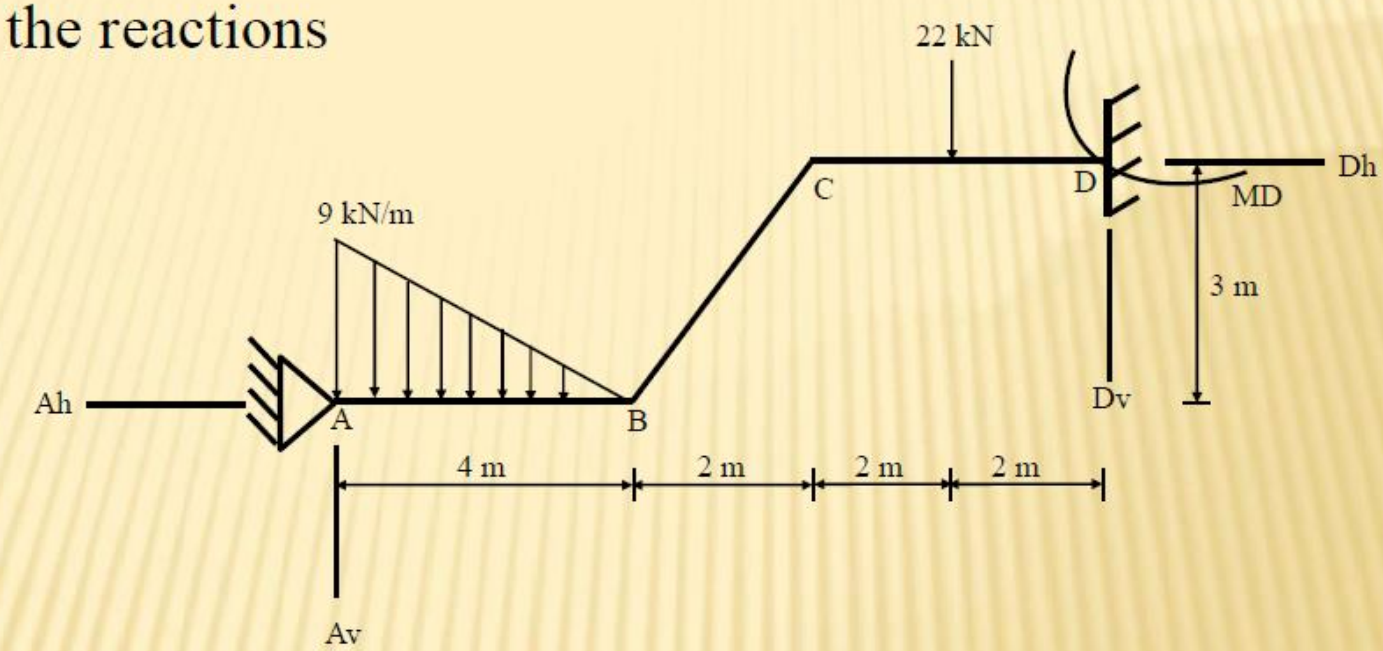


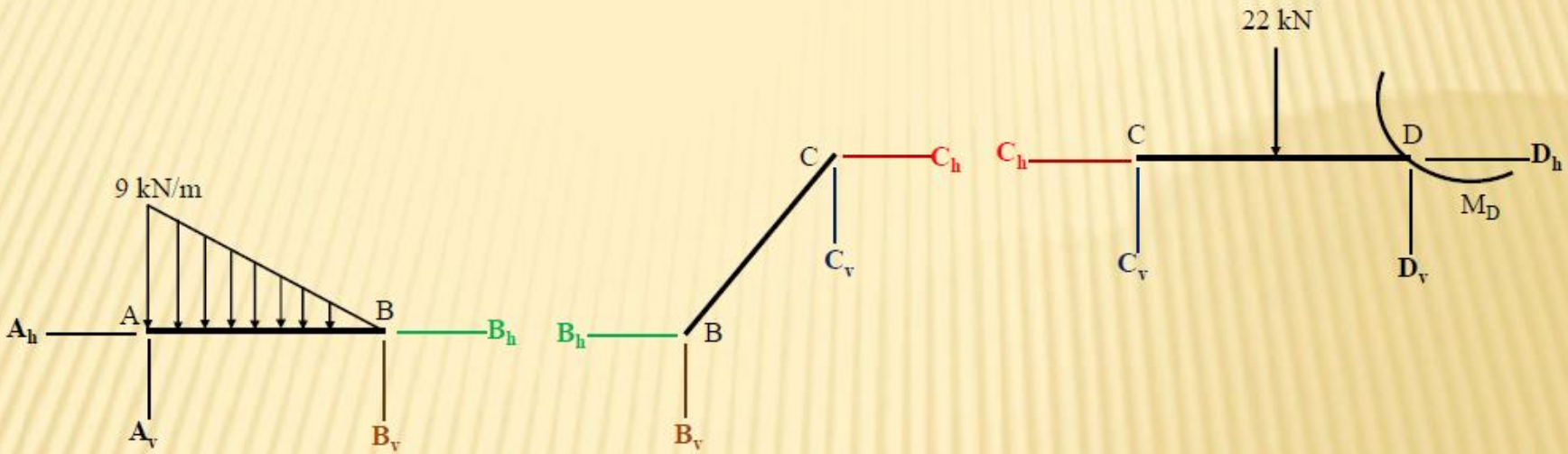



**Example:** For the frame shown in figure below, calculate the forces in all members. Joints B and C are pin connected.

Calculate the reactions



# Draw FBD




Apply the equations of equilibrium on each member:

Member AB

Member BC

Member CD

$$\sum F_x = 0$$

$$-4 + D_h = 0$$

$$D_h = 4\ kN \rightarrow$$

$$C_V = 6\ kN$$

$$B_v = 6kN$$

$$\sum F_y = 0$$

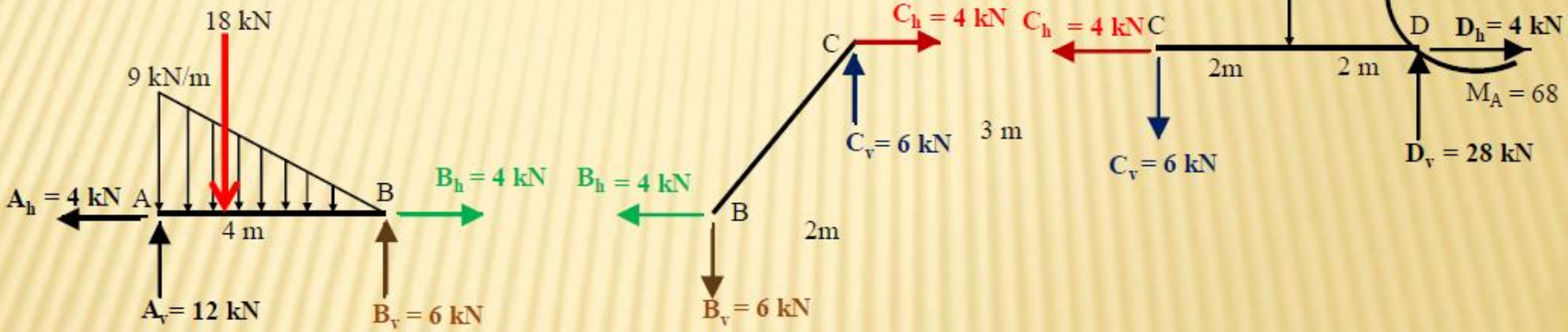
$$-22 - 6 + D_v = 0$$

$$D_v = 28\ kN \uparrow$$

$$\sum M@_D = 0$$

$$-6(4) - 22(2) + M_D = 0$$

$$M_D = 68\ kN.m\ clock\ wise$$



**Example**: For the frame shown in figure. Compute the forces at pin connected joints A, B, and C.

- Calculate the reactions:

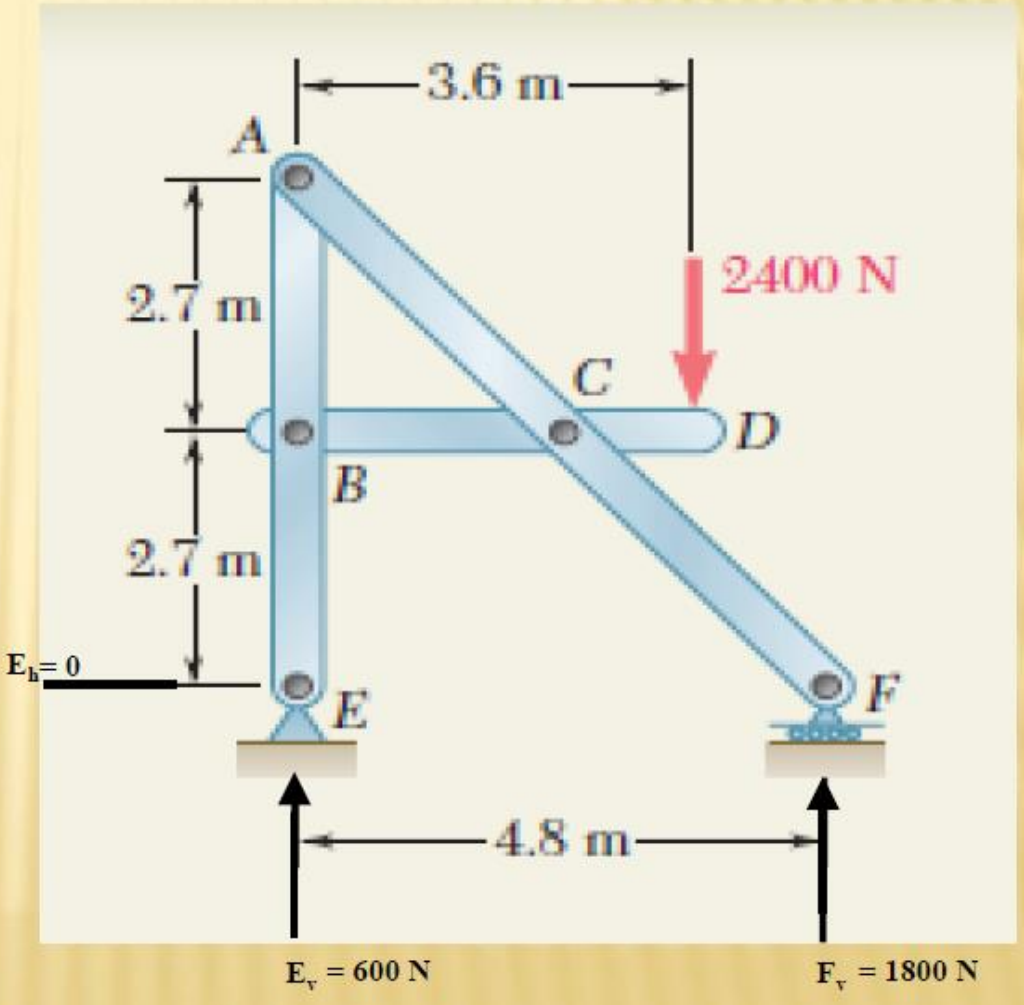
$$\sum M@E = 0$$

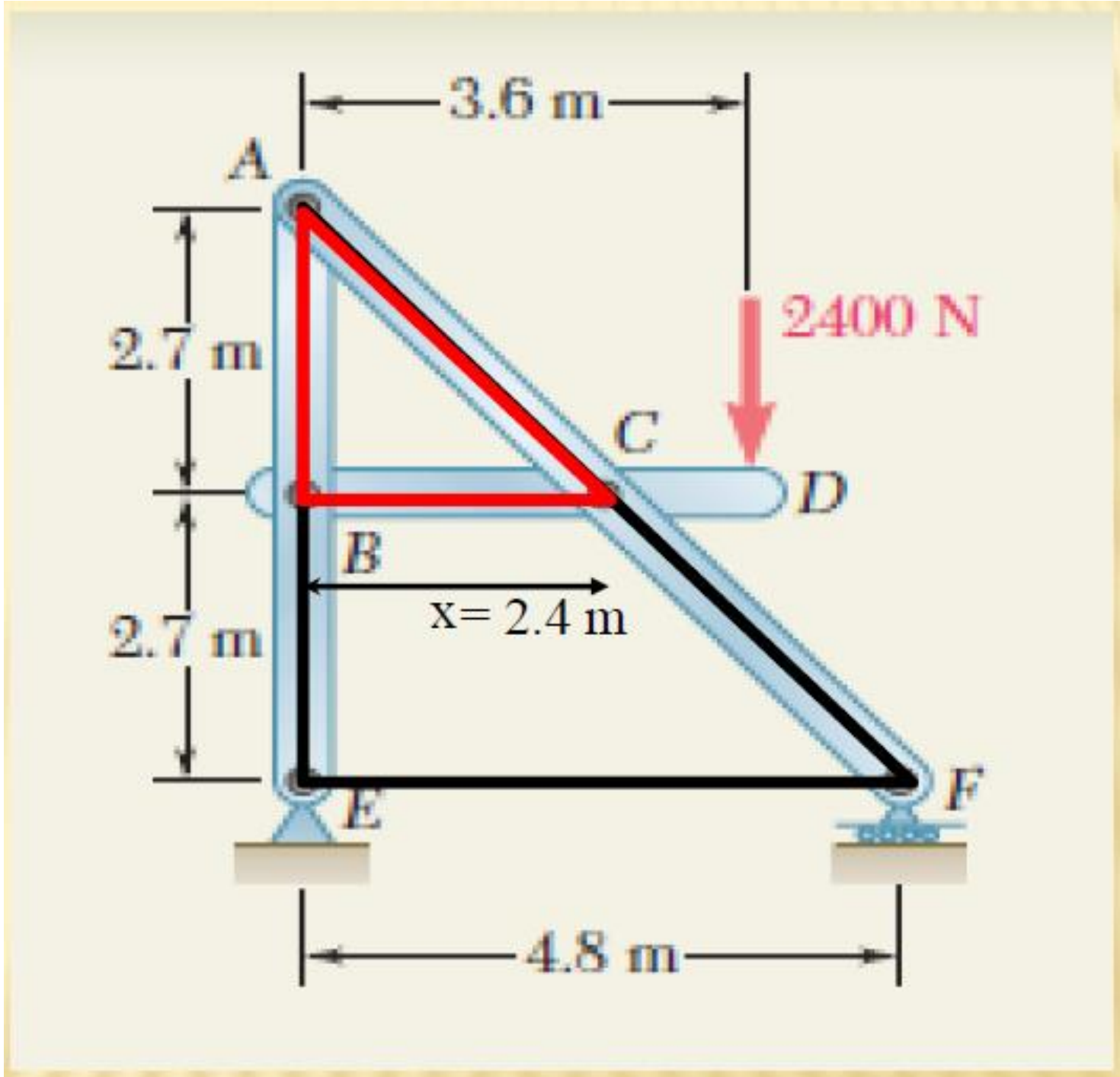
$$2400(3.6) - F_v(4.8) = 0 \rightarrow F_v = 1800\ N\ \uparrow$$

$$\sum M@F = 0$$

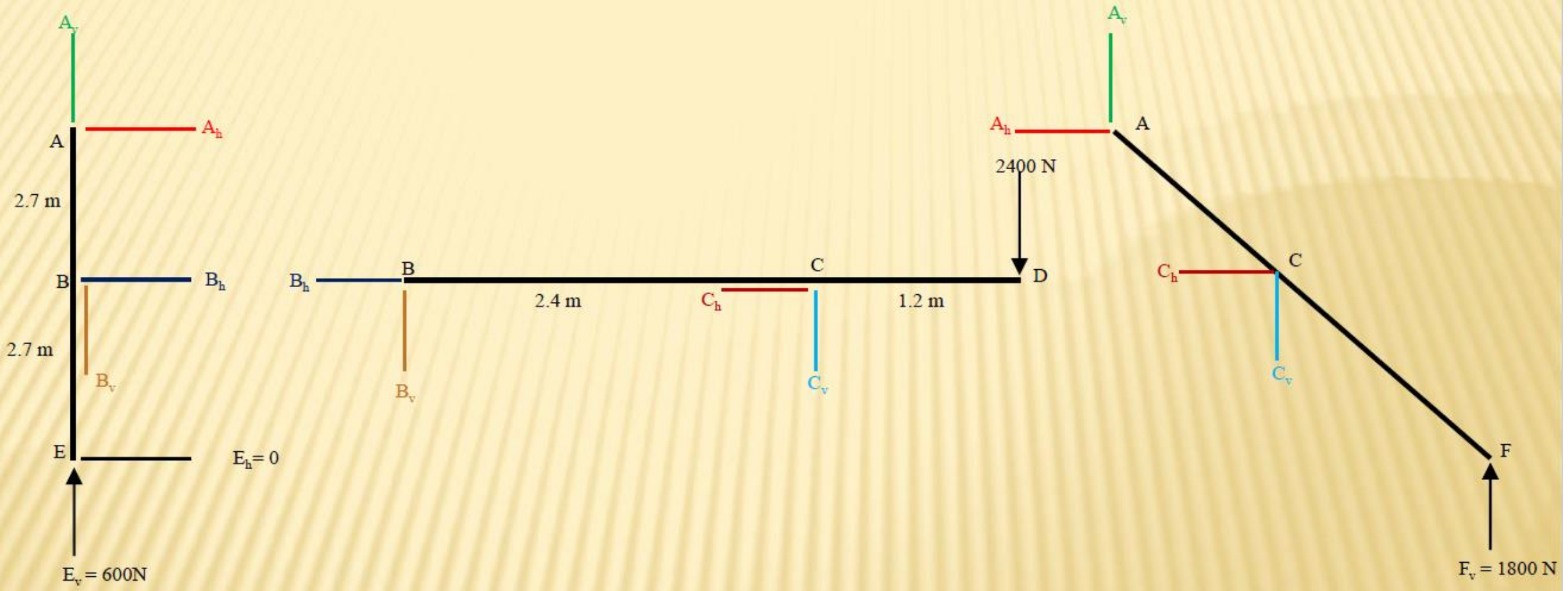
$$-2400(1.2) + E_v(4.8) = 0 \rightarrow E_v = 600\ N\ \uparrow$$

$$\sum Fx = 0 \rightarrow E_h = 0$$




3.6 m
A
2.7 m
2400 N
C
D
B
x= 2.4 m
2.7 m
E
F
4.8 m

- Draw FBD:




- Apply the equations of equilibrium on each member:

## Member ABE:

$$\sum M@A = 0 \rightarrow B_h = 0$$

$$\sum F_X = 0 \rightarrow A_h =$$

## Member BCD

$$\sum F_X = 0 \rightarrow C_h = 0$$

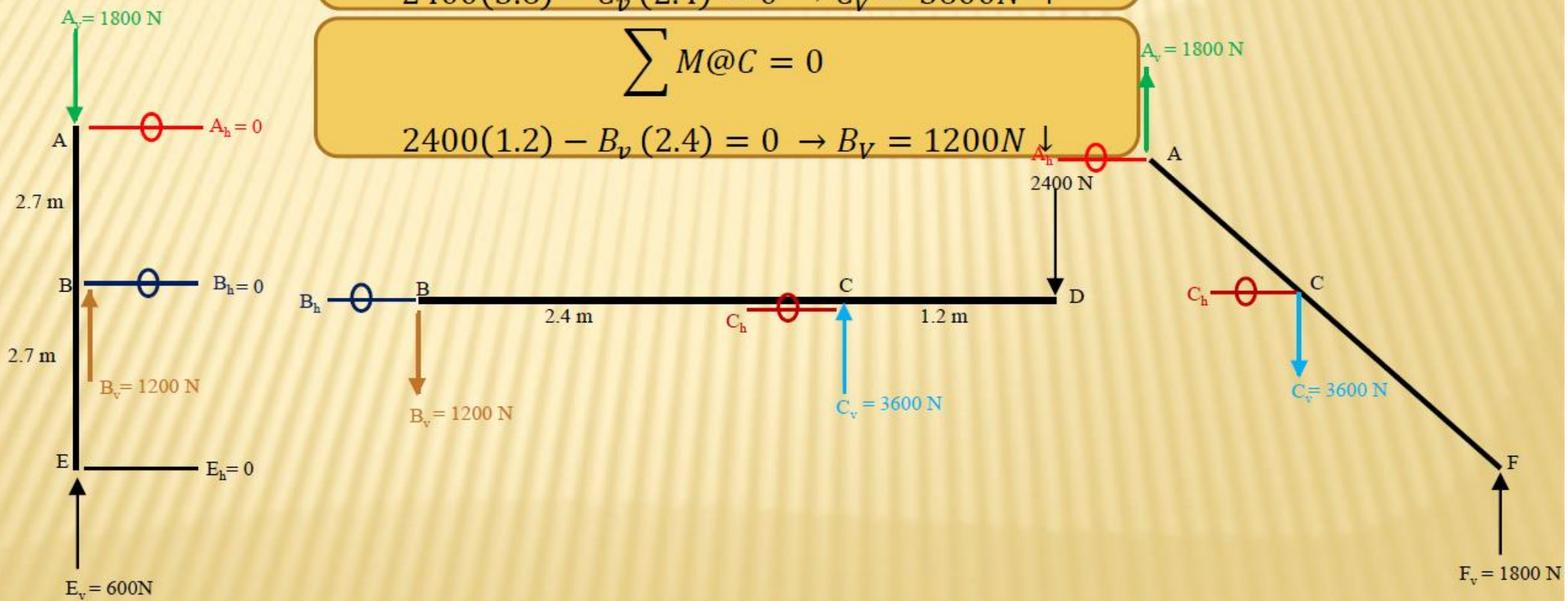
$$\sum M@B = 0$$

$$2400(3.6) - C_v\,(2.4) = 0 \rightarrow C_V = 3600N \uparrow$$

$$\sum M@C = 0$$

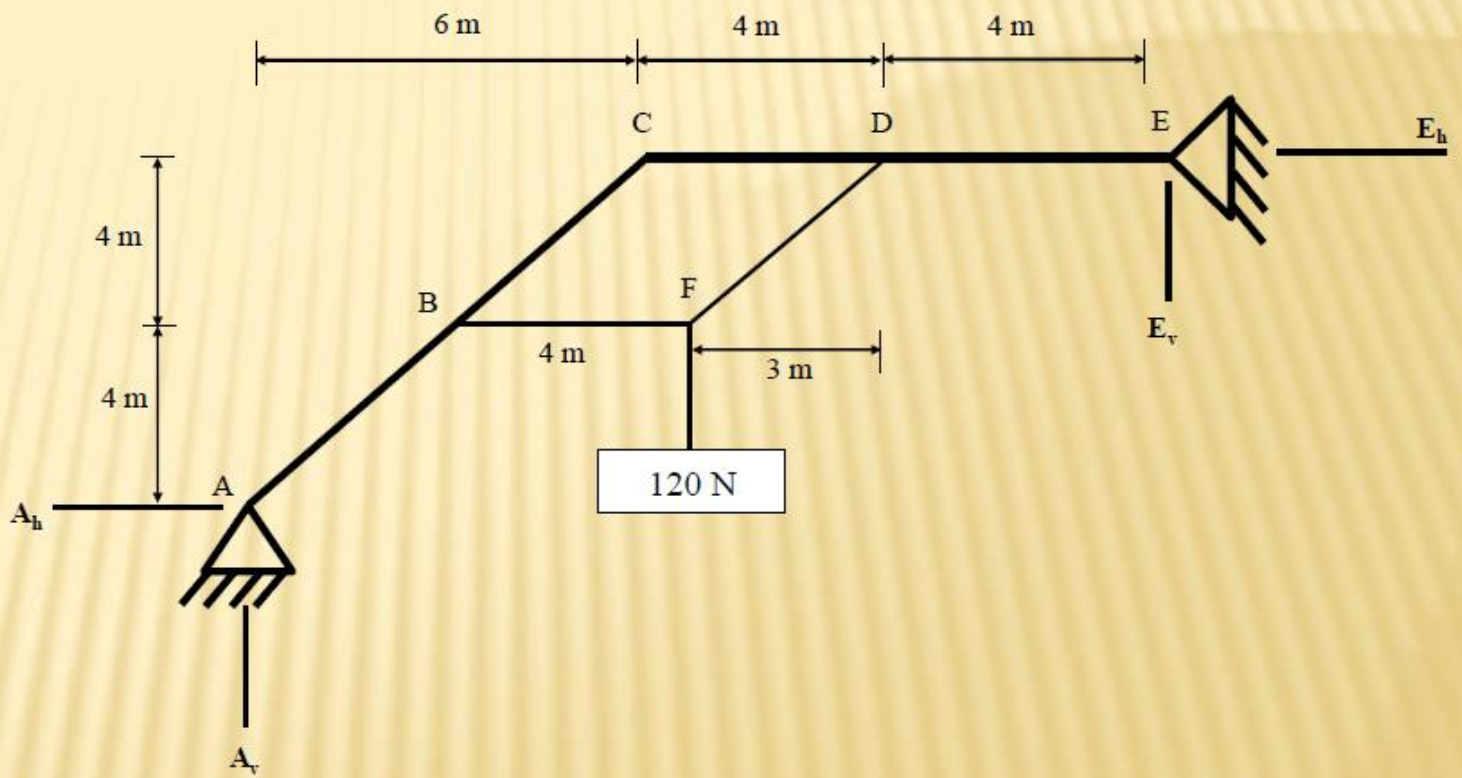
$$2400(1.2) - B_v\,(2.4) = 0 \rightarrow B_V = 1200N \downarrow$$

## Member ACF

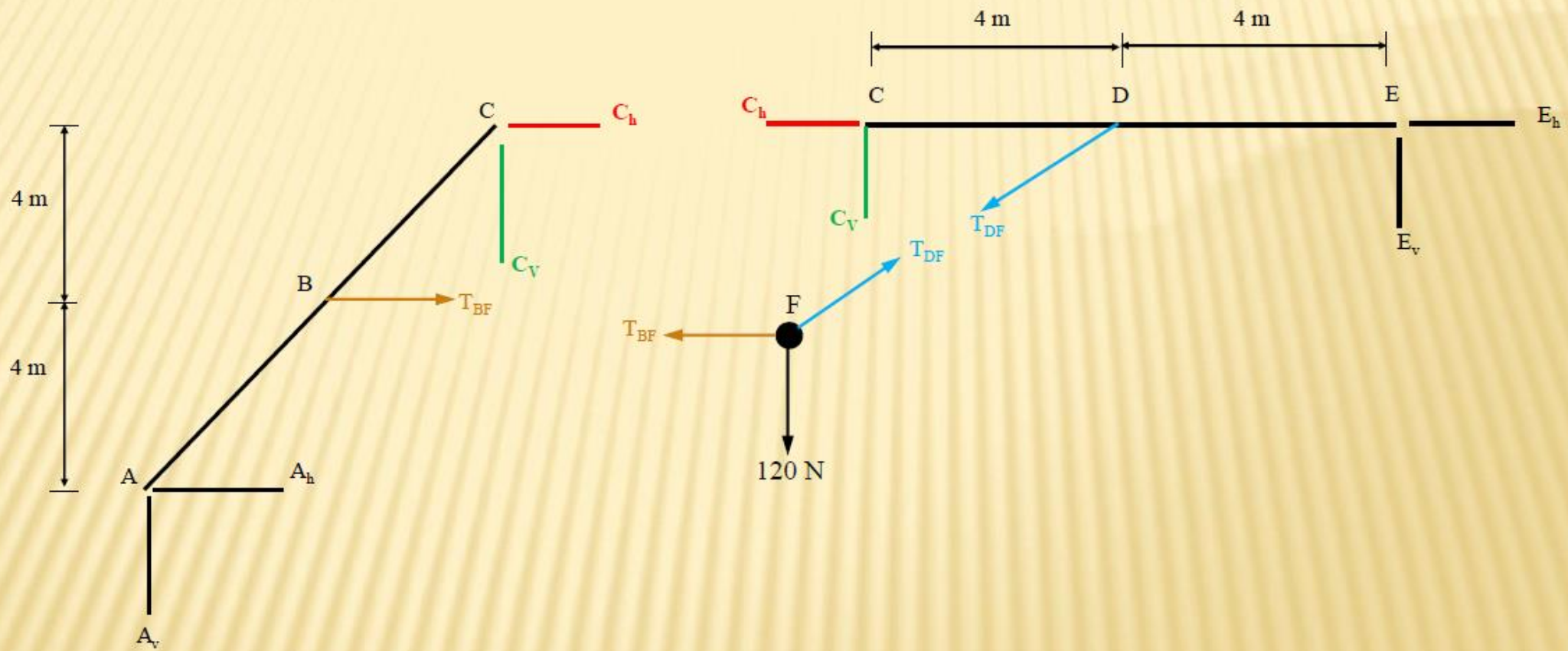


**Example:** Calculate the components of the pin force at C of the frame shown in figure.

- Calculate the reactions:



- Draw FBD:




- Apply the equations of equilibrium on each member:

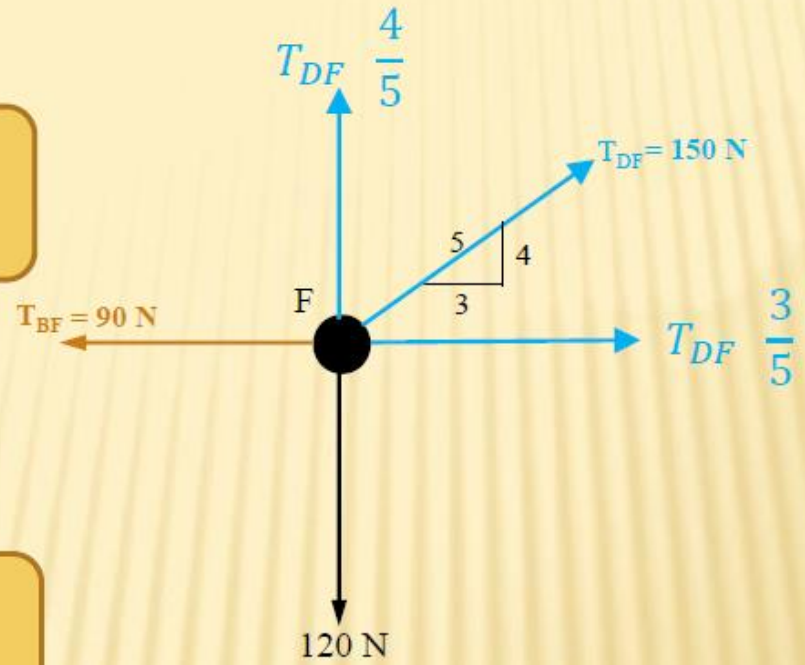
Node F:

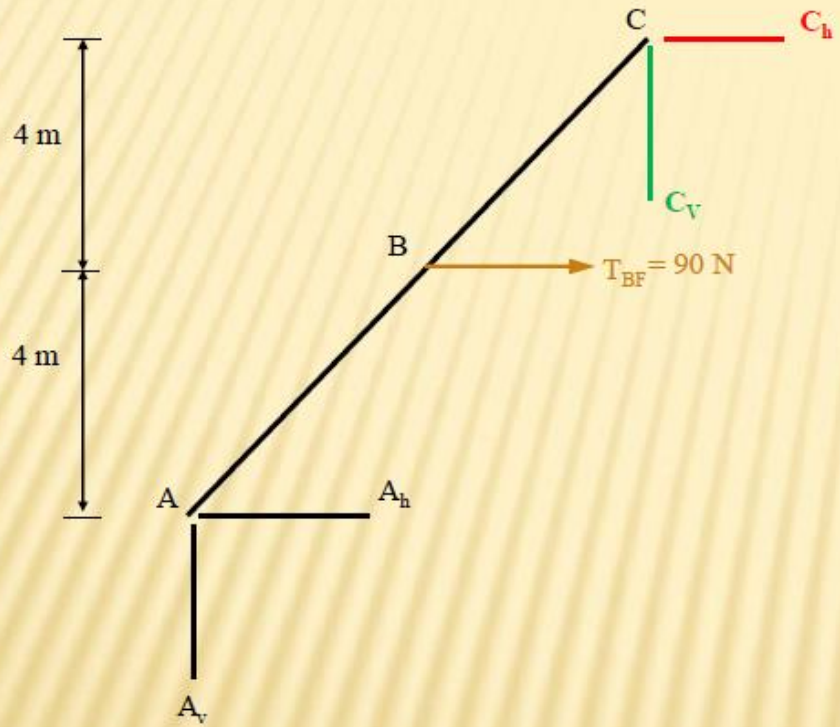
$$\sum Fy = 0$$

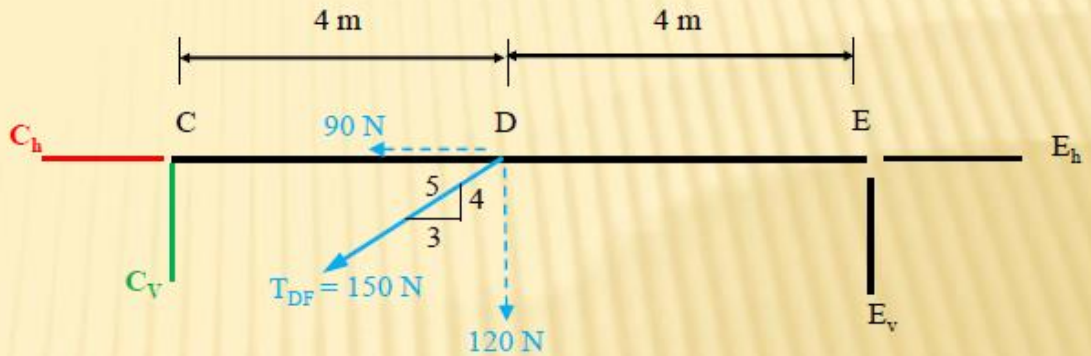
$$T_{DF}\frac{4}{5} - 120 = 0 \rightarrow T_{DF} = 150\ N$$

$$\sum F_X = 0$$

$$150\frac{3}{5} - T_{BF} = 0 \rightarrow T_{BF} = 90\ N$$



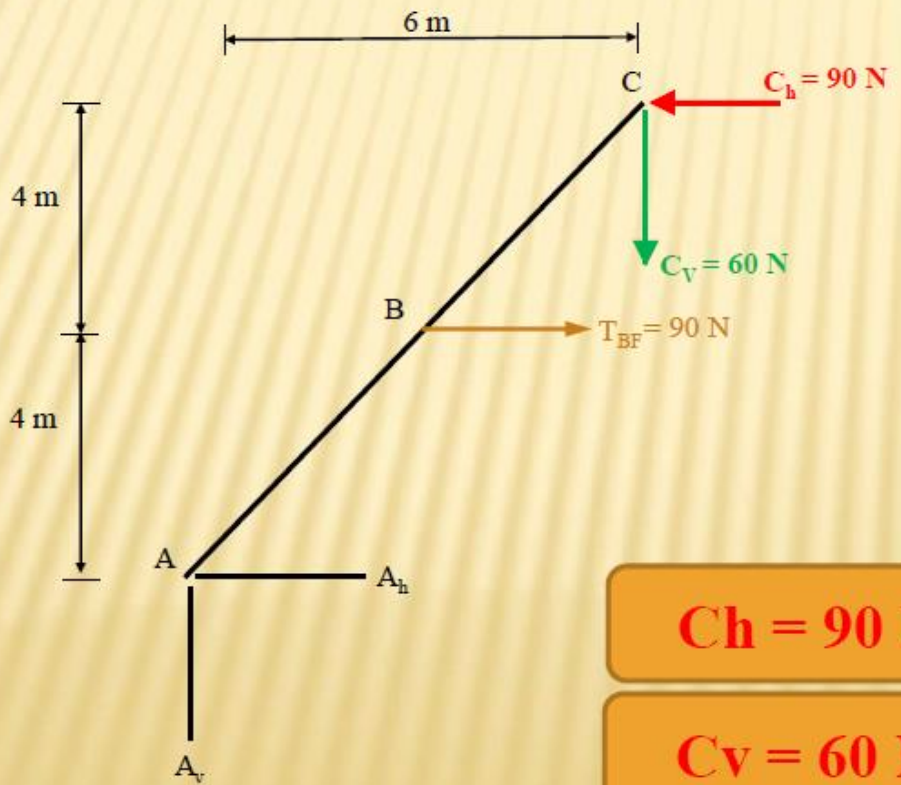

4 m
4 m
C
$C_h$
$C_V$
B
$T_{BF}$ = 90 N
A
$A_h$
$A_v$


4 m
4 m
$C_h$
C
90 N
D
E
$E_h$
5
4
3
$C_V$
$T_{DF}$ = 150 N
120 N
$E_v$

## Member ABC

$$\sum M@\ A = 0$$

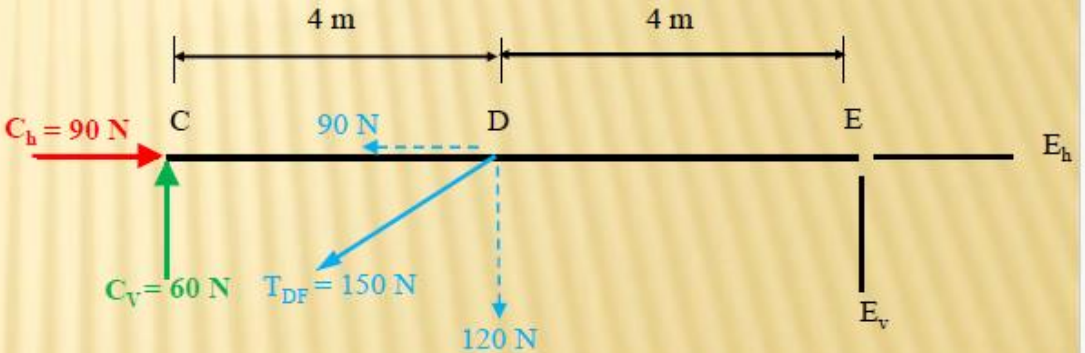
$$60(6) + 90\ (4) - Ch(8) = 0 \ \rightarrow Ch = 90\ \text{N}\leftarrow$$




## Member CDE

$$\sum M@\ E = 0$$

$$-120(4) + Cv\ (8) = 0 \ \rightarrow Cv = 60\ \text{N}\uparrow$$




**Ch = 90 N**

**Cv = 60 N**

**H.W.**

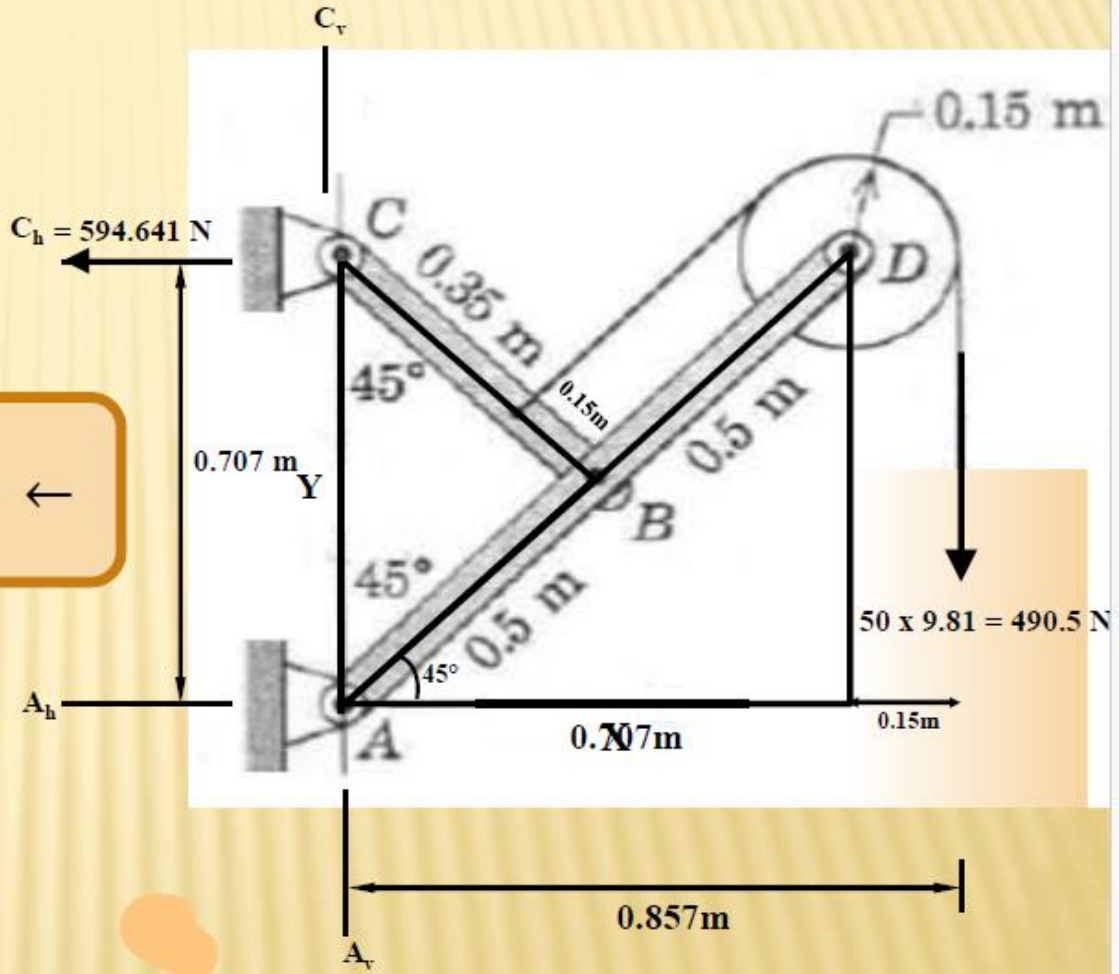
**Calculate the reactions**

**Example**: For the loaded beam as shown in figure, calculate the force components at pin B. A and C are hinge supports D is pin joint.

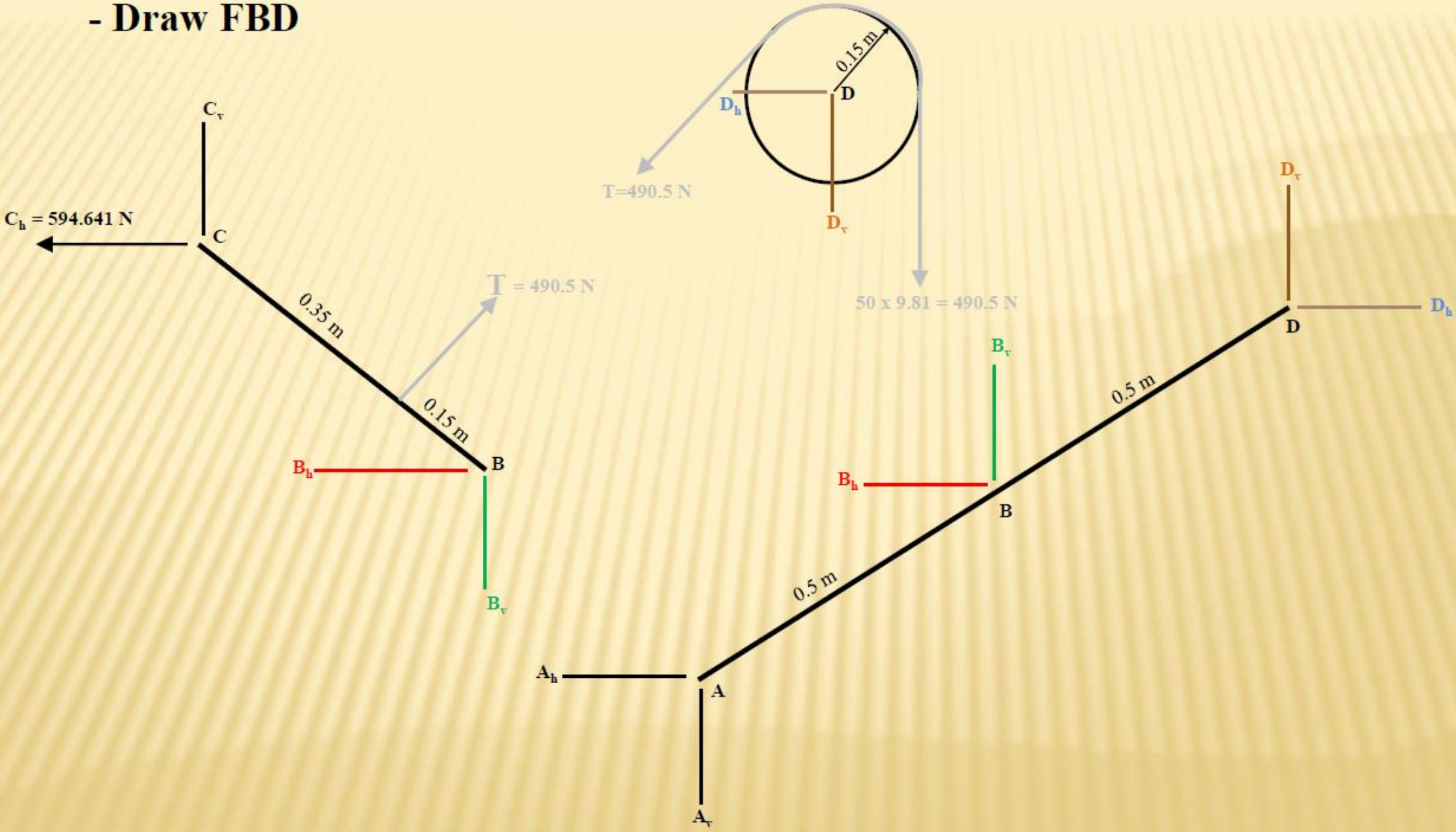
- **Calculate the reactions:**

$$\sum M@A = 0$$

$$490.5(0.857) - C_h(0.707) = 0 \rightarrow C_h = 594.641N \leftarrow$$



- **Draw FBD**




- **Apply the equations of equilibrium on each member:**

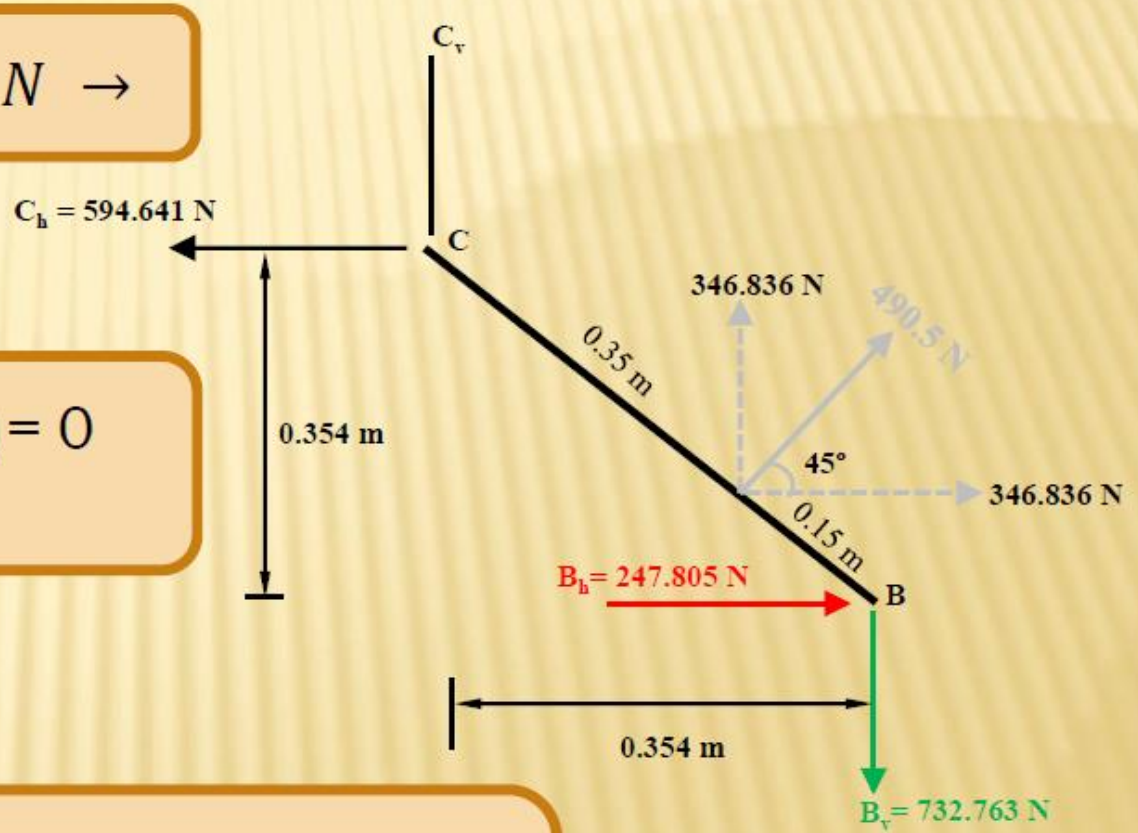
## Member BC

$$\sum Fx = 0$$

$$346.836 - 594.641 + B_h = 0 \rightarrow B_h = 247.805\ N \rightarrow$$

$$\sum M @ C = 0$$

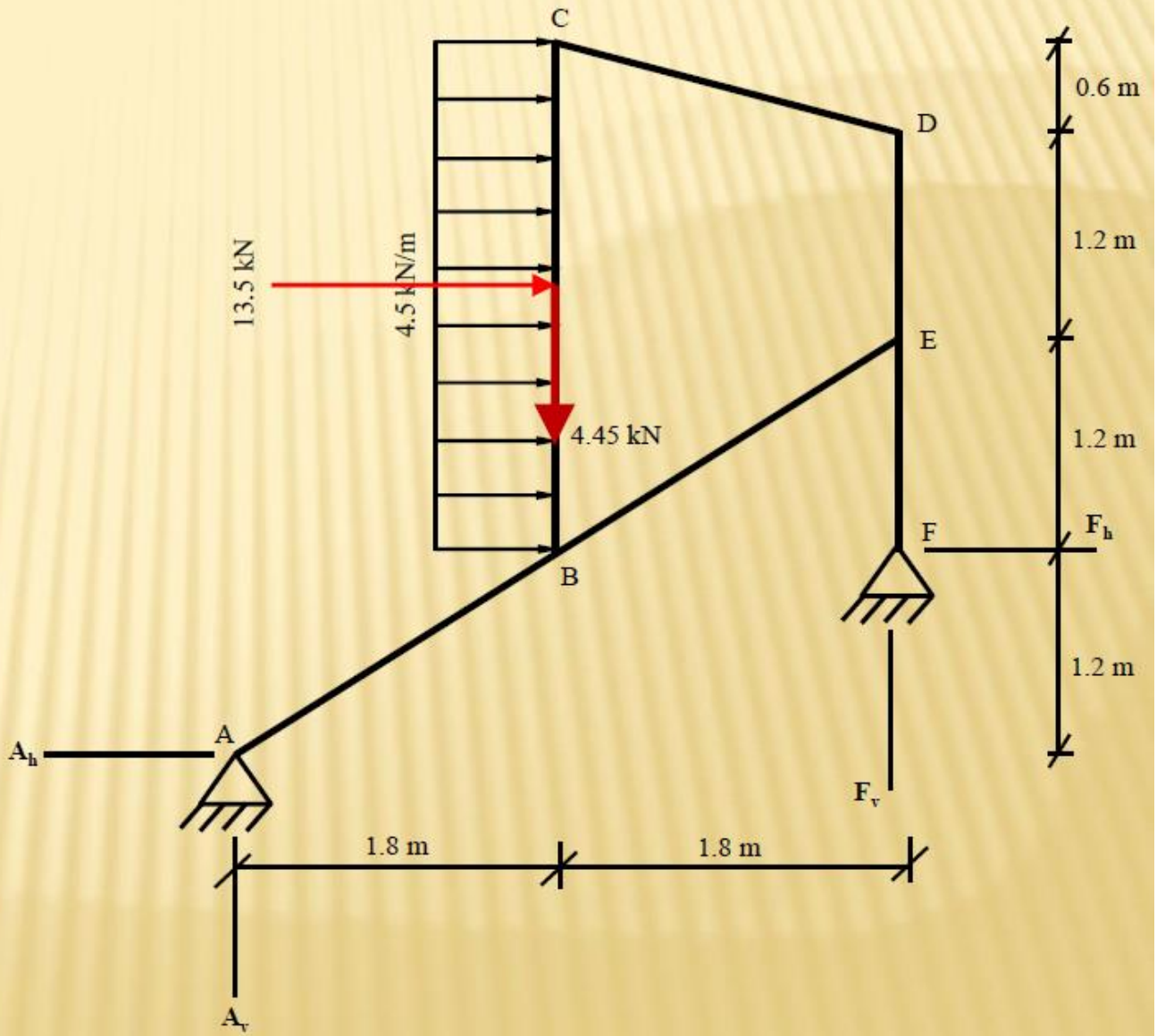
$$-247.805(0.354) - 490.5(0.35) + B_v\ (0.354) = 0$$
$$\rightarrow B_v = 732.763\ N \downarrow$$



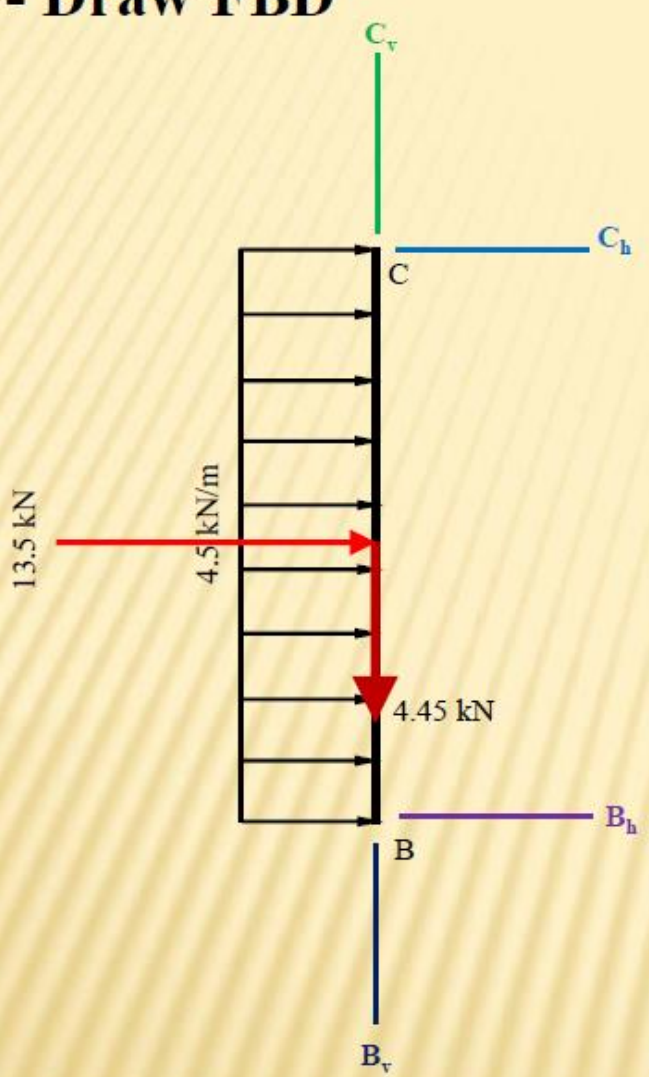

***H.W.***
***Calculate the forces at A, and D***

**Example:** A billboard BC weighting **4.45 kN** is subjected to a wind pressure of **4.5 kN/m** as shown in figure. Calculate the reactions at A and F. Neglect the weight of the other members.

- **Calculate the reactions:**

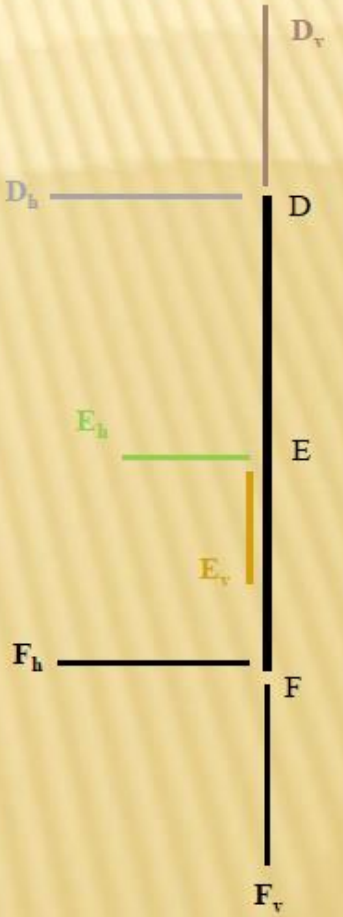


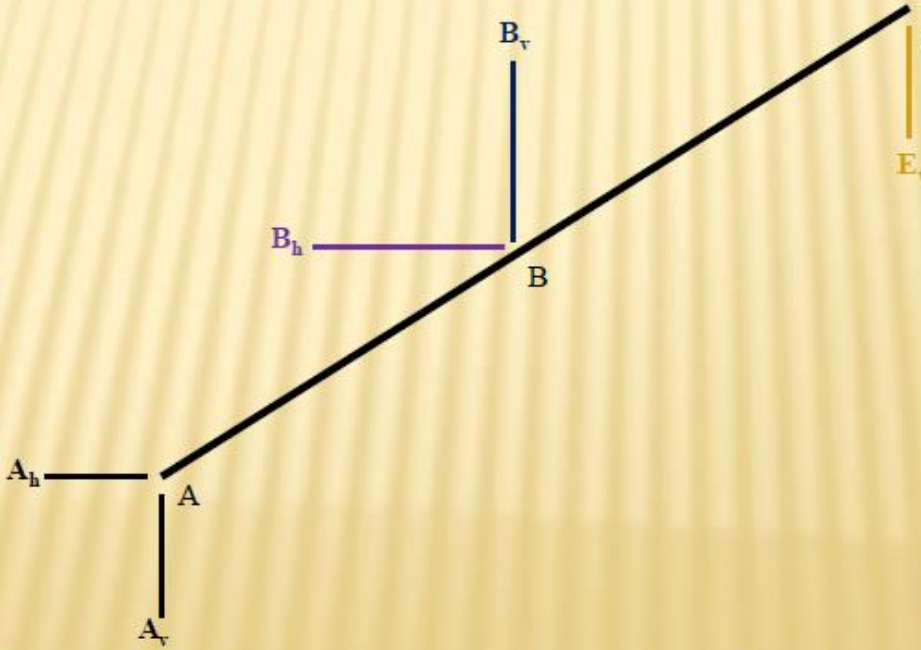
- **Draw FBD**







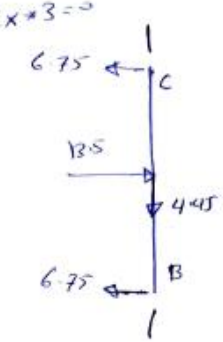


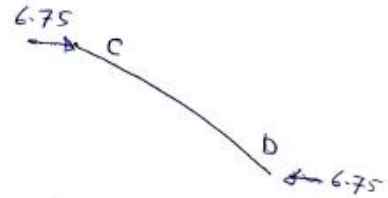




- **Apply the equations of equilibrium on each member:**

$\Sigma M_B = 0 = 13.5 * 1.5 - Cx * 3 = 0$

$Cx = 6.75 \Rightarrow Bx = 6.75$

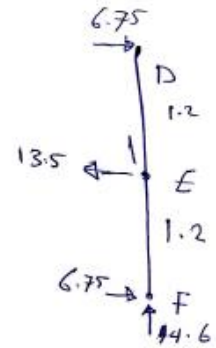


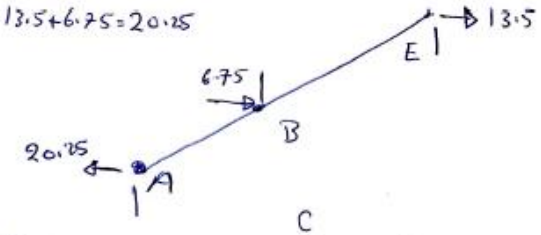

$\Sigma M_E = 0 = 6.75 * 1.2 - Fx * 1.2 = 0$

$Fx = 6.75$

$\Sigma Fx = 0 \Rightarrow Ex = 13.5$




$\Sigma Fx = 0 \Rightarrow Ax = 13.5 + 6.75 = 20.25$




$\Sigma M_A = 0 = 13.5 * 2.7 + 4.45 * 1.8 + 6.75 * 1.2 - Fy * 3.6 = 0$

$Fy = 14.6$

$\Sigma Fy = 0 \Rightarrow Ay = 14.6 - 4.45 = 10.15$

