Mechanical Engineering

First – year

Second semester

2025-2026

Method of Sections

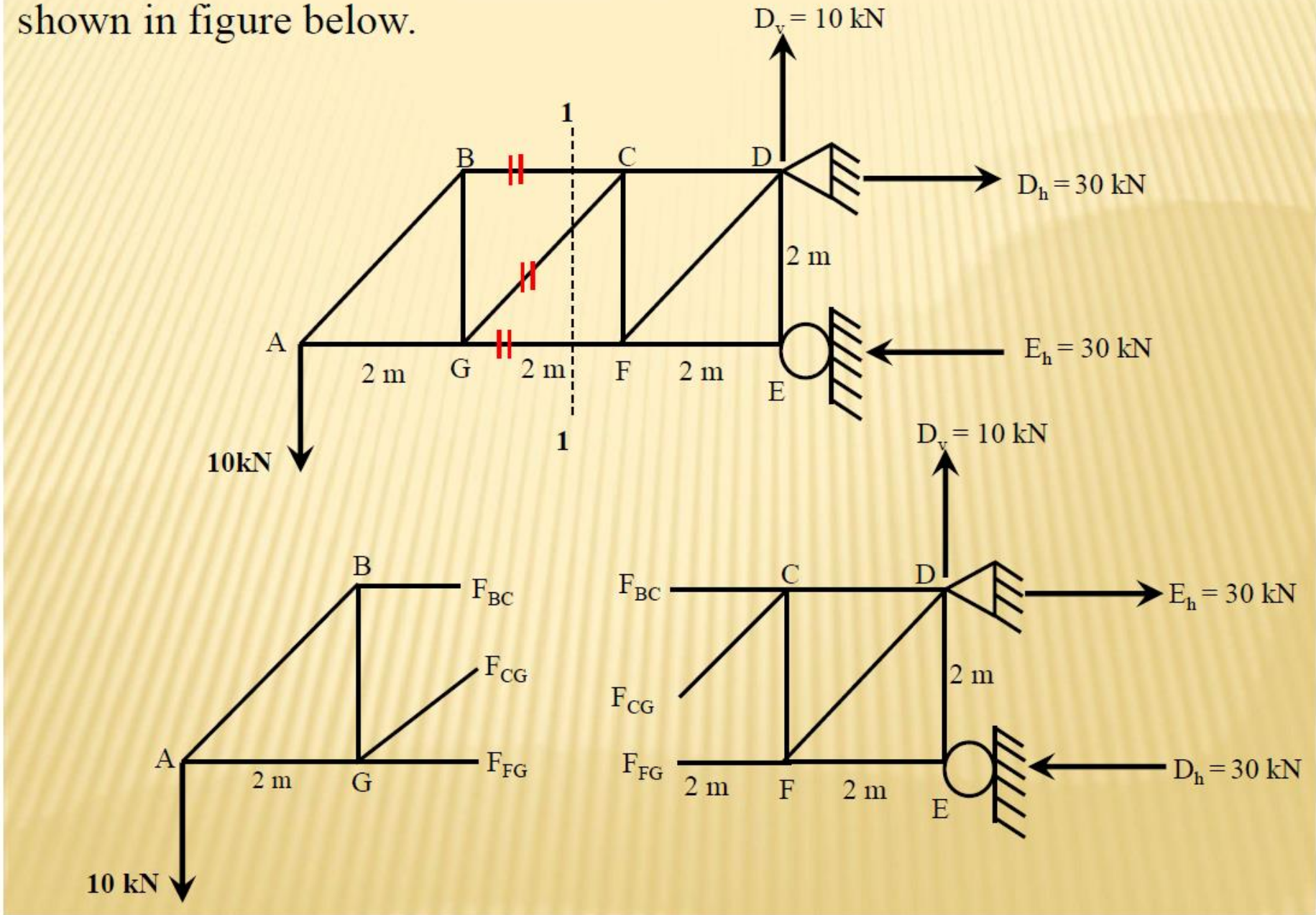
The method involves dividing the truss into sections by cutting through the selected members and analyzing the section as a rigid body. The advantage of the Method of Sections is that the only internal member forces exposed are those which you have cut through, the remaining internal forces are not exposed and thus ignored.



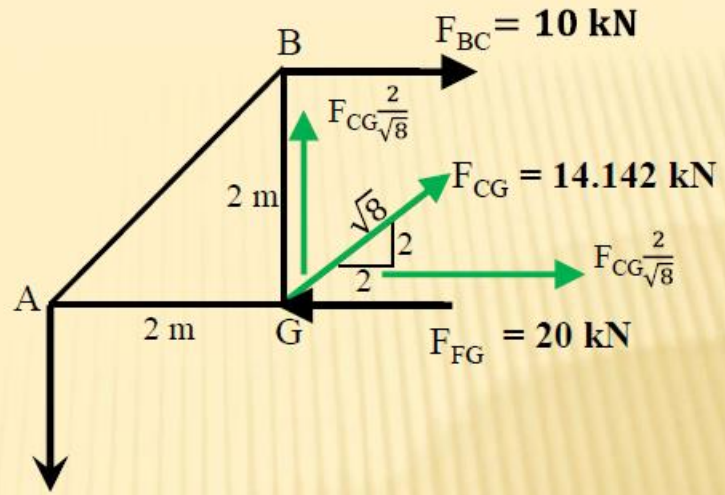
Method of Sections can be used to determine **three unknowns** member forces per FBD since all **three equations** can be used.

$$\sum F_x = 0 \qquad \sum F_Y = 0 \qquad \sum M_o = 0$$

Example: Determine the force in members **BC**, **CG**, and **FG** of the truss shown in figure below.



Section 1 – 1 left



$$\sum F_Y = 0$$

$$-10 + F_{CG}\frac{2}{\sqrt{8}} = 0 \Rightarrow F_{CG} = 14.142 \text{ kN (T)}$$

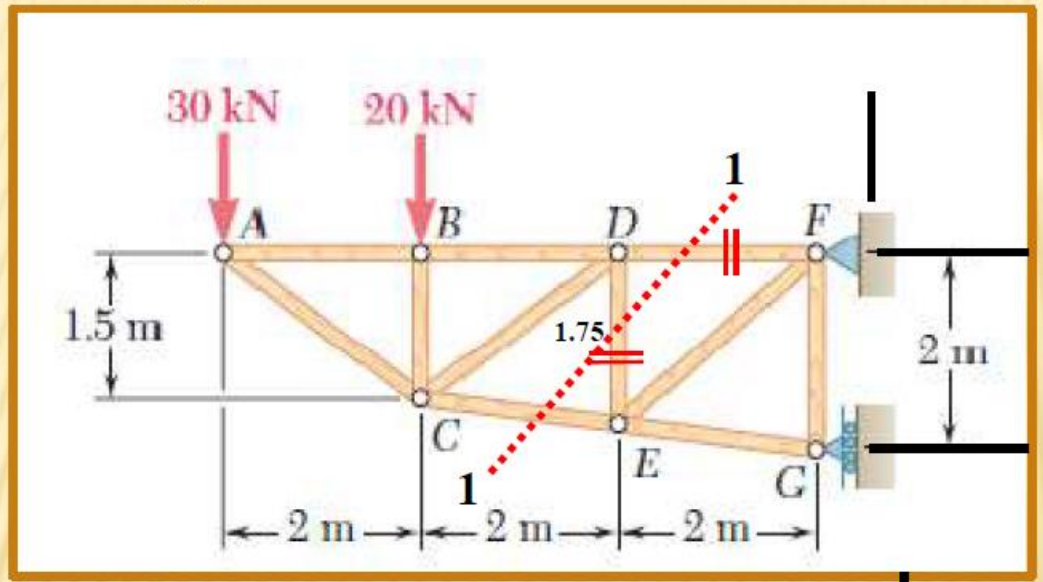
$$\sum M_G = 0$$

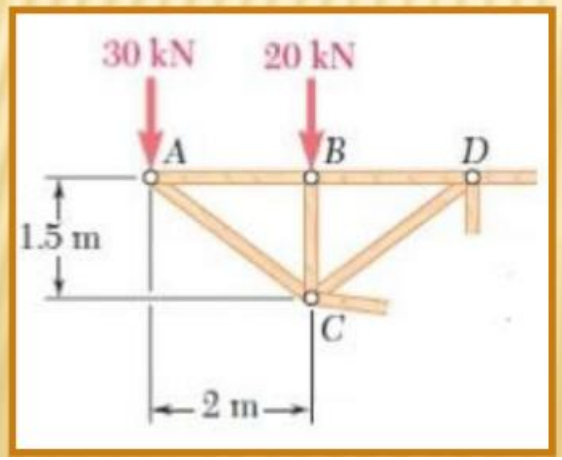
$$-10\,(2) + F_{BC}\,(2) = 0 \Rightarrow F_{BC} = 10 \text{ kN (T)}$$

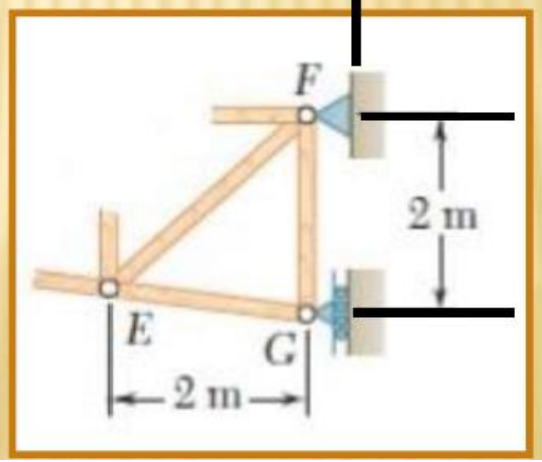
$$\sum F_X = 0$$

$$10 + 14.142\frac{2}{\sqrt{8}} + F_{FG} = 0 \Rightarrow F_{FG} = -20 \text{ kN (C)}$$

Example: Determine the force in members ***DF*** and ***DE*** of the truss shown in figure. **Using section method**. State whether each member is in tension or compression.







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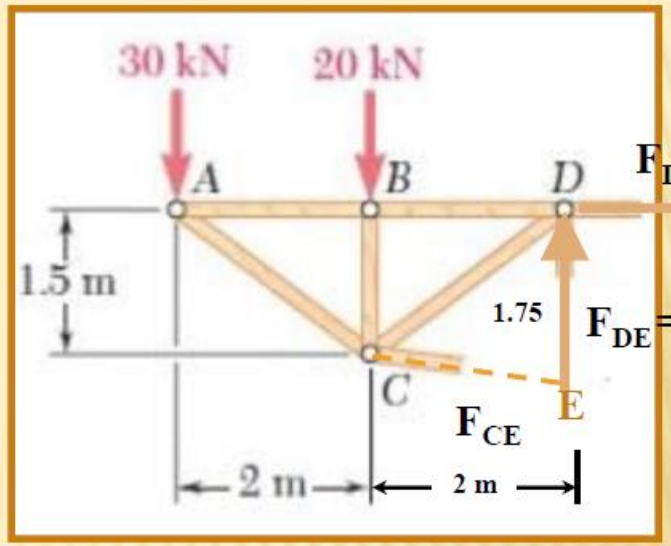


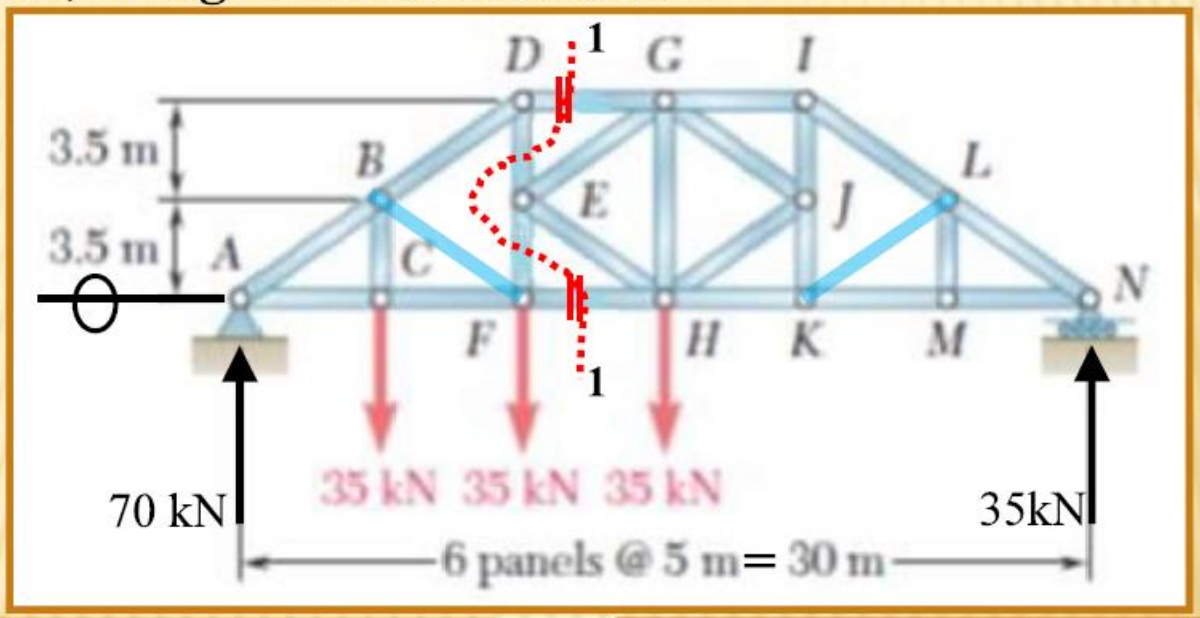
$$\sum M_E = 0$$

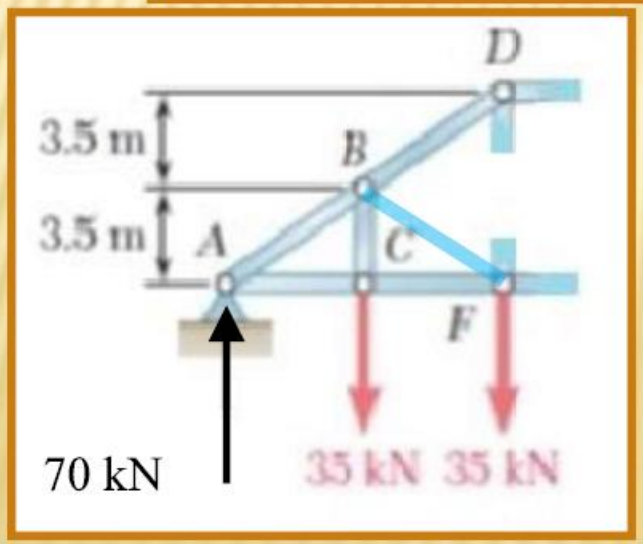
$$-20(2) - 30(4) + F_{DF}(1.75) = 0 \Rightarrow F_{DF} = 91.429 \text{ kN (T)}$$

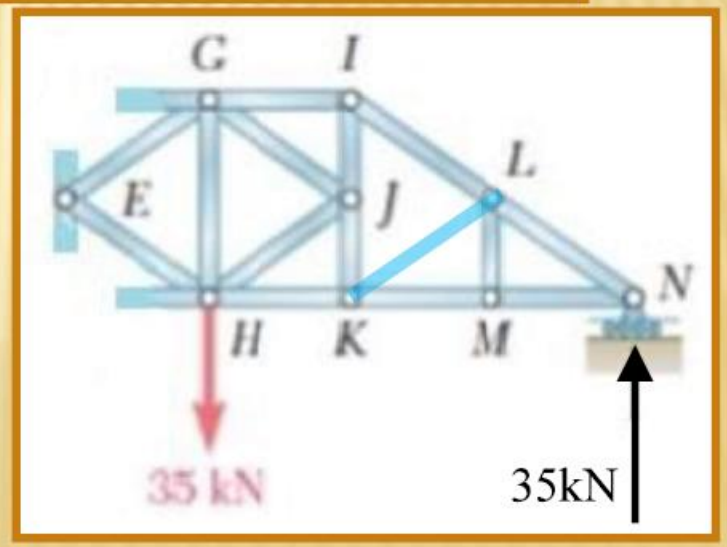
$$\sum M_C = 0$$

$$91.429(1.5) - 30(2) + F_{DE}(2) = 0 \Rightarrow F_{DE} = -38.572 \text{ kN (C)}$$

Example: Determine the force in members ***DG*** and ***FH*** of the truss shown in figure, **using method of sections** .







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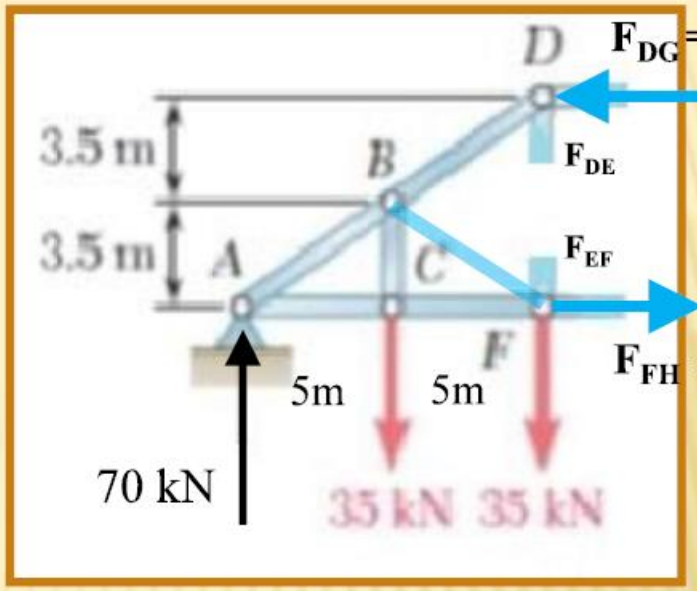


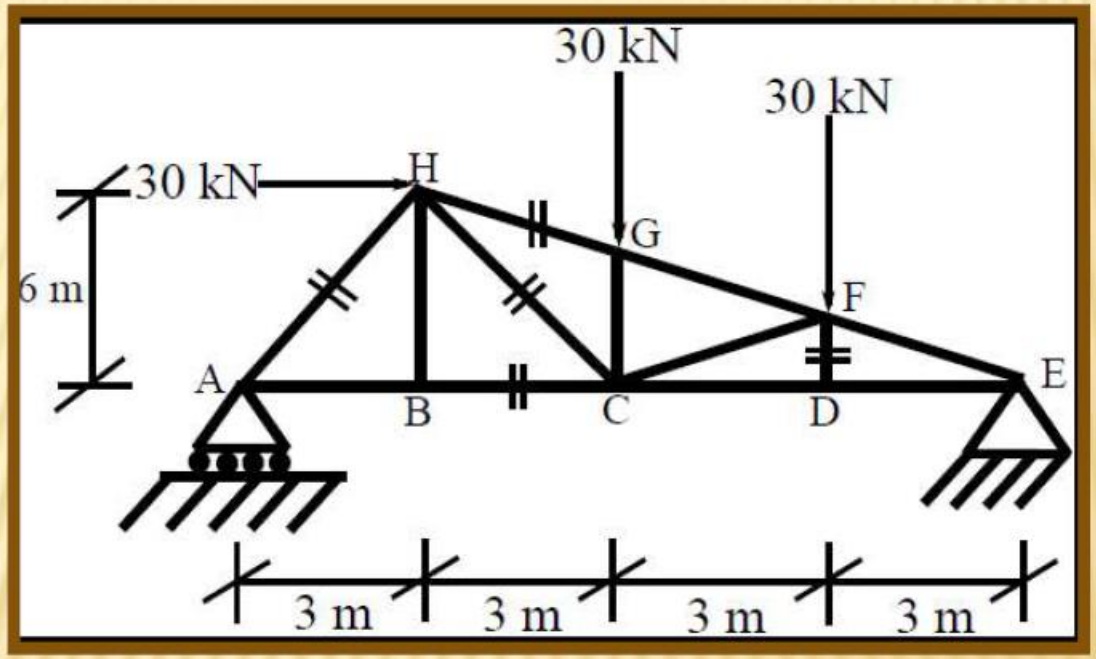
$$\sum M_F = 0$$

$$70\,(10) - 35\,(5) + F_{DG}\,(7) = 0 \Rightarrow F_{DG} = -75 \text{ kN (C)}$$

$$\sum F_X = 0$$

$$-75 + F_{FH} = 0 \Rightarrow F_{FH} = 75 \text{ kN (T)}$$

Example: Compute the force in members (AH, BC, CH, GH, and DF) of the truss shown in figure below. State if the members are in tension or compression.



Frames

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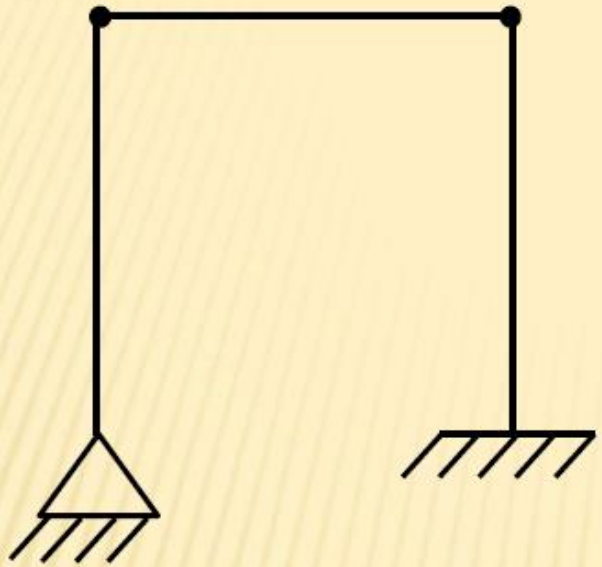
Frame: Is composed of several connected members that are either fixed (rigid) or pin (smooth) connected at their ends.

fixed (rigid) → Three Forces

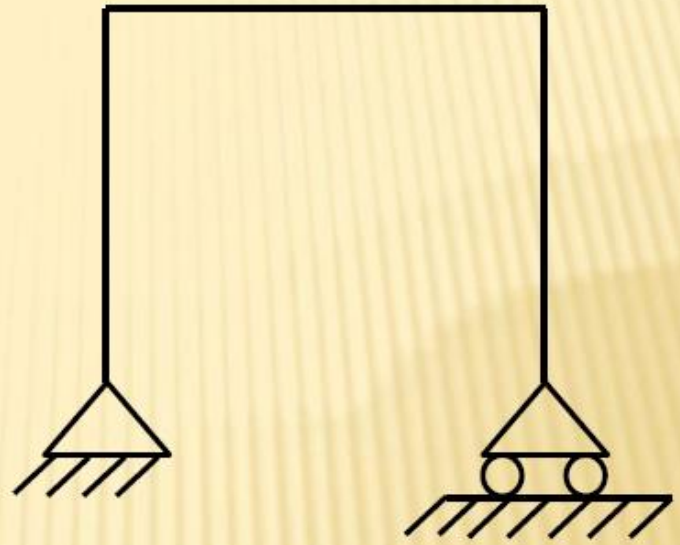
pin (smooth) → Two Forces

difference between trusses and frames

Truss	Frame
• In truss forces act only along the axis of the axis the members. Members are having transverse tension or compression.	• In frames forces are acting along the axis the members. . In addition to transverse forces.
• Each member is acted upon by two equal and opposite forces frame having line of action forces along the Centre of members. i.e. every member of truss is a two force member.	• One or more than one member of is subjected to more than two forces member. (Multiple force members).
• Member does not bend.	• Members may bend/may not bend.
• Forces are applied at the joints only.	• Forces may act anywhere on the member.



Smooth Frame



Rigid Frame

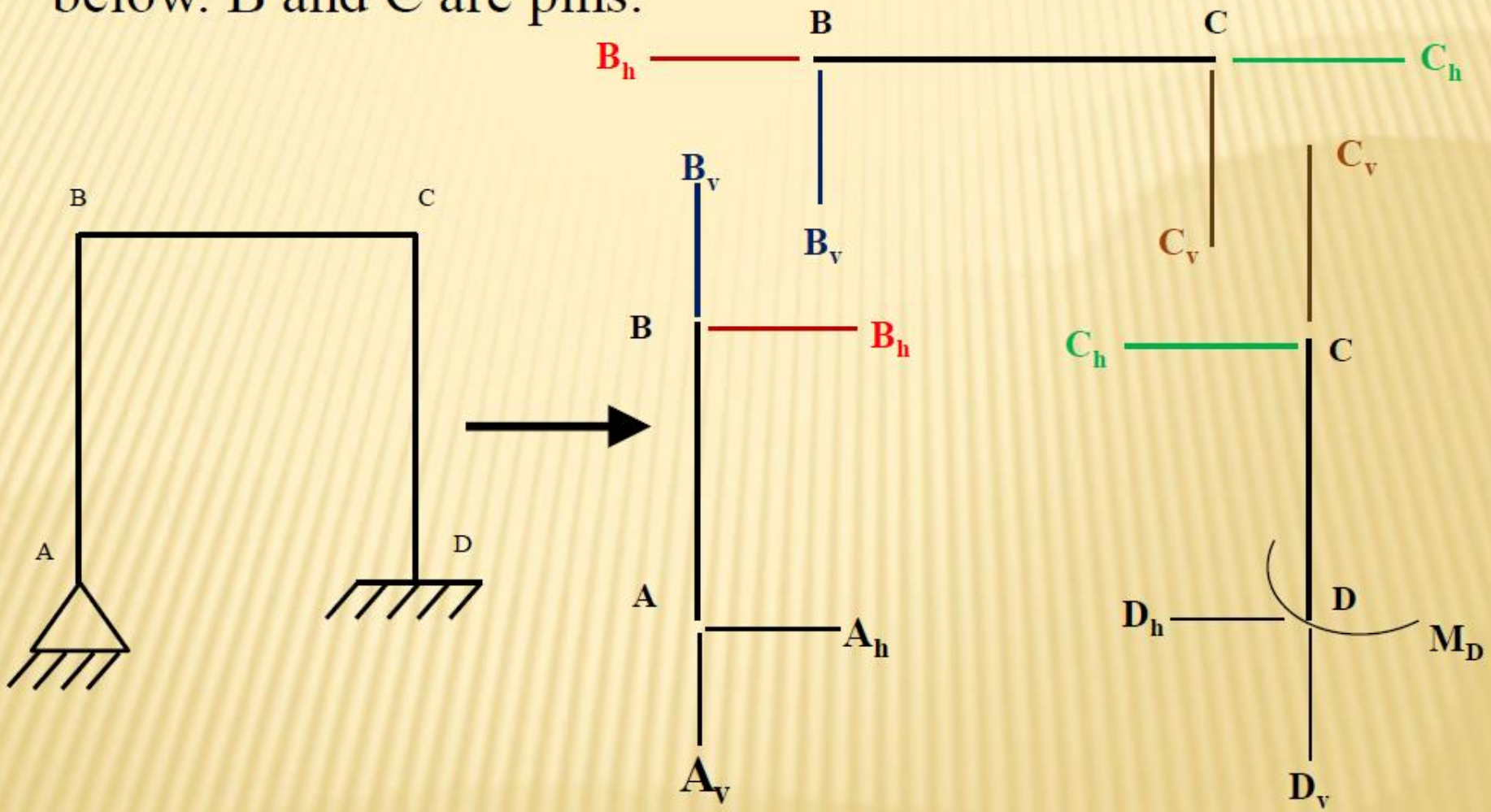
Analysis of frames: Using Method of member

1. Compute the reactions from the all frame (if possible).

2. Draw the Free Body Diagram (FBD).

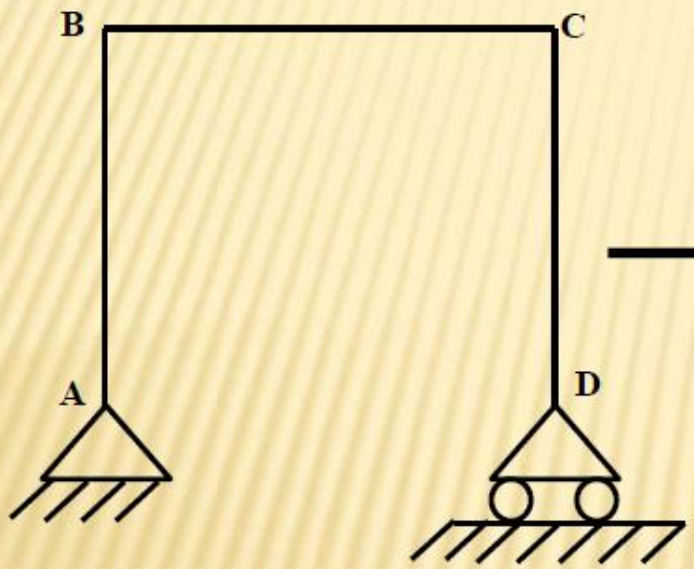
3. Apply the equilibrium equations on each member. ($\sum F_x=0$, ($\sum F_y=0$, ($\sum M=0$).

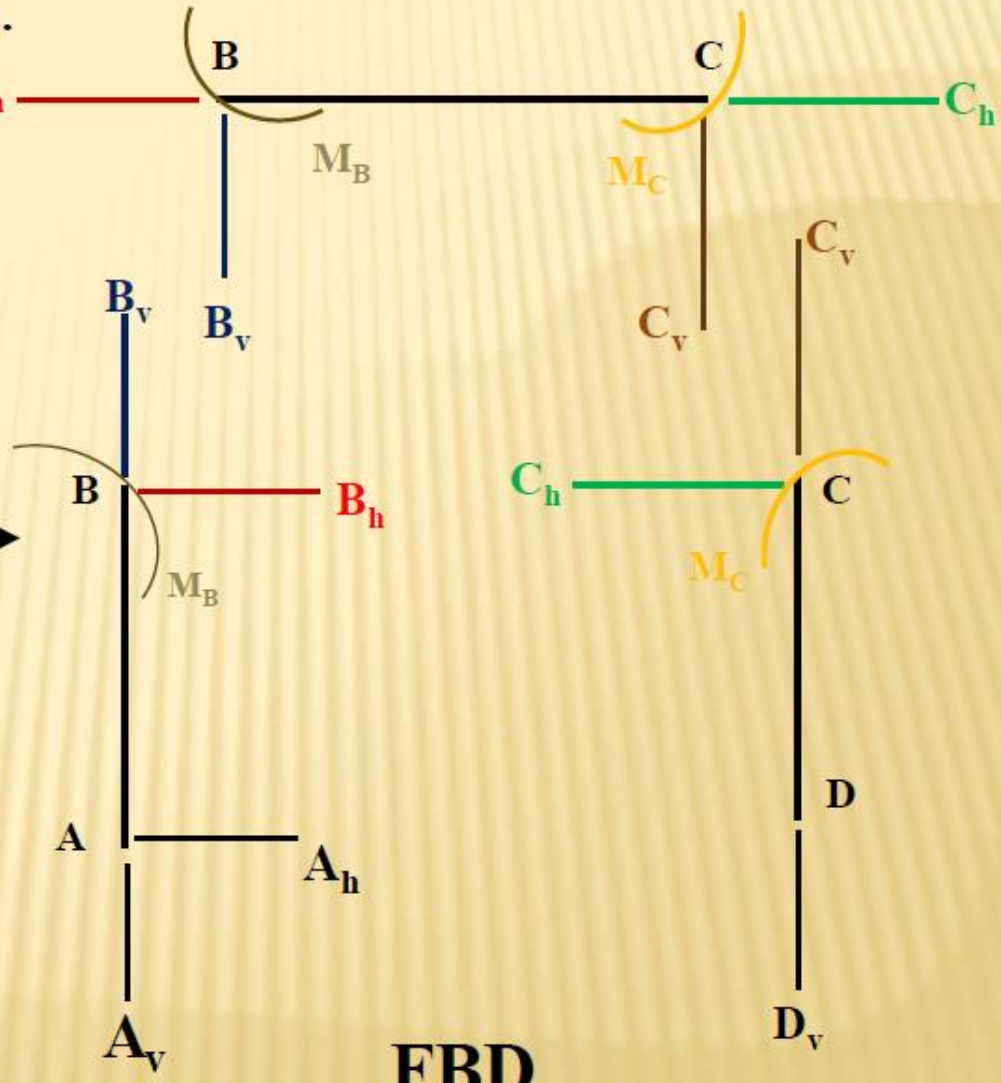
Example: Draw the FBD of the frame shown in figure below. B and C are pins.



FBD

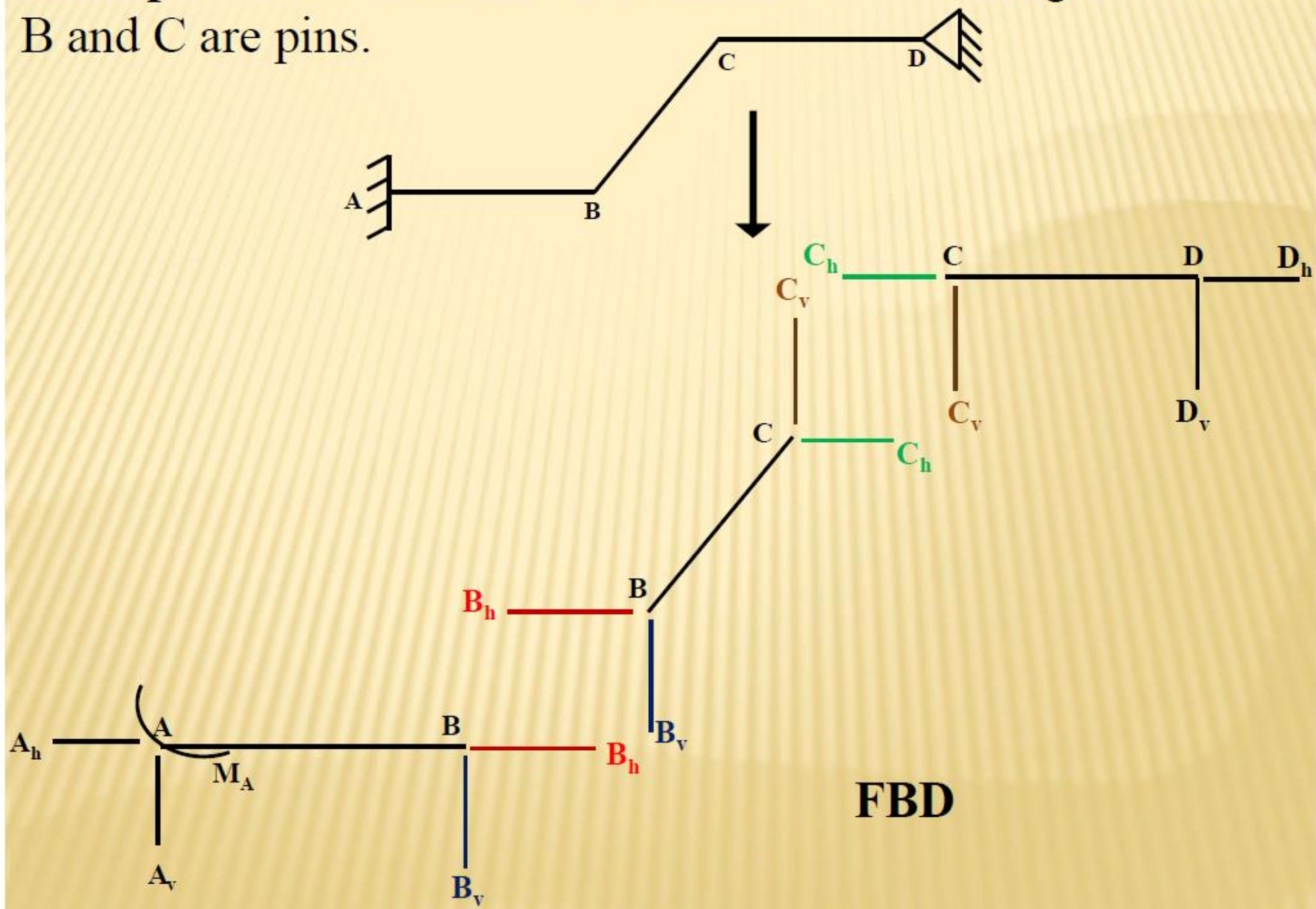
Example: Draw the FBD of the frame shown in figure below. B and C are rigid.





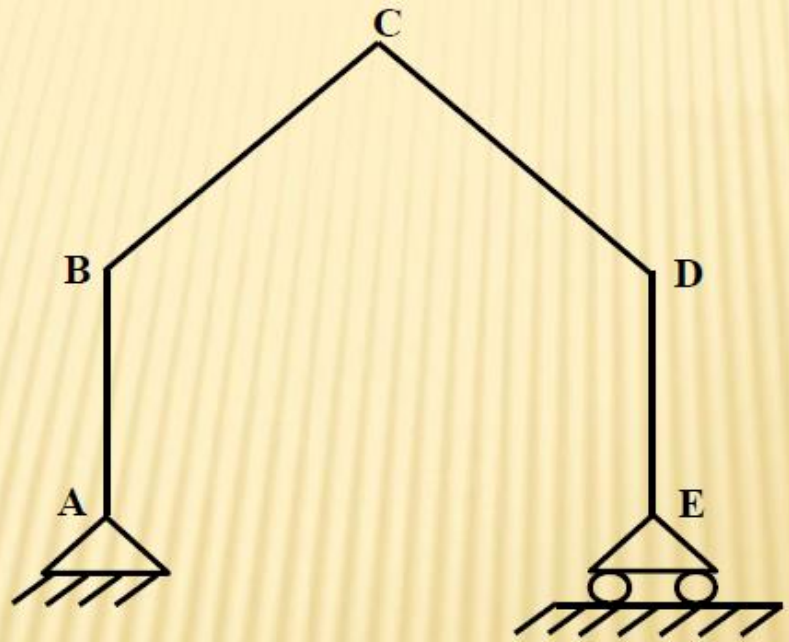
FBD

Example: Draw the FBD of the frame shown in figure below. B and C are pins.



H.W.

Draw the FBD of the frame shown in figure below. B, C and D are rigid.



Analysis of Frames
Method of Member

- Compute the reactions from the all frame (if possible).
- Draw the Free Body Diagram (FBD).
- Apply the equilibrium equations on each member. ($\sum F_x=0$, ($\sum F_y=0$, ($\sum M=0$).

Example: For the frame shown in figure below, calculate the forces in all members. Joints B and C are rigid connected.

Calculate the reactions

$$\sum M@A = 0$$

$$40(2) + 20(3) - D_v(4) = 0$$

$$\rightarrow D_v = 35\ kN \uparrow$$

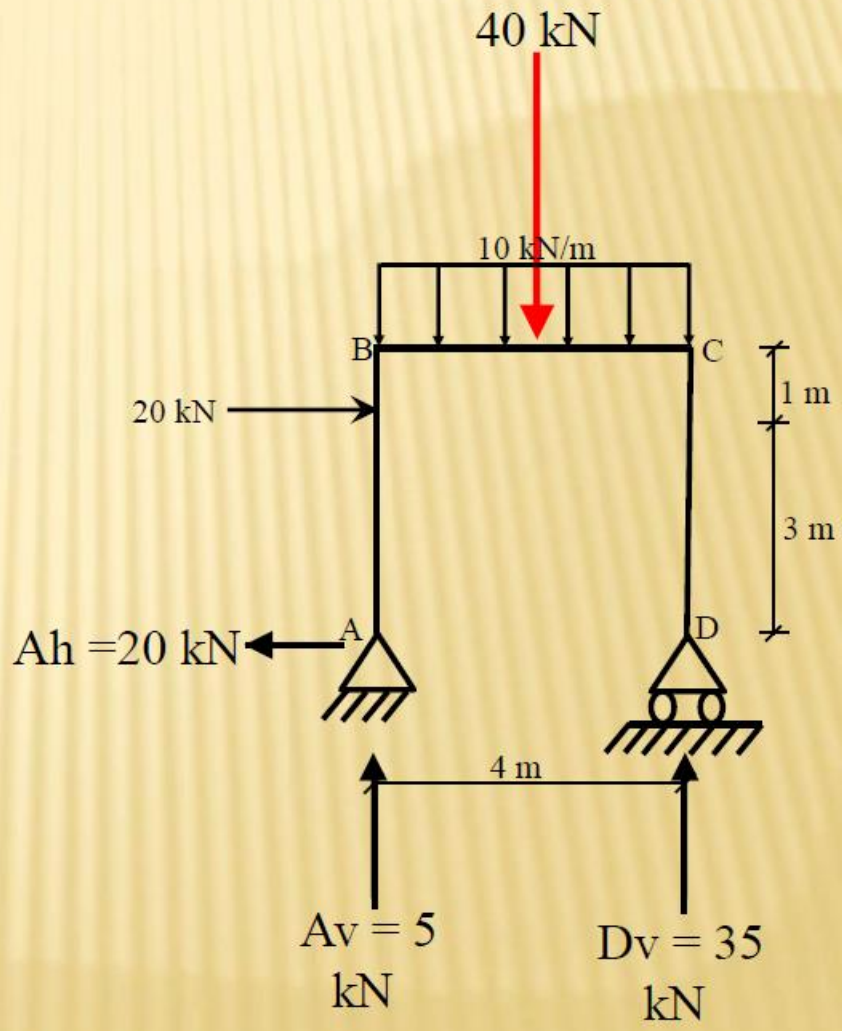
$$\sum M@D = 0$$

$$-40(2) + 20(3) + A_v(4) = 0$$

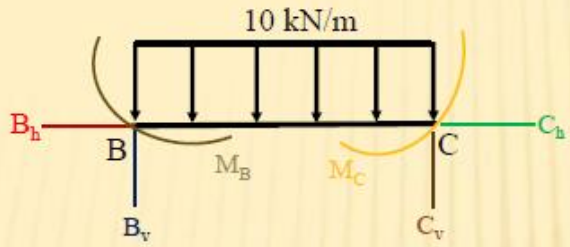
$$\rightarrow A_v = 5\ kN \uparrow$$

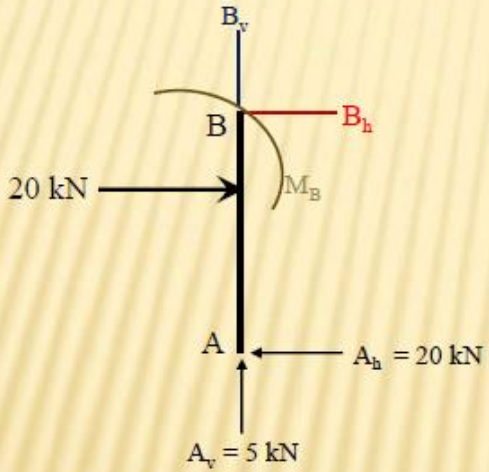
$$\sum f_x = 0$$

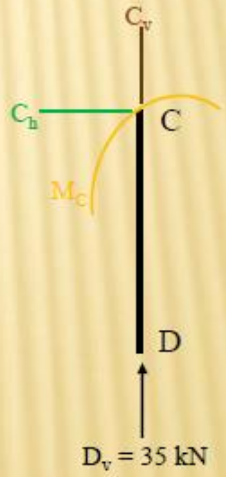
$$20 - A_h = 0 \rightarrow A_h = 20\ kN \leftarrow$$



- Draw FBD of each member:







- Apply the equations of equilibrium on each member:

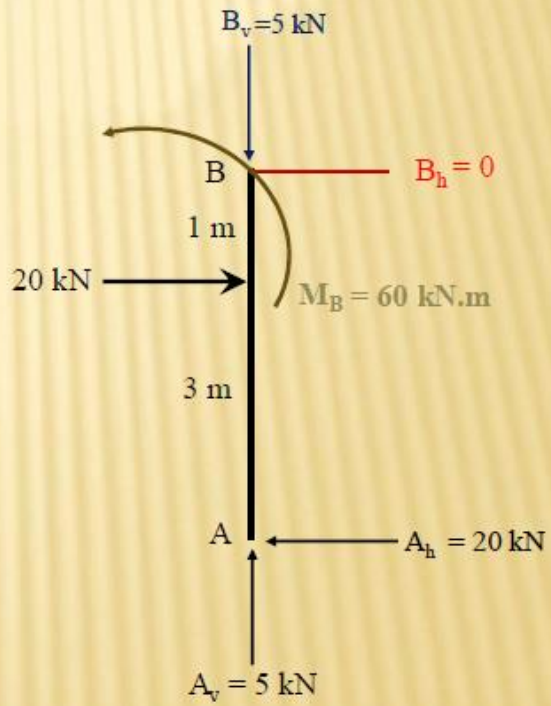
- Apply the equations of equilibrium on each member:

Member AB:

$$\sum f_x = 0$$
$$20 - 20 + B_h = 0 \rightarrow B_h = 0$$

$$\sum f_y = 0$$
$$5 - B_v = 0 \rightarrow B_v = 5 \downarrow$$

$$\sum M@A = 0$$
$$20(3) - M_B = 0 \rightarrow M_B$$
$$= 60\ kN.m \text{ Counter clock wise}$$

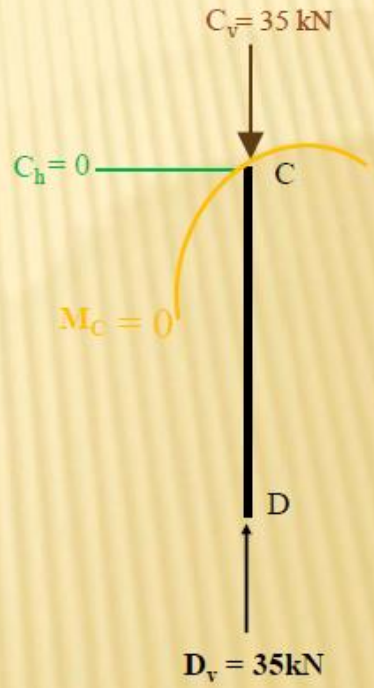


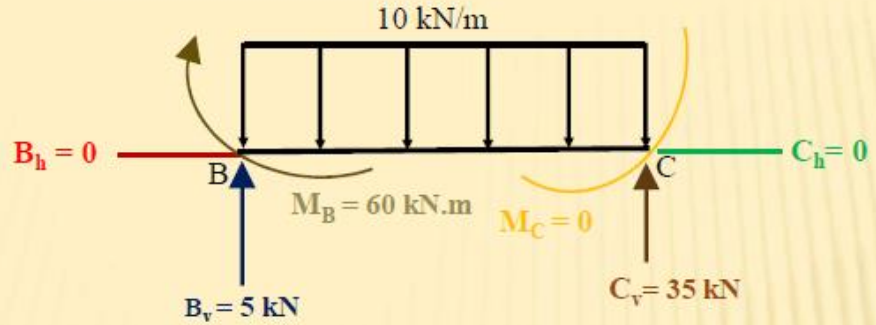
Member CD:

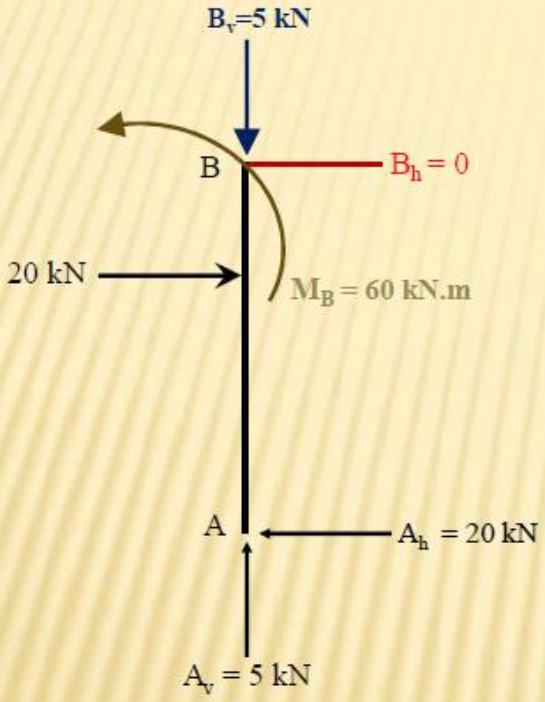
$$\sum f_x = 0$$
$$C_h = 0$$

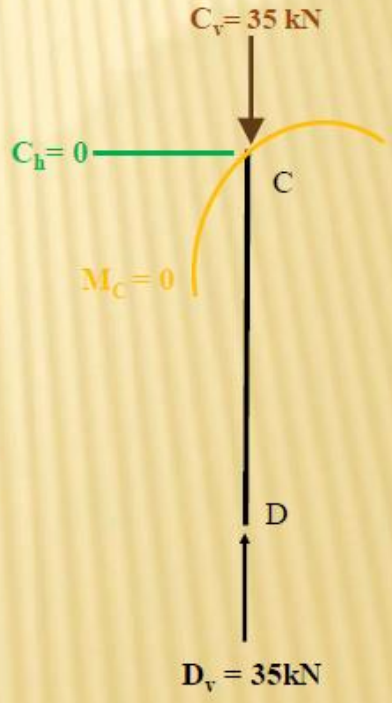
$$\sum f_y = 0$$
$$35 - c_v = \rightarrow c_v$$
$$= 35kN \downarrow$$

$$\sum M@D = 0 \rightarrow M_c = 0$$



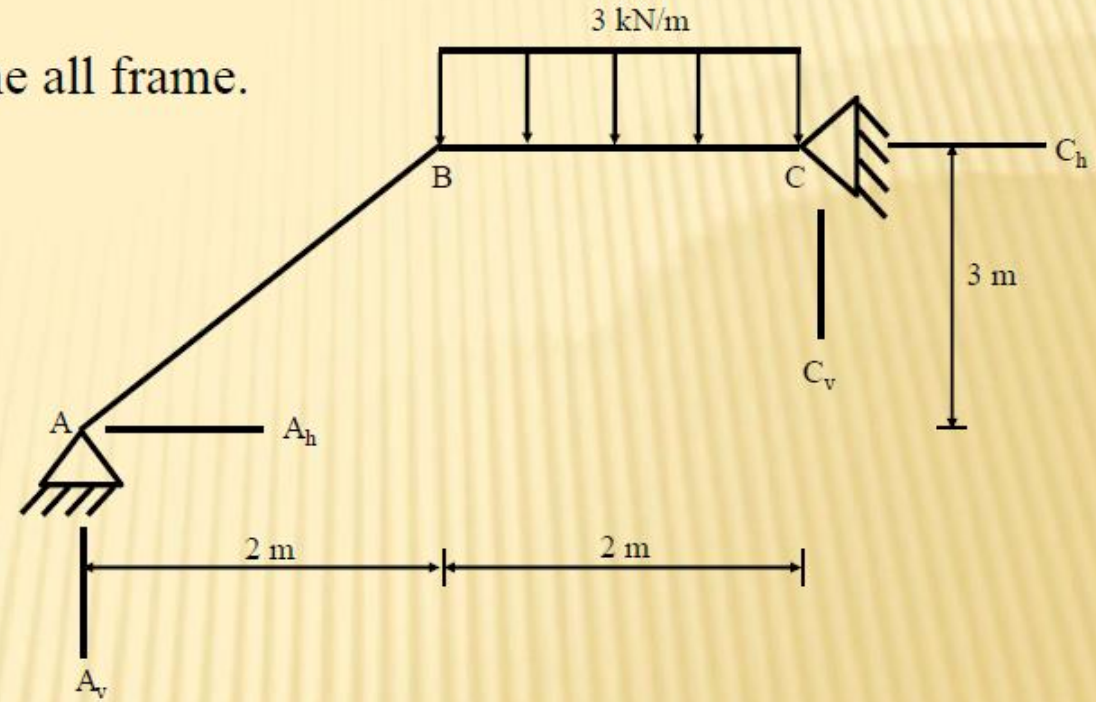

10 kN/m
$B_h = 0$
B
C
$C_h = 0$
$M_B = 60$ kN.m
$M_C = 0$
$B_v = 5$ kN
$C_v = 35$ kN


$B_v = 5$ kN
B
$B_h = 0$
20 kN
$M_B = 60$ kN.m
A
$A_h = 20$ kN
$A_v = 5$ kN

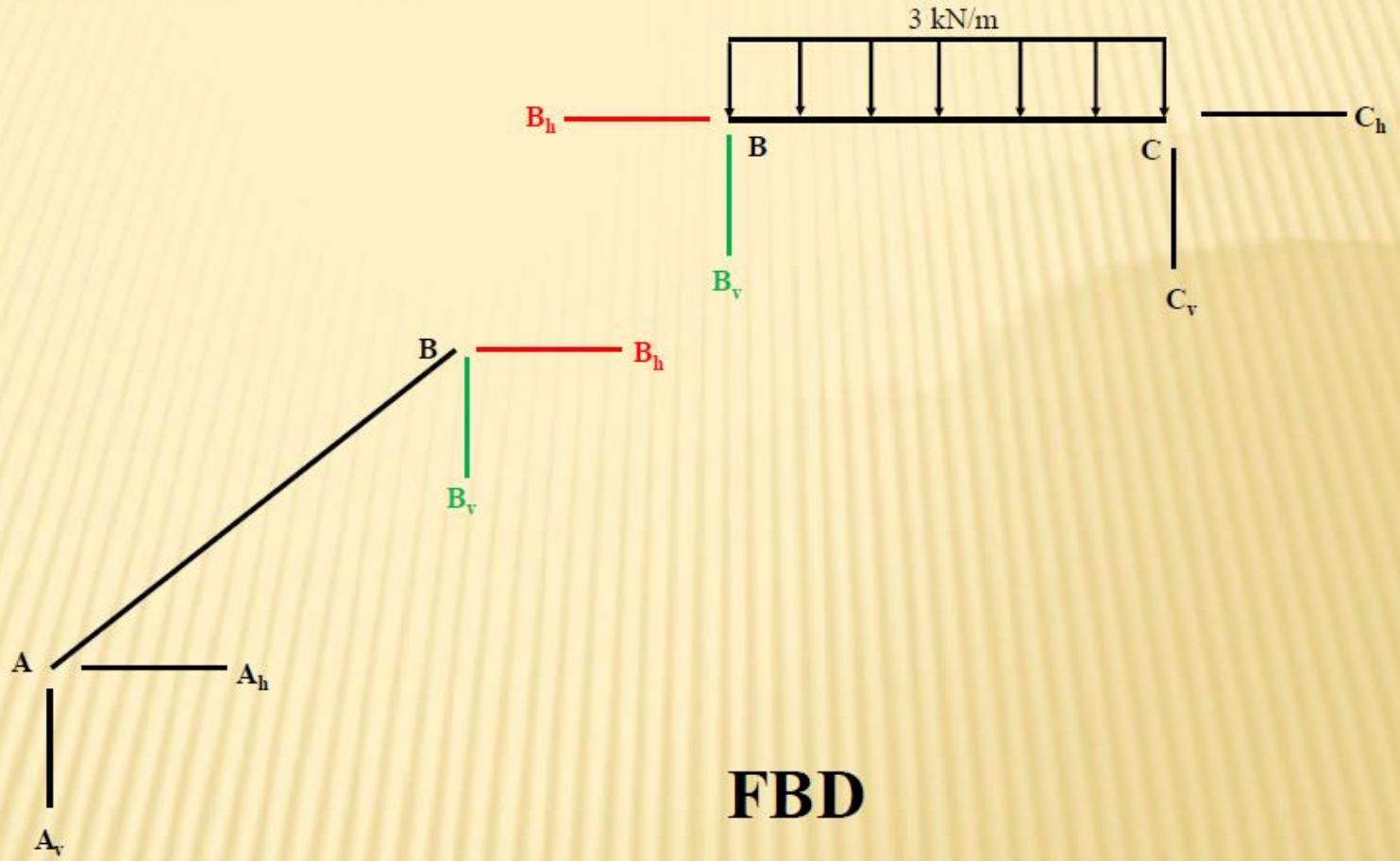

$C_v = 35$ kN
$C_h = 0$
C
$M_C = 0$
D
$D_v = 35$kN

Example: Determine the horizontal and vertical components of reaction at the pins *A*, *B*, and *C* of the two-member frame shown in the figure

- Calculate reactions from the all frame.



- Draw the FBD of the frame:



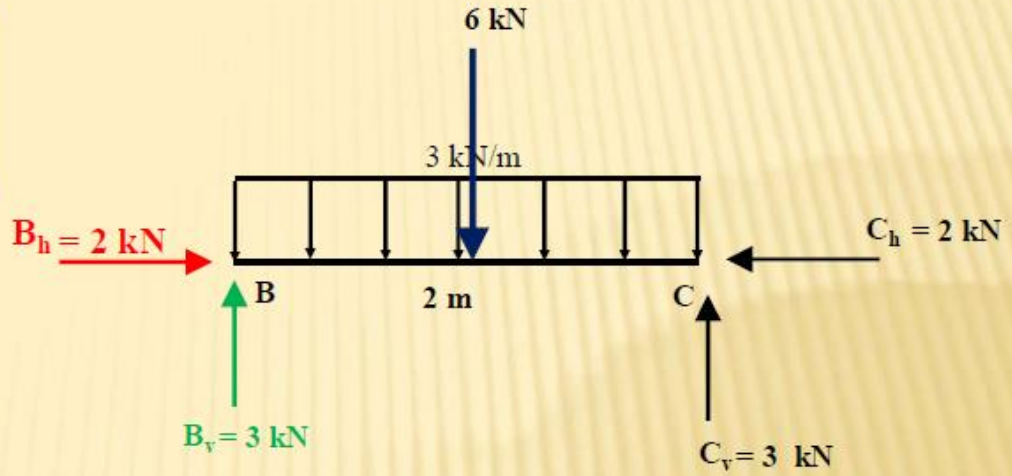
FBD

- Apply the equations of equilibrium on each member:

Member BC

$$\sum M@B = 0$$
$$6(1) - C_v(2) = 0 \rightarrow C_v$$
$$= 3\ kN\ \uparrow$$

$$\sum M@C = 0$$
$$-6(1) + B_v(2) = 0 \rightarrow B_v$$
$$= 3\ kN\ \uparrow$$

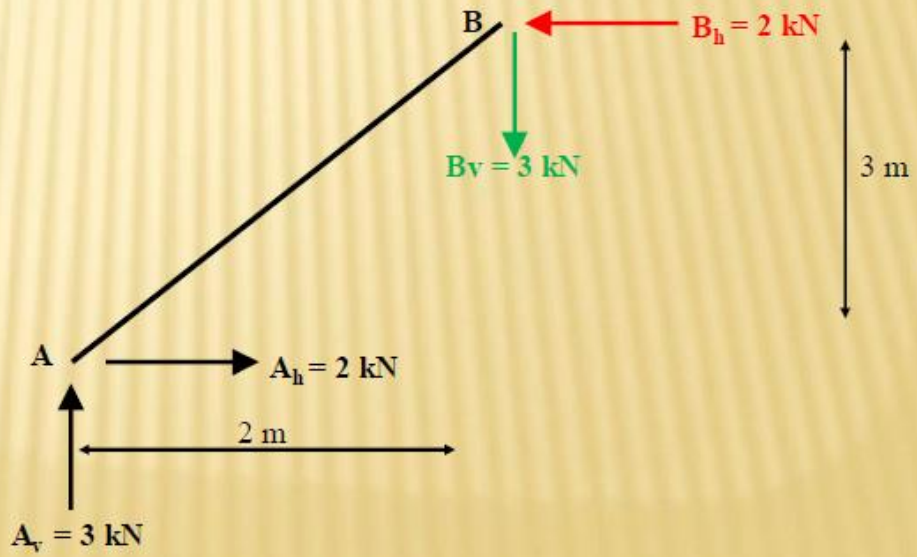


Member AB

$$\sum F_y = 0$$
$$A_v - 3 = 0 \rightarrow A_v = 3\ kN\ \uparrow$$

$$\sum M@A = 0$$
$$3(2) - B_h(3) = 0 \rightarrow B_h$$
$$= 2\ kN\ \leftarrow$$

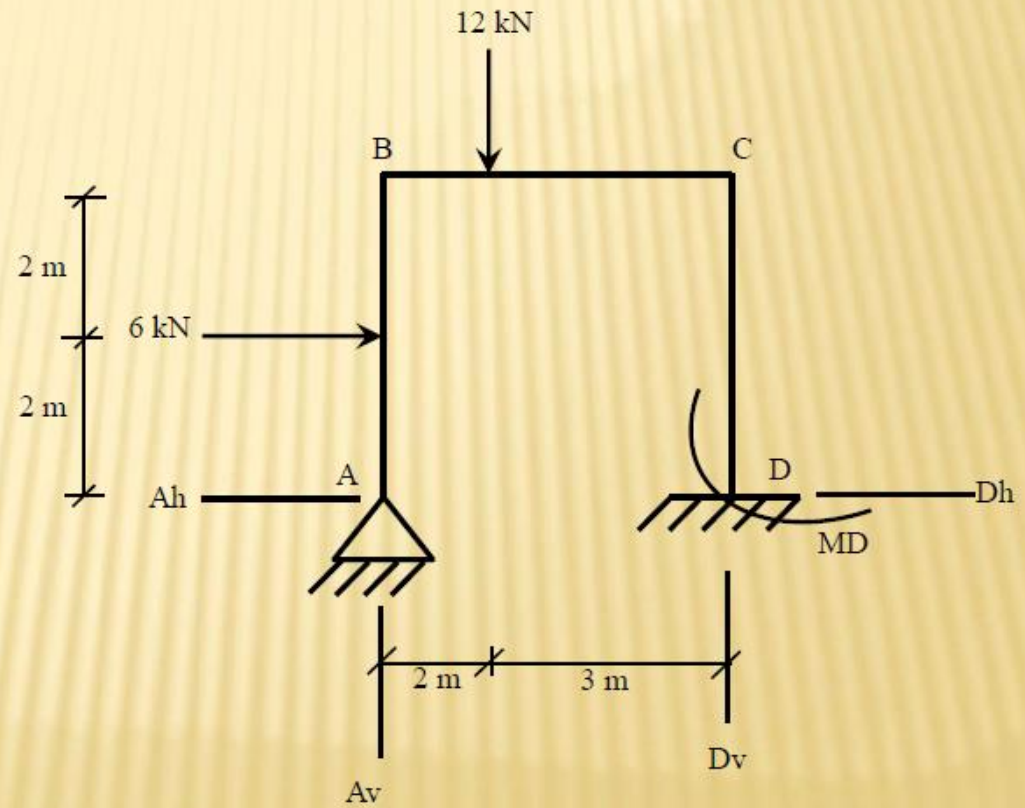
$$\sum F_x = 0$$
$$-2 + A_h = 0 \rightarrow A_h = 2\ kN\ \rightarrow$$



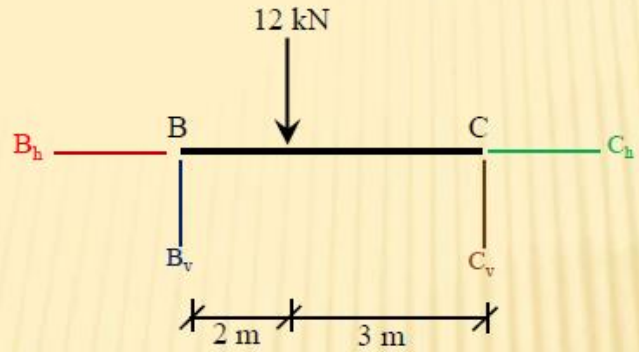
Analysis of Frames
Method of Member

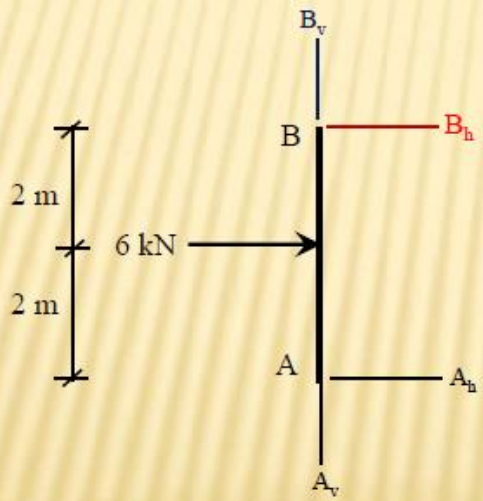
Example: For the frame shown in figure below, calculate the forces in all members. Joints B and C are pin connected.

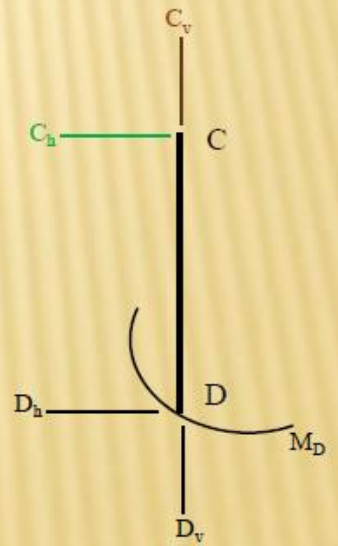
Calculate the reactions



Draw FBD







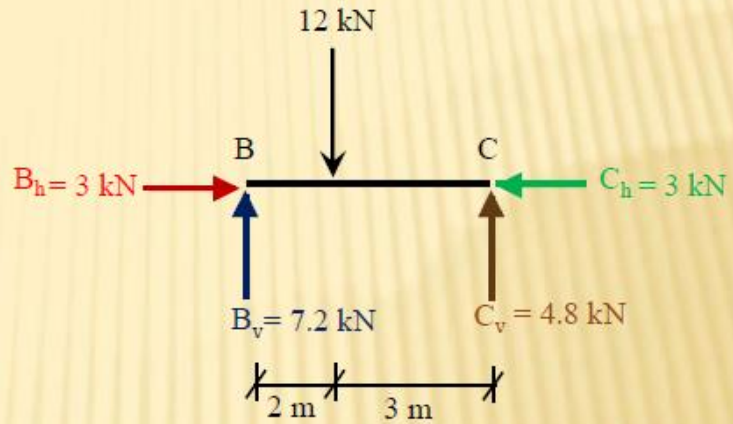
Apply the equations of equilibrium on each member:

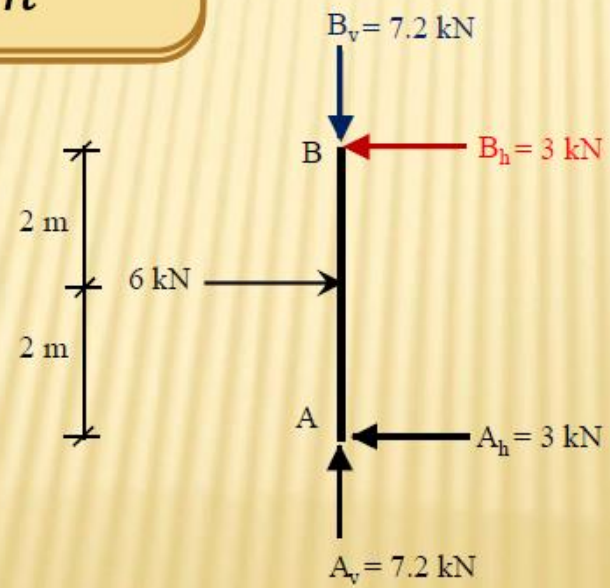
Member BC

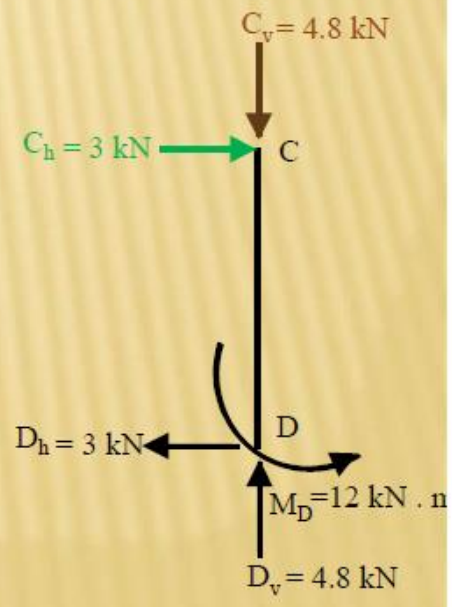
$$\sum F_y = 0$$
$$-4.8 + D_V = 0$$
$$D_V = 4.8\ kN\ \uparrow$$

$$\sum M@_D = 0$$
$$3(4) - M_D = 0$$
$$M_D = 12\ kN.m$$

$$\sum F_x = 0$$
$$3 - D_h = 0$$
$$D_h = 3\ kN\ \leftarrow$$

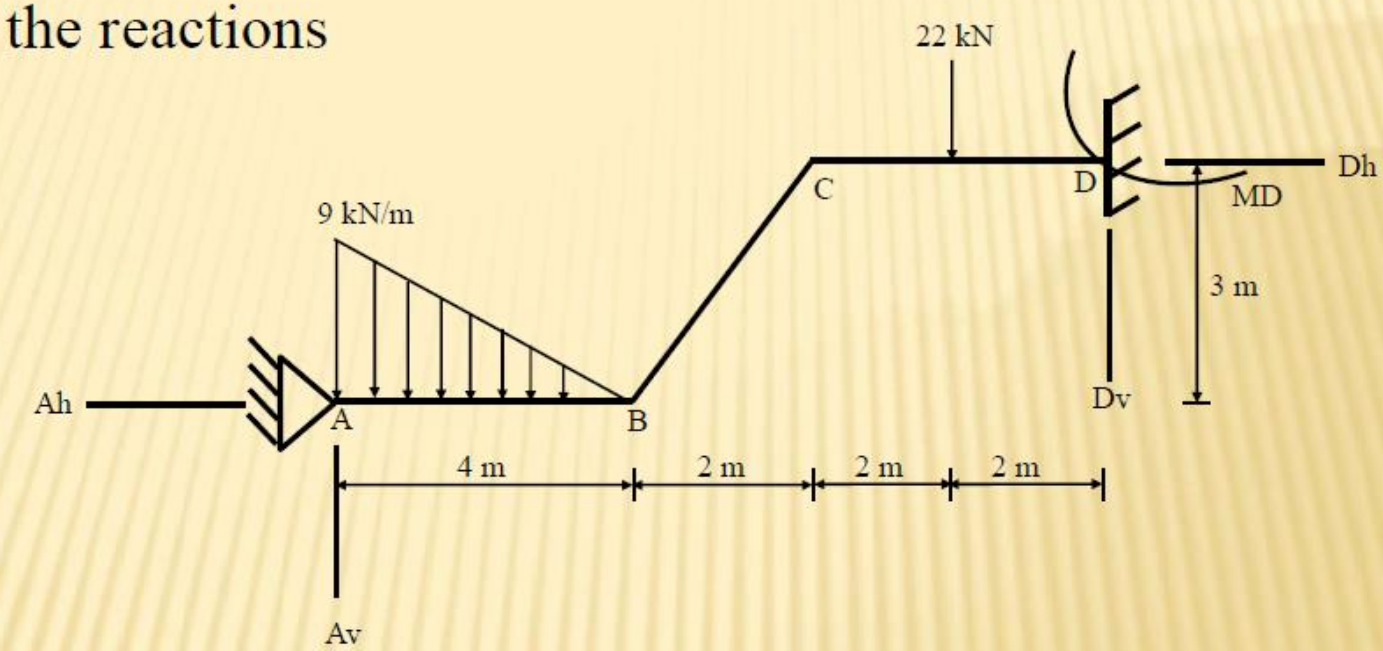




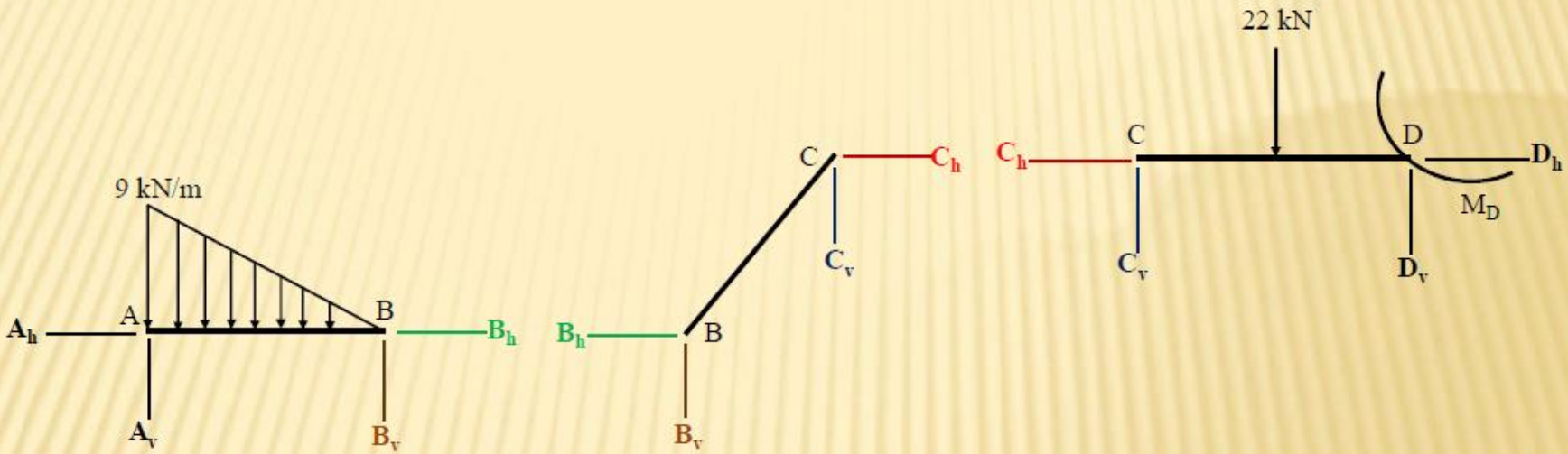


Example: For the frame shown in figure below, calculate the forces in all members. Joints B and C are pin connected.

Calculate the reactions



Draw FBD



Apply the equations of equilibrium on each member:

Member AB

Member BC

Member CD

$$\sum F_x = 0$$

$$-4 + D_h = 0$$

$$D_h = 4\ kN \rightarrow$$

$$C_V = 6\ kN$$

$$B_v = 6kN$$

$$\sum F_y = 0$$

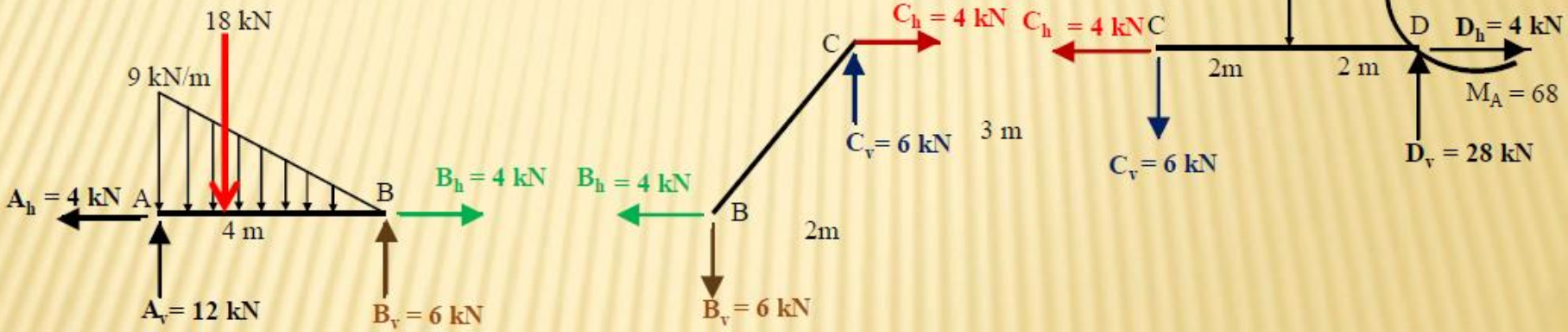
$$-22 - 6 + D_v = 0$$

$$D_v = 28\ kN \uparrow$$

$$\sum M@_D = 0$$

$$-6(4) - 22(2) + M_D = 0$$

$$M_D = 68\ kN.m\ clock\ wise$$



Example: For the frame shown in figure. Compute the forces at pin connected joints A, B, and C.

- Calculate the reactions:

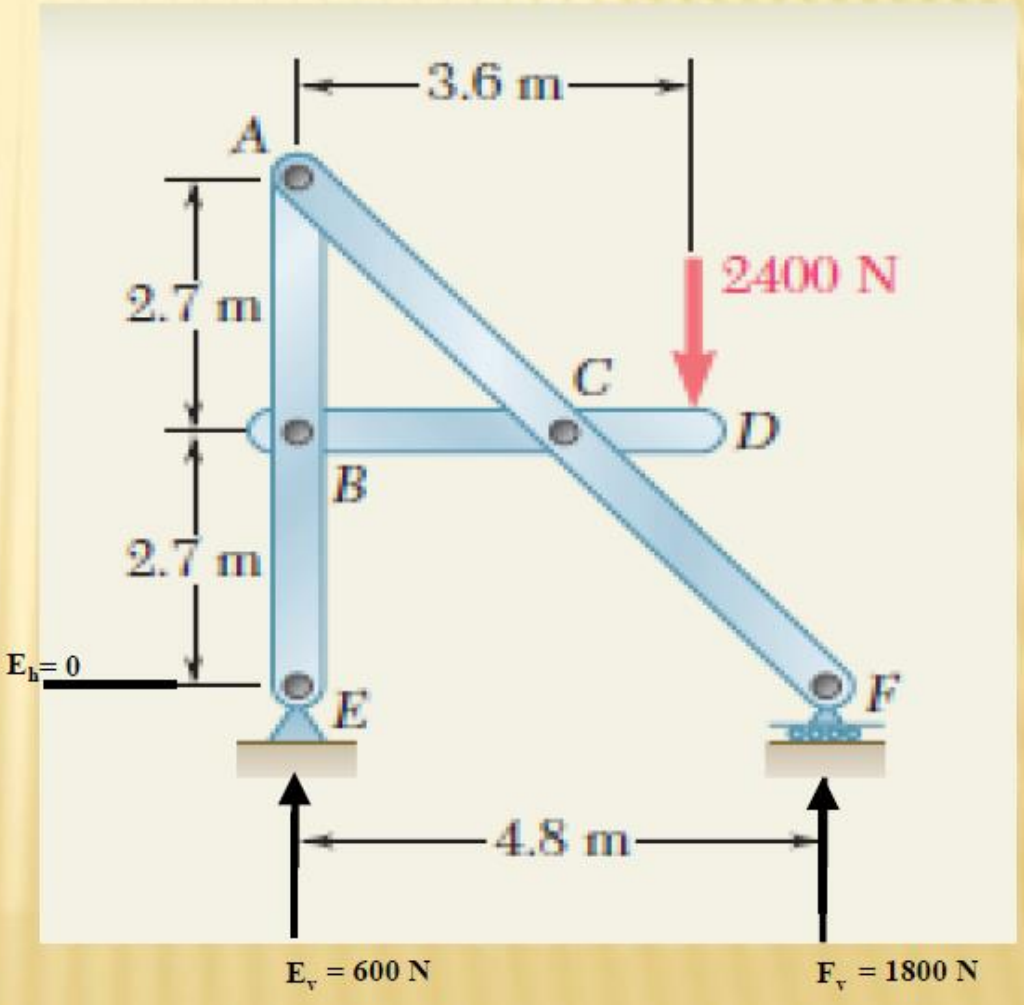
$$\sum M@E = 0$$

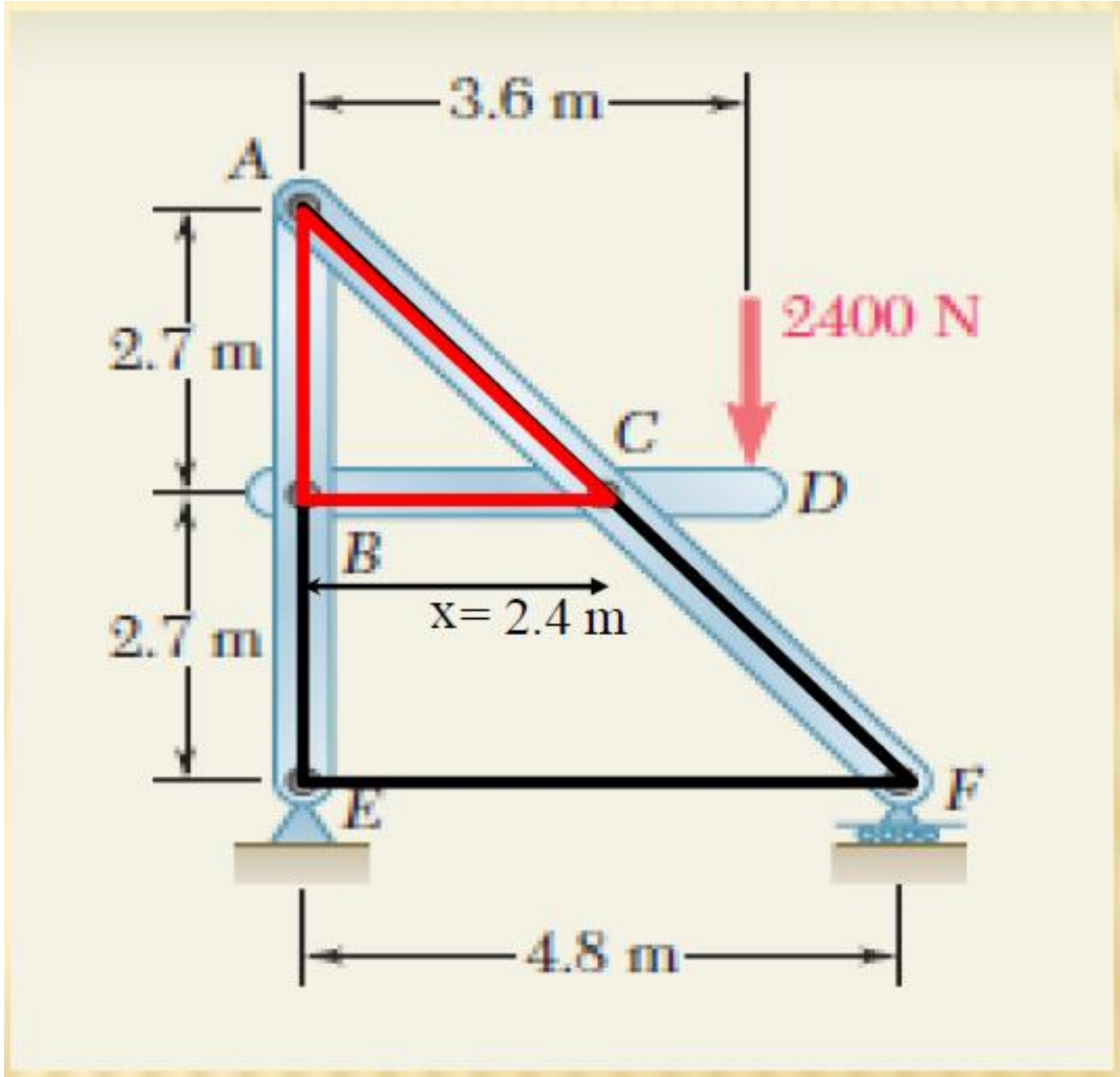
$$2400(3.6) - F_v(4.8) = 0 \rightarrow F_v = 1800\ N\ \uparrow$$

$$\sum M@F = 0$$

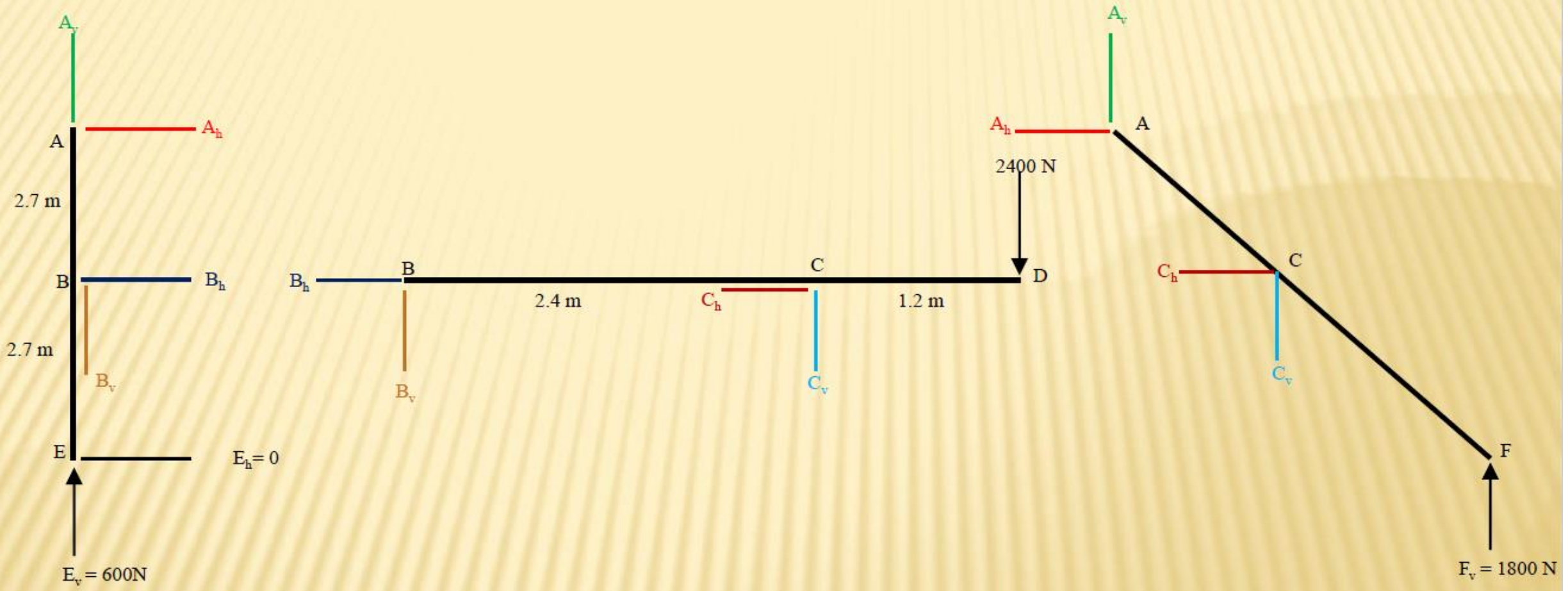
$$-2400(1.2) + E_v(4.8) = 0 \rightarrow E_v = 600\ N\ \uparrow$$

$$\sum Fx = 0 \rightarrow E_h = 0$$




3.6 m
A
2.7 m
2400 N
C
D
B
x= 2.4 m
2.7 m
E
F
4.8 m

- Draw FBD:



- Apply the equations of equilibrium on each member:

Member ABE:

$$\sum M@A = 0 \rightarrow B_h = 0$$

$$\sum F_X = 0 \rightarrow A_h =$$

Member BCD

$$\sum F_X = 0 \rightarrow C_h = 0$$

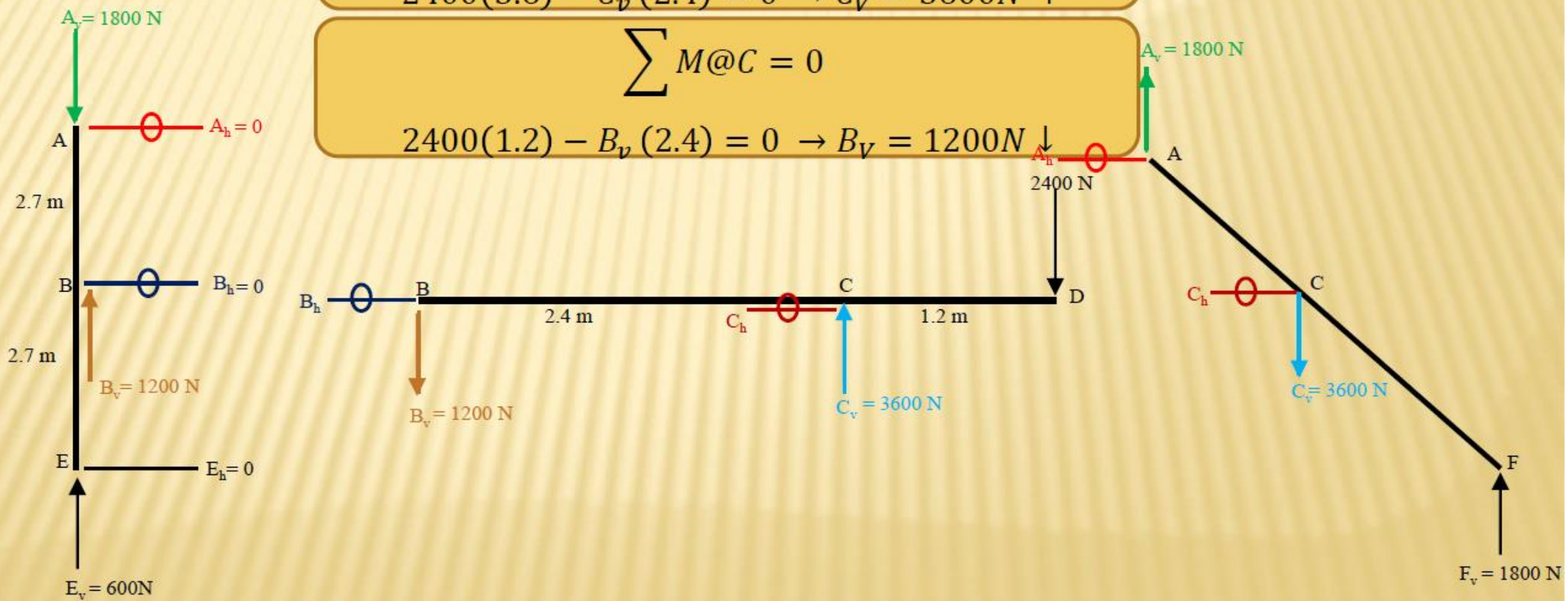
$$\sum M@B = 0$$

$$2400(3.6) - C_v\,(2.4) = 0 \rightarrow C_V = 3600N \uparrow$$

$$\sum M@C = 0$$

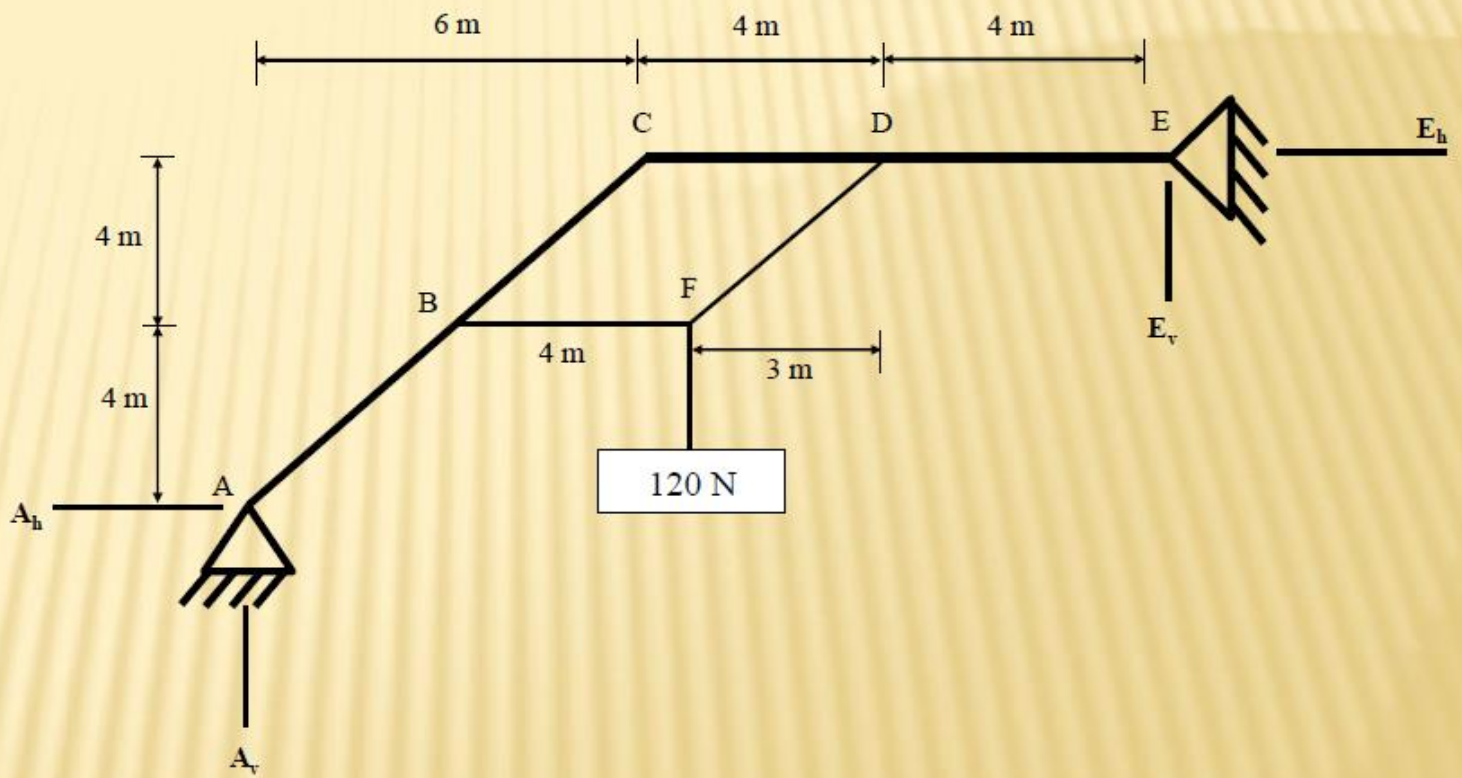
$$2400(1.2) - B_v\,(2.4) = 0 \rightarrow B_V = 1200N \downarrow$$

Member ACF

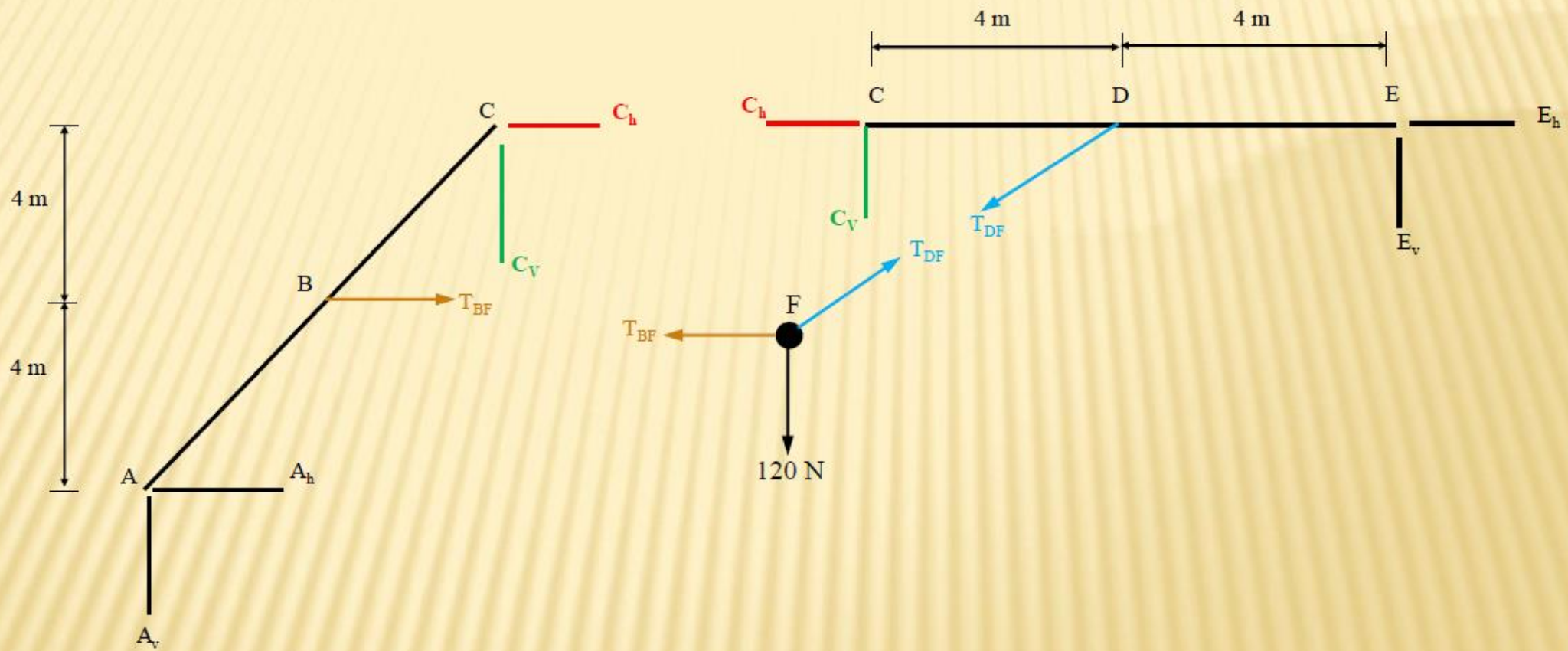


Example: Calculate the components of the pin force at C of the frame shown in figure.

- Calculate the reactions:



- Draw FBD:



- Apply the equations of equilibrium on each member:

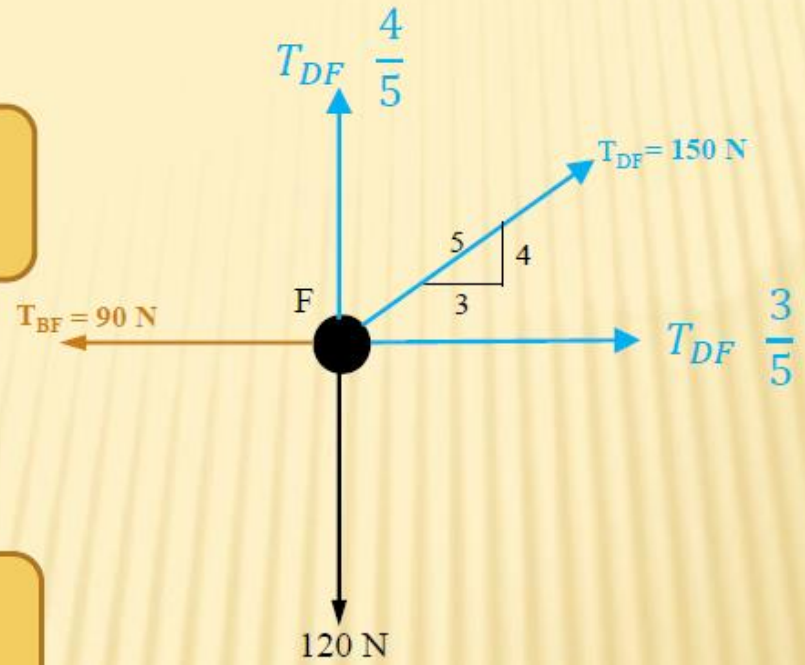
Node F:

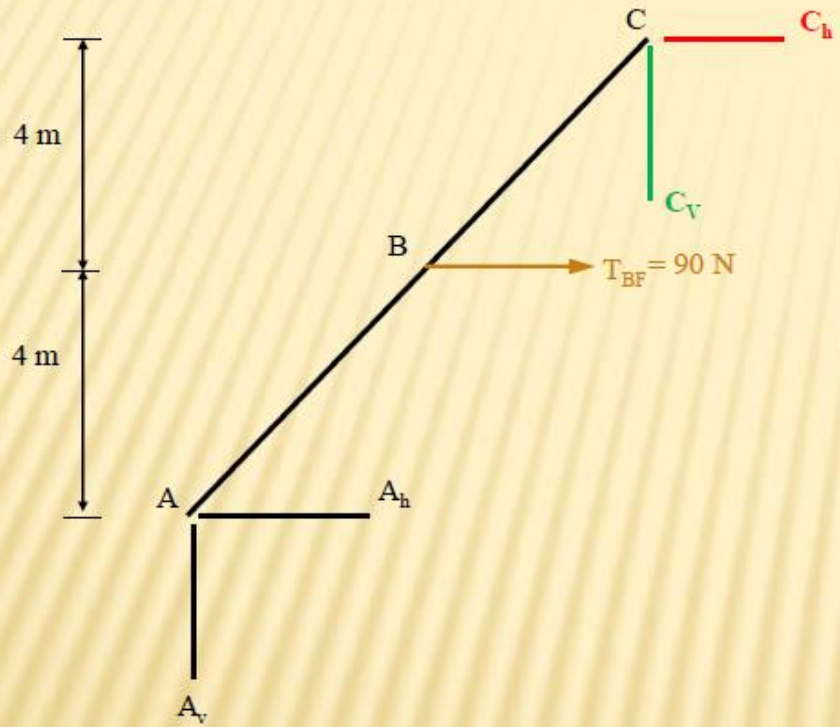
$$\sum Fy = 0$$

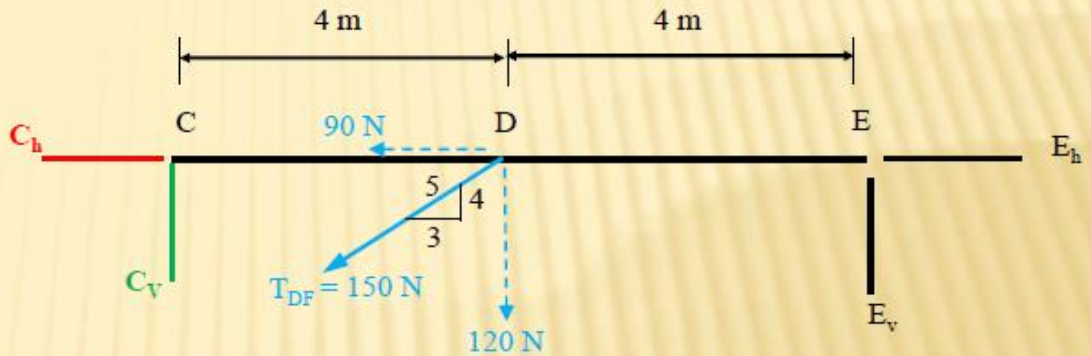
$$T_{DF}\frac{4}{5} - 120 = 0 \rightarrow T_{DF} = 150\ N$$

$$\sum F_X = 0$$

$$150\frac{3}{5} - T_{BF} = 0 \rightarrow T_{BF} = 90\ N$$



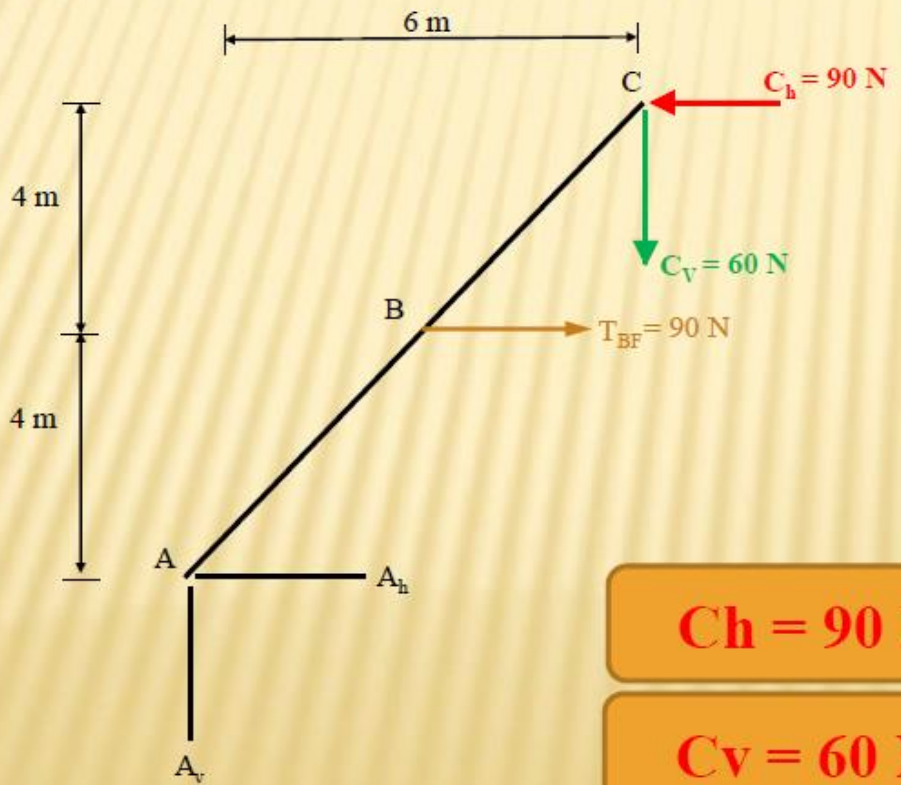

4 m
4 m
C
C_h
C_V
B
T_{BF} = 90 N
A
A_h
A_v


4 m
4 m
C_h
C
90 N
D
E
E_h
5
4
3
C_V
T_{DF} = 150 N
120 N
E_v

Member ABC

$$\sum M@\ A = 0$$

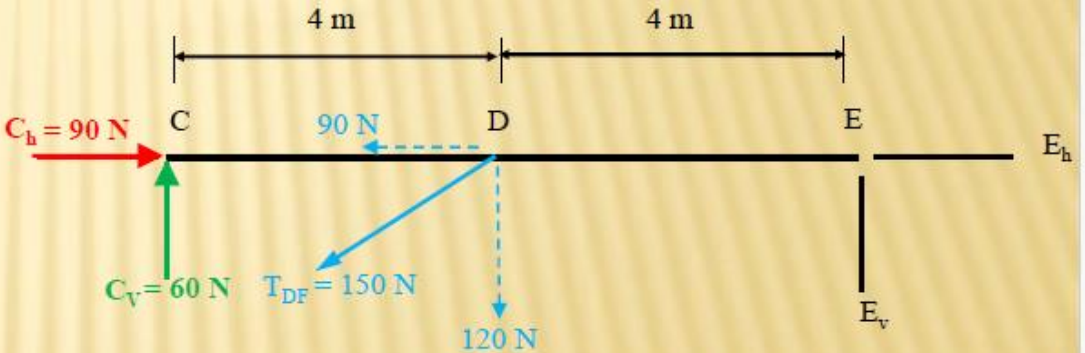
$$60(6) + 90\ (4) - Ch(8) = 0 \ \rightarrow Ch = 90\ \text{N}\leftarrow$$



Member CDE

$$\sum M@\ E = 0$$

$$-120(4) + Cv\ (8) = 0 \ \rightarrow Cv = 60\ \text{N}\uparrow$$



Ch = 90 N

Cv = 60 N

H.W.

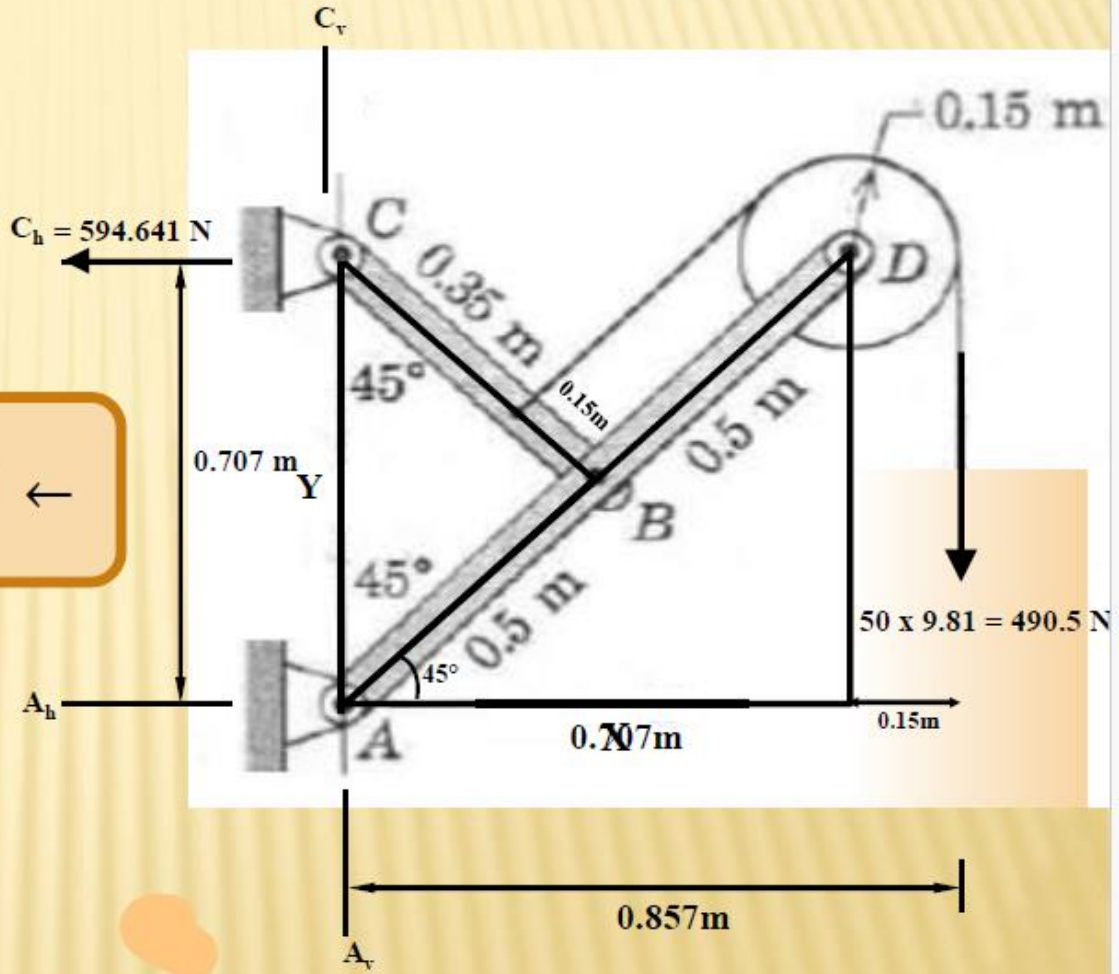
Calculate the reactions

Example: For the loaded beam as shown in figure, calculate the force components at pin B. A and C are hinge supports D is pin joint.

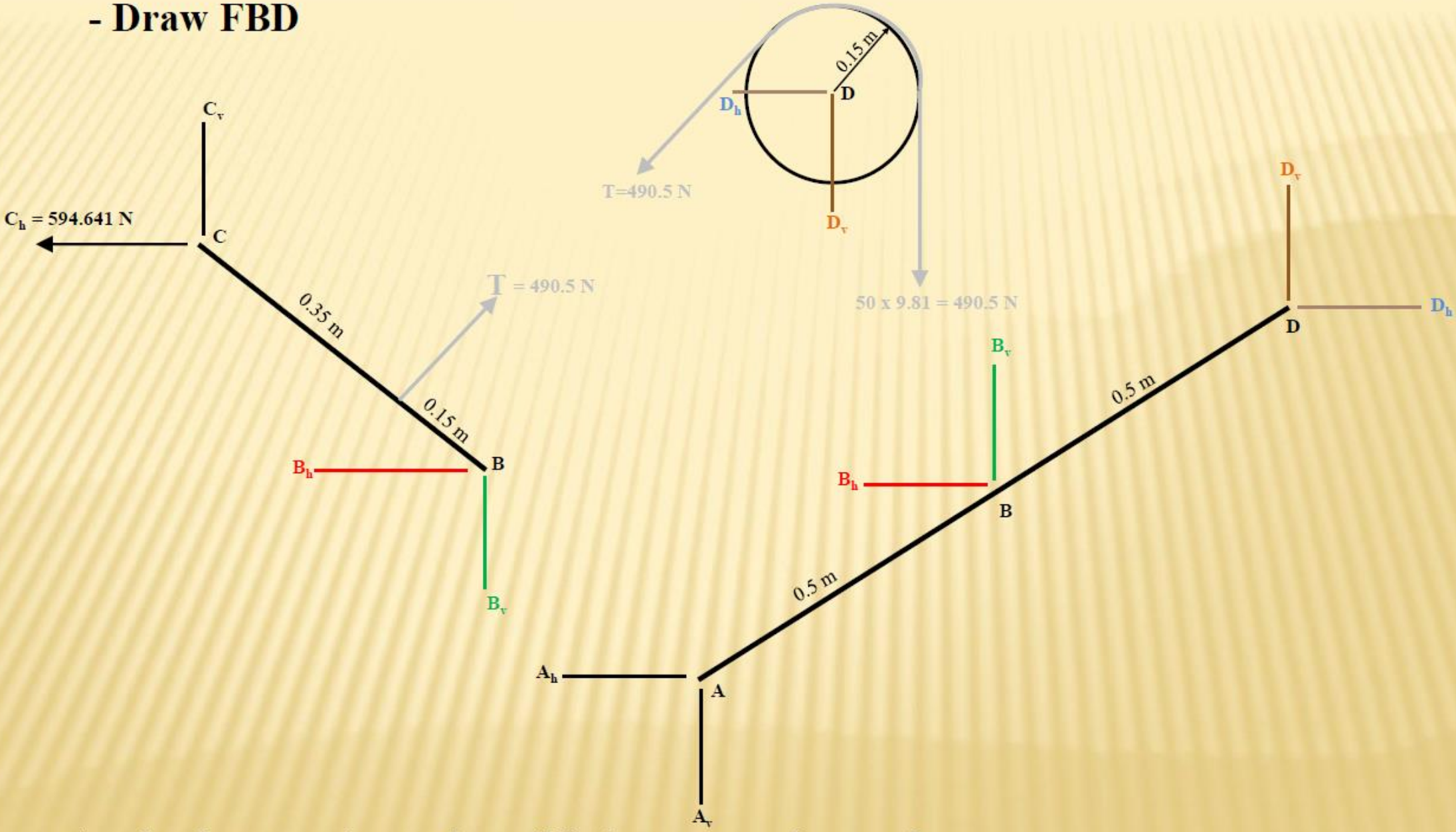
- **Calculate the reactions:**

$$\sum M@A = 0$$

$$490.5(0.857) - C_h(0.707) = 0 \rightarrow C_h = 594.641N \leftarrow$$



- **Draw FBD**



- **Apply the equations of equilibrium on each member:**

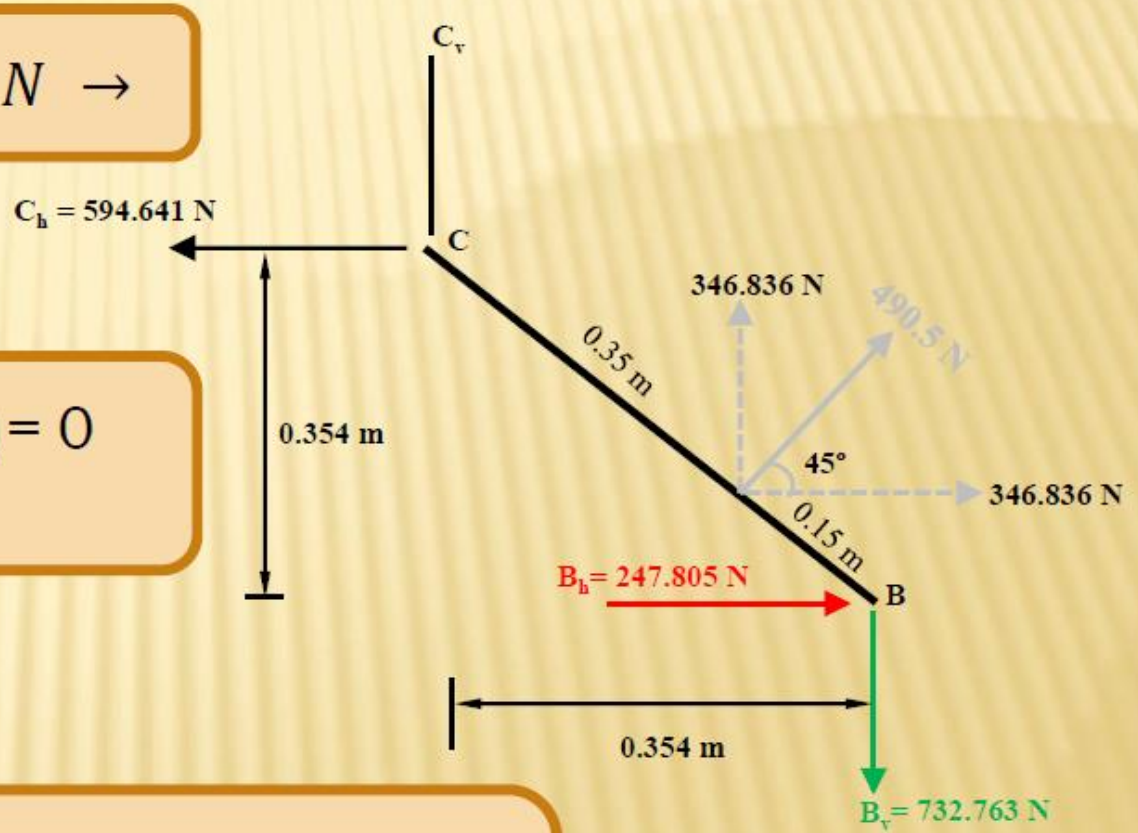
Member BC

$$\sum Fx = 0$$

$$346.836 - 594.641 + B_h = 0 \rightarrow B_h = 247.805\ N \rightarrow$$

$$\sum M @ C = 0$$

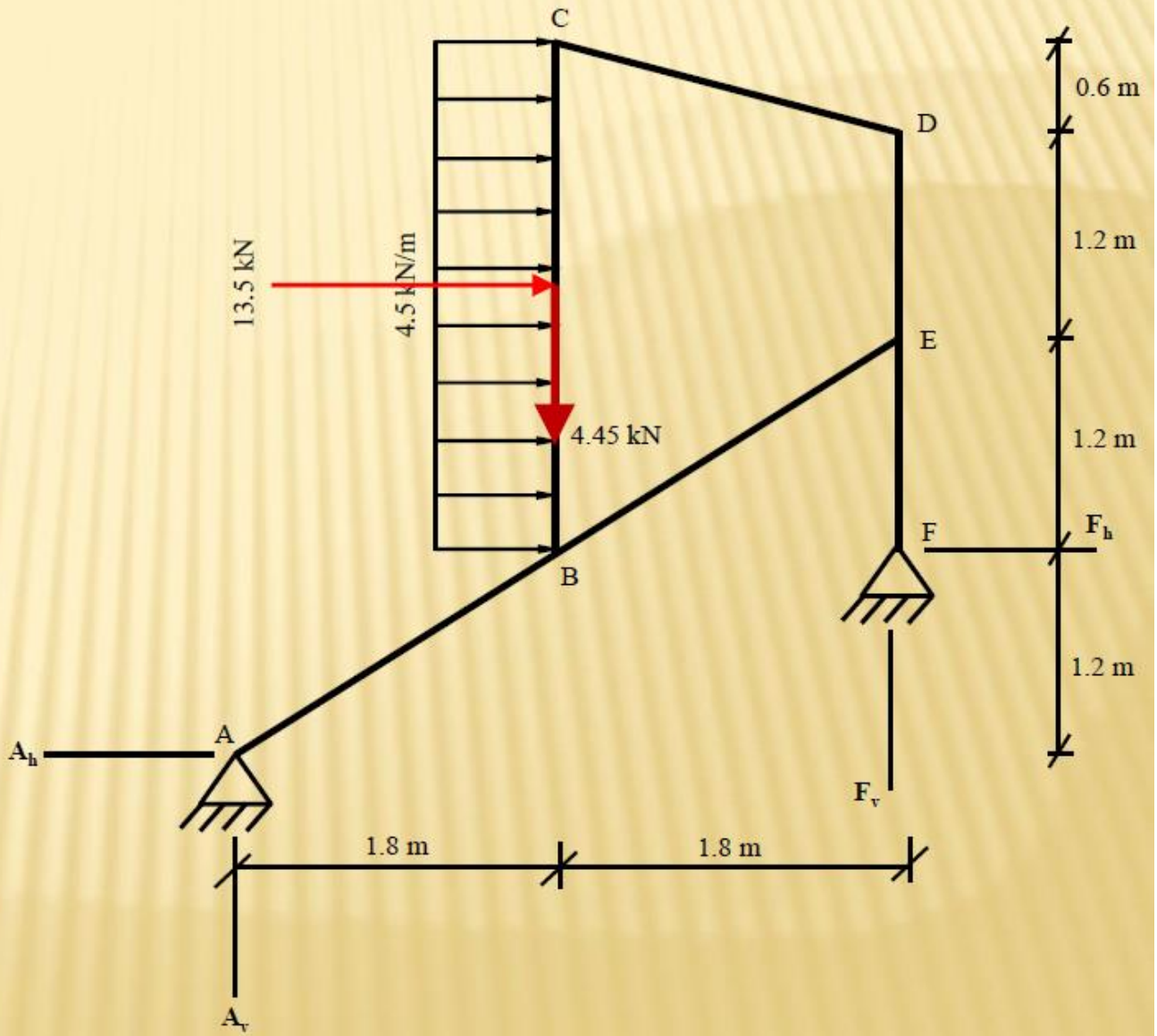
$$-247.805(0.354) - 490.5(0.35) + B_v\ (0.354) = 0$$
$$\rightarrow B_v = 732.763\ N \downarrow$$



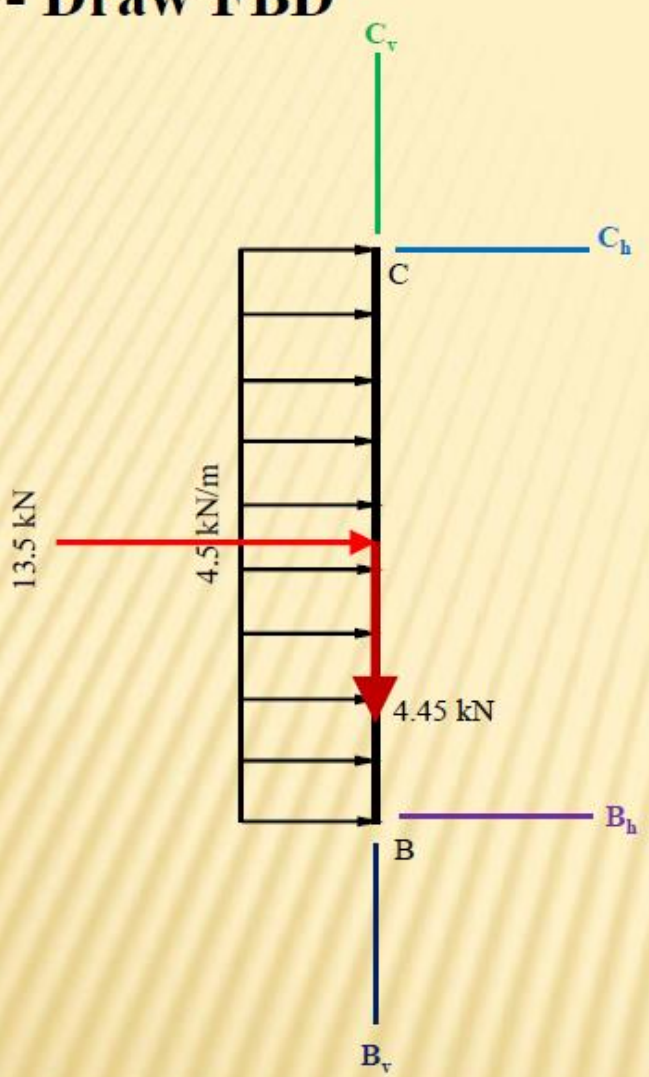
H.W.
Calculate the forces at A, and D

Example: A billboard BC weighting **4.45 kN** is subjected to a wind pressure of **4.5 kN/m** as shown in figure. Calculate the reactions at A and F. Neglect the weight of the other members.

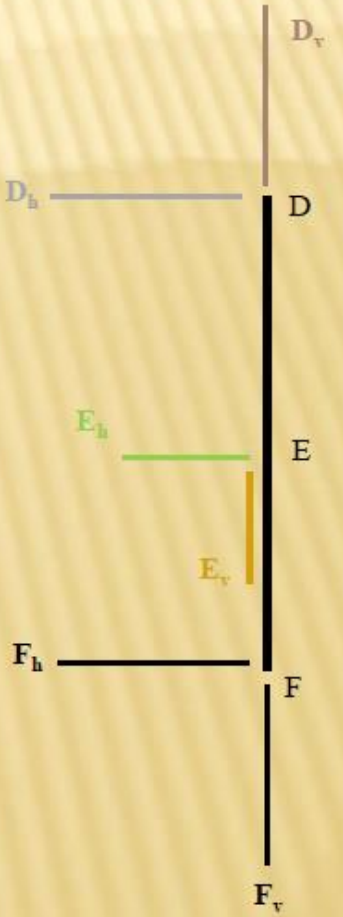
- **Calculate the reactions:**

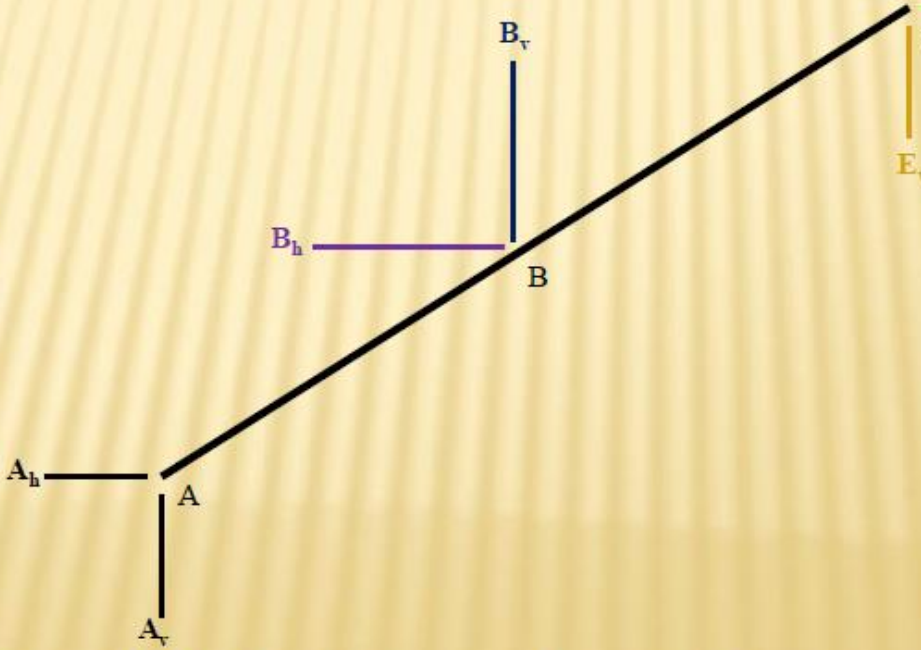


- **Draw FBD**





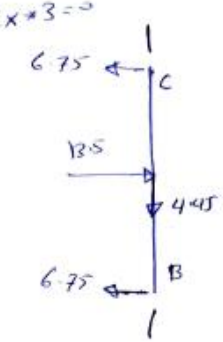


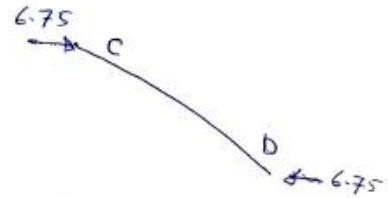


- **Apply the equations of equilibrium on each member:**

$\Sigma M_B = 0 = 13.5 * 1.5 - Cx * 3 = 0$

$Cx = 6.75 \Rightarrow Bx = 6.75$

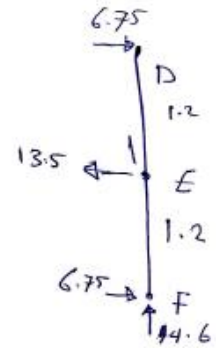




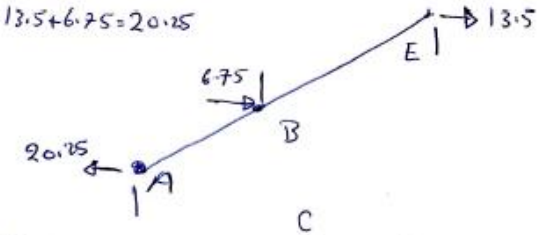
$\Sigma M_E = 0 = 6.75 * 1.2 - Fx * 1.2 = 0$

$Fx = 6.75$

$\Sigma Fx = 0 \Rightarrow Ex = 13.5$



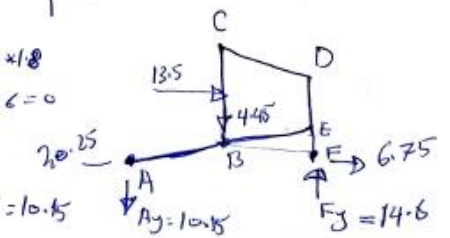
$\Sigma Fx = 0 \Rightarrow Ax = 13.5 + 6.75 = 20.25$



$\Sigma M_A = 0 = 13.5 * 2.7 + 4.45 * 1.8 + 6.75 * 1.2 - Fy * 3.6 = 0$

$Fy = 14.6$

$\Sigma Fy = 0 \Rightarrow Ay = 14.6 - 4.45 = 10.15$



6.F1 For the frame and loading shown, draw the free-body diagram(s) needed to determine the force in member BD and the components of the reaction at C.

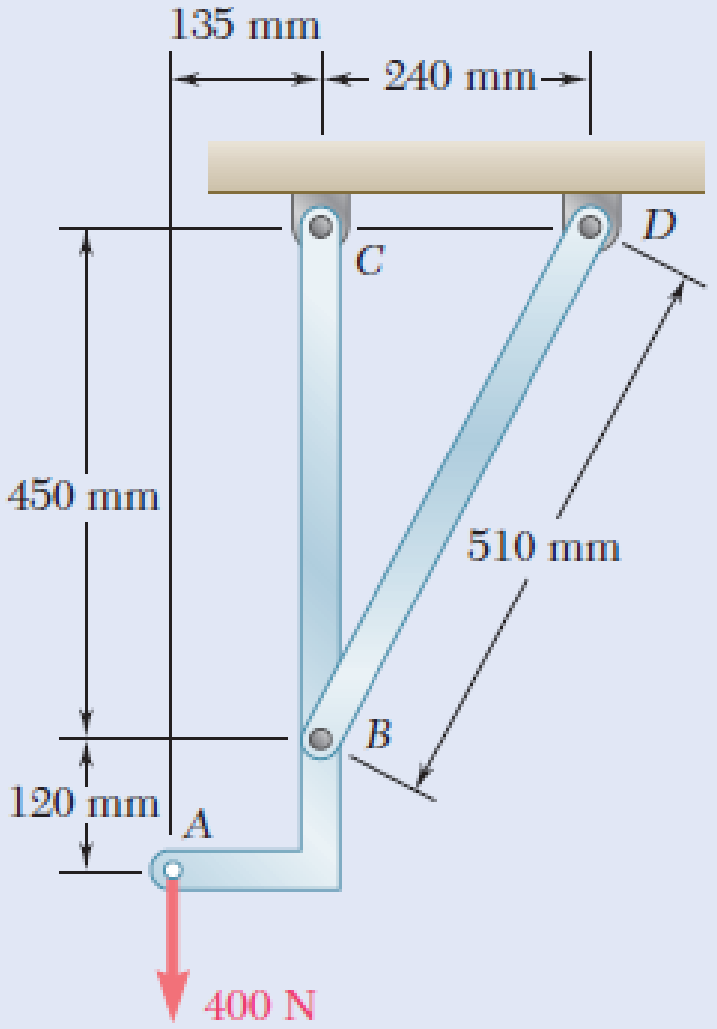


Fig. P6.F1

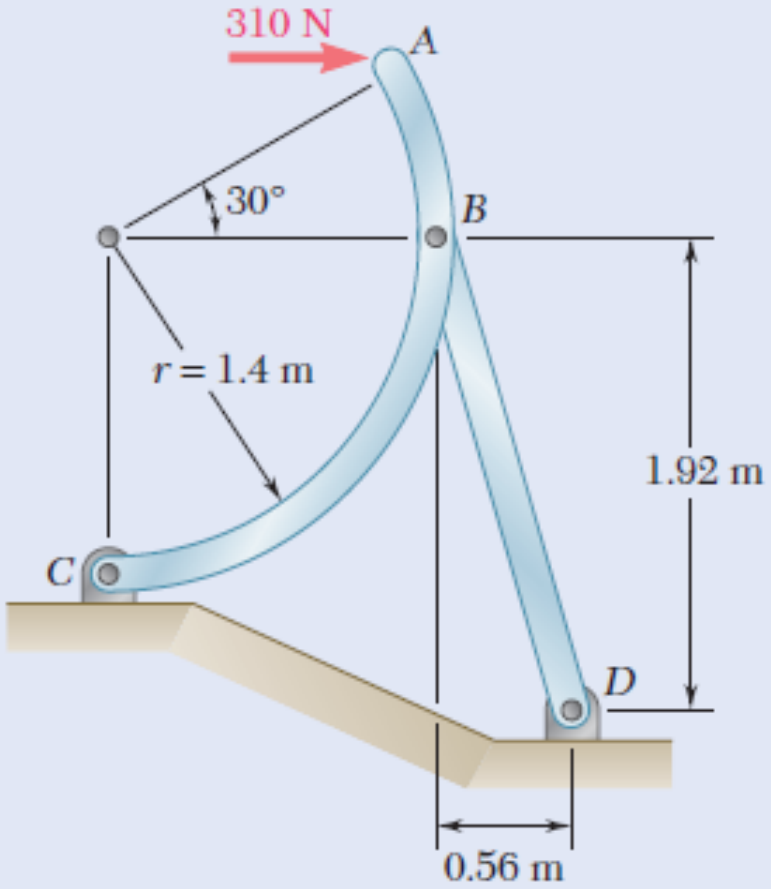


Fig. P6.75

END-OF-SECTION PROBLEMS

6.75 and 6.76 Determine the force in member *BD* and the components of the reaction at *C*.

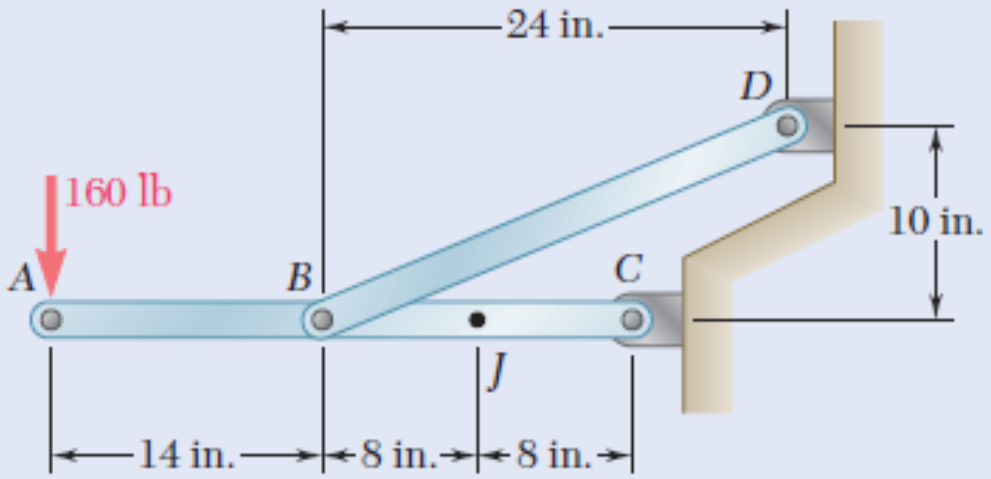


Fig. P6.76

6.77 For the frame and loading shown, determine the force acting on member ABC (a) at B, (b) at C.

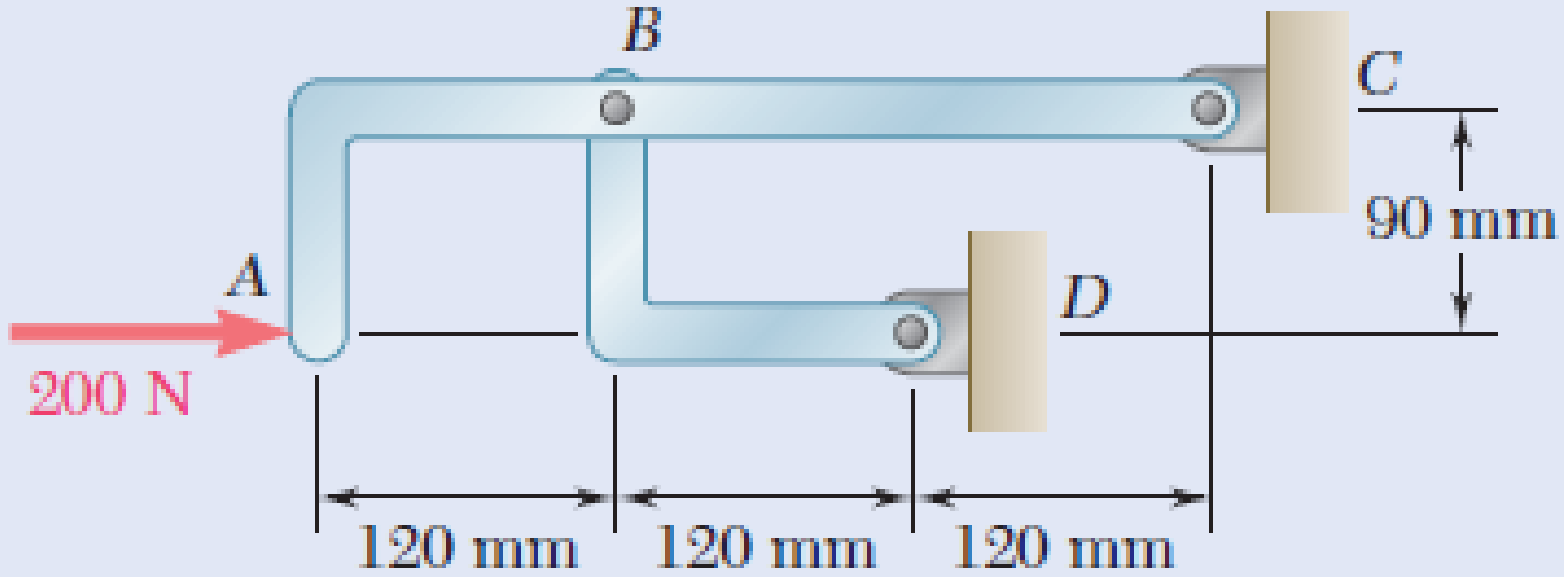


Fig. P6.77

6.79 For the frame and loading shown, determine the components of all forces acting on member *ABC*.

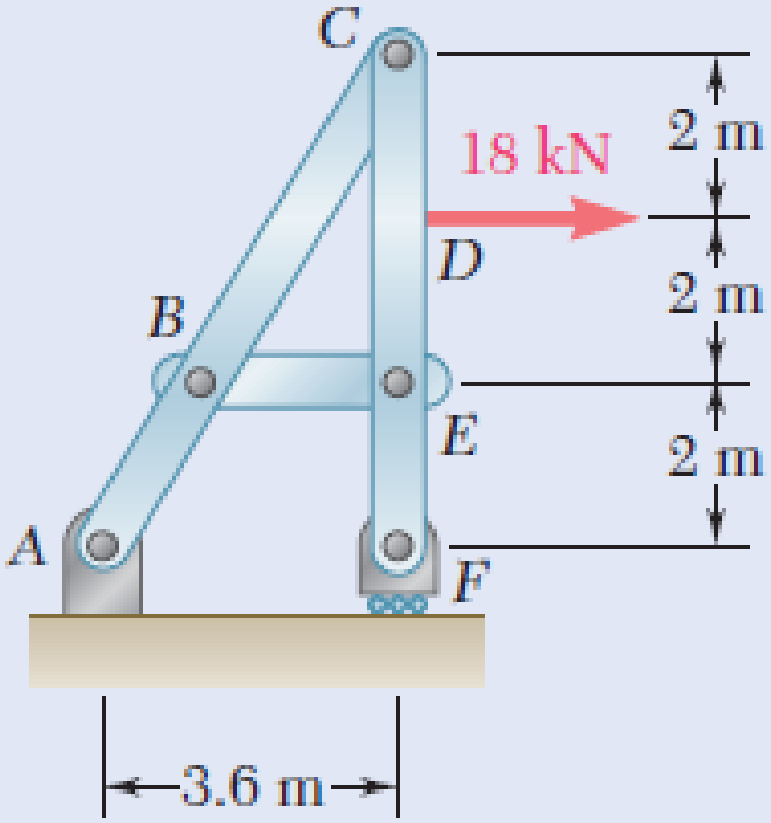


Fig. P6.79

Applications to Physics and Engineering



Among the many applications of integral calculus to physics and engineering, we consider Three here:

1.centroid.

2. moment of inertia

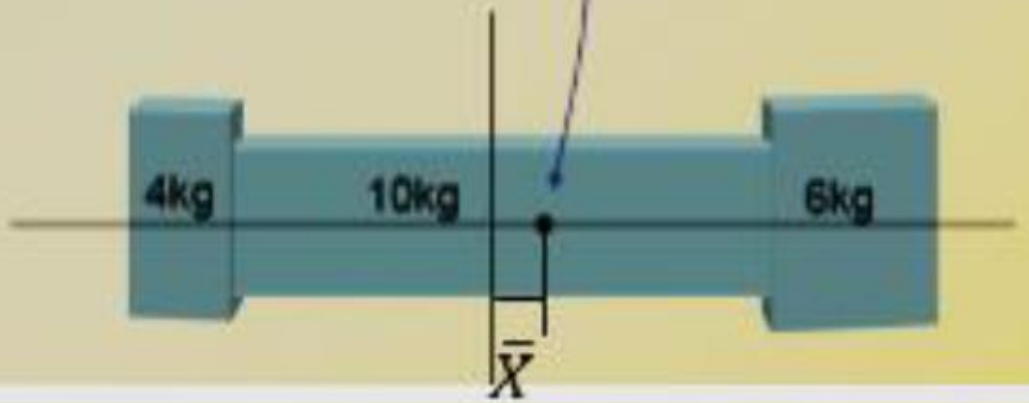
3. friction.

Centroids

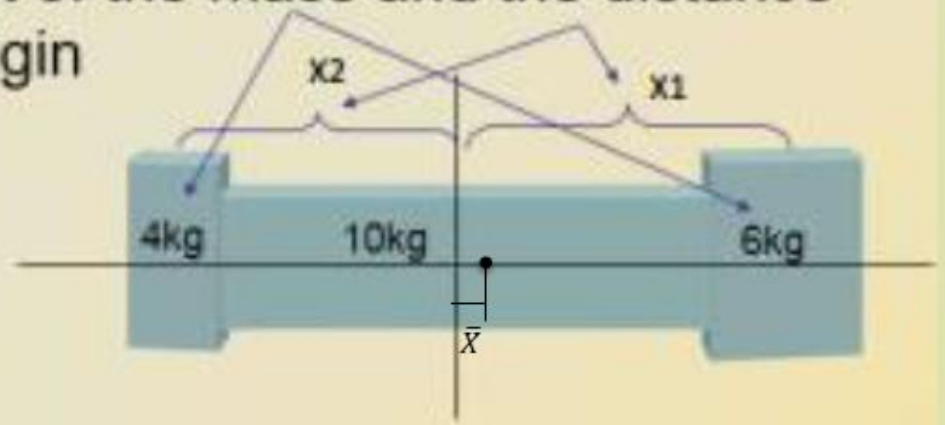
Centroid: is the point at which the body is balanced with respect to the three axis, also called the center of gravity.

١. مركز الثقل بالنسبة لمجموعة من كتل

- Center of mass for a system
  - The point where all the mass seems to be concentrated
  - If the mass is of constant density this point is called the centroid



- Each mass in the system has a "moment"
  - The product of the mass and the distance from the origin

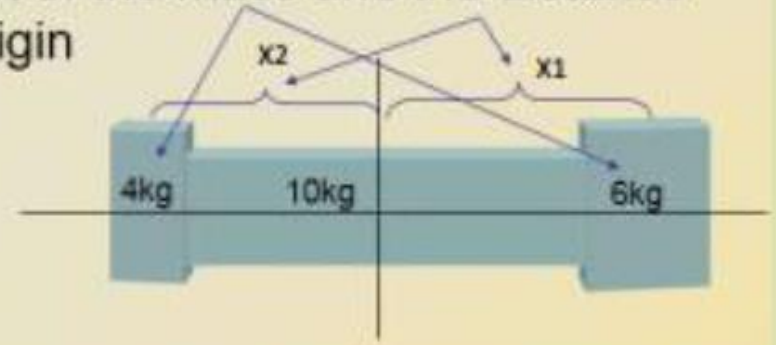


$$(m_1 + m_2) * \bar{X} = m_1 * x_1 + m_2 * x_2$$

$$\bar{X} = \frac{m_1 * x_1 + m_2 * x_2}{(m_1 + m_2)} \quad \rightarrow \quad \bar{X} = \frac{M_y}{m}$$

m: مجموع الكتلتين

- Each mass in the system has a "moment"
  - The product of the mass and the distance from the origin



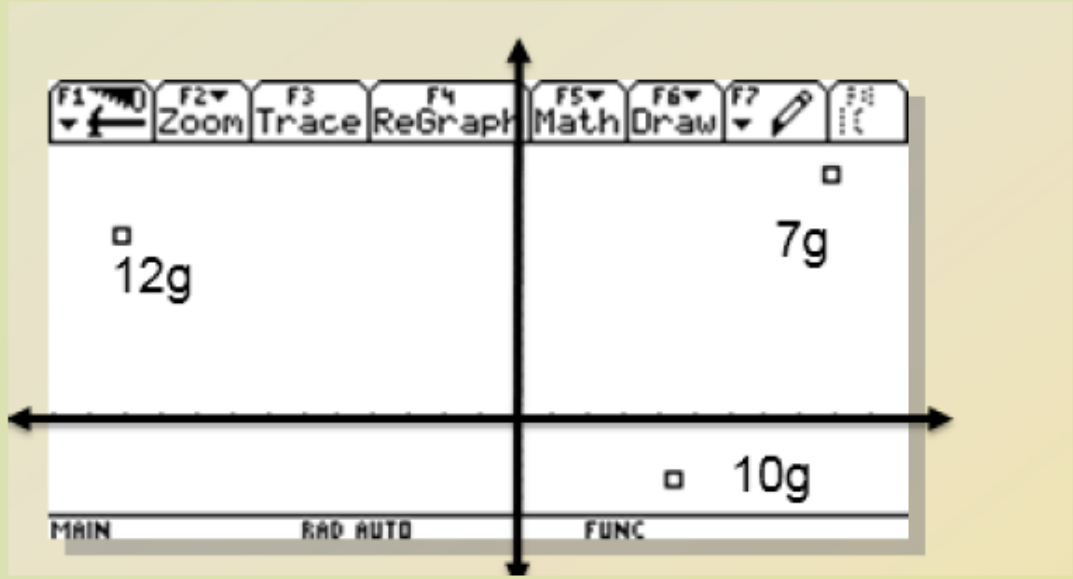
$$\bar{y} * (m_1 + m_2) = m_1 * y_1 + m_2 * y_2$$

$$\bar{y} = \frac{m_1 * y_1 + m_2 * y_2}{(m_1 + m_2)} \quad \rightarrow \quad \bar{y} = \frac{M_x}{m}$$

The location of the centroid is the ordered pair $(\overline{x}, \overline{y})$

Ex/1-Consider a system with 10g at (2,-1), 7g at (4, 3), and 12g at (-5,2)

- What is the center of mass?



Given 10g at (2,-1),
7g at (4, 3),
12g at (-5,2)

solution:

sample	m (g)	X (cm)	m.x (g.cm)	Y (cm)	m.y (g.cm)
m1	10	2	20	-1	**-10**
m2	7	4	28	3	**21**
m3	12	-5	-60	2	**24**
	$\sum m$= 29		$My = \sum \mathrm{m.x} = -12$		$Mx = \sum \mathrm{m.y} = 35$

$$\bar{X} = \frac{M_y}{m} \rightarrow \quad \bar{X} = \frac{\sum \mathrm{m.x}}{\sum m} \rightarrow \quad \bar{X} = \frac{-12}{29}\ \mathrm{cm}$$

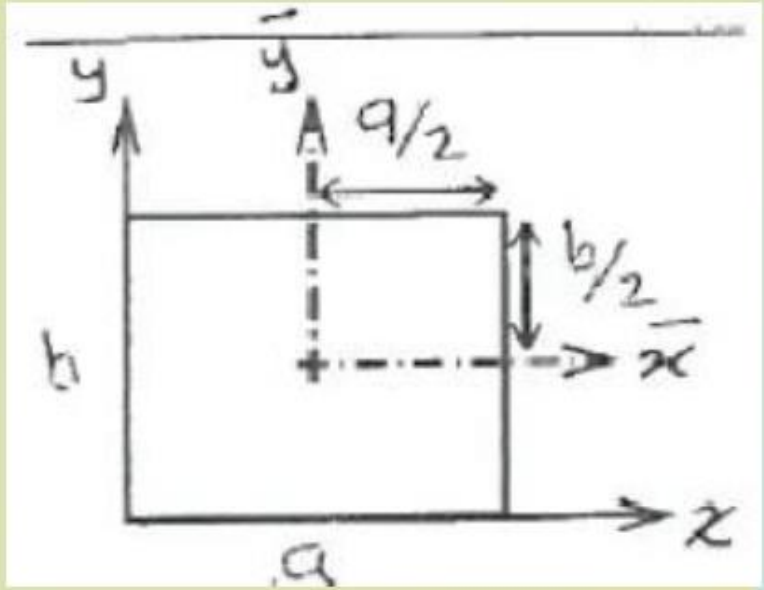
$$\bar{y} = \frac{M_x}{m} \rightarrow \quad \bar{y} = \frac{\sum \mathrm{m.y}}{\sum m} \rightarrow \quad \bar{y} = \frac{35}{29}\ \mathrm{cm}$$

2.Centroids of Areas and lines

- Area

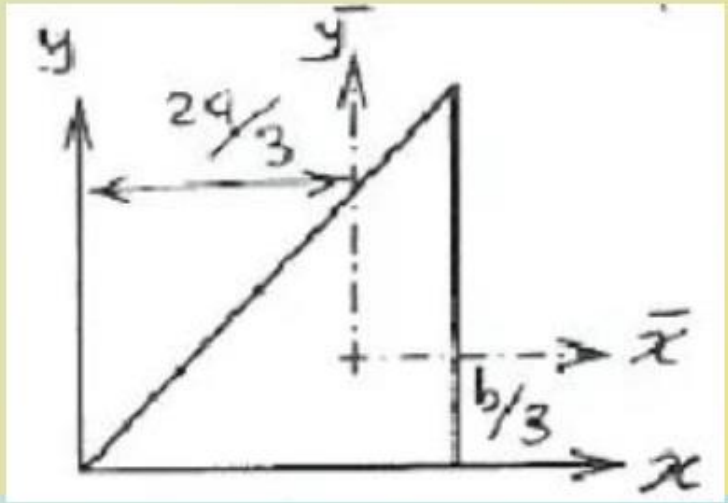
$$\bar{X} = \frac{a}{2} \qquad \bar{y} = \frac{b}{2}$$

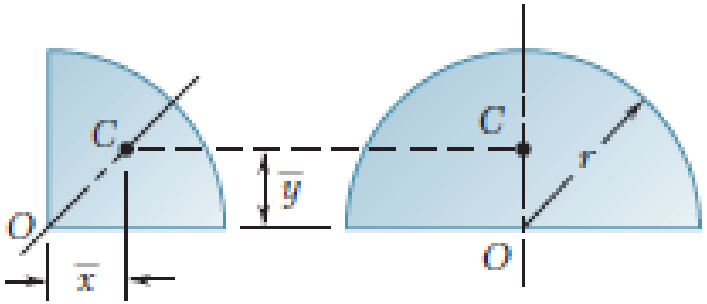
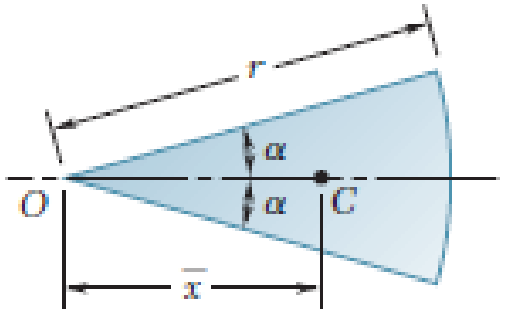
$$A = b * a$$

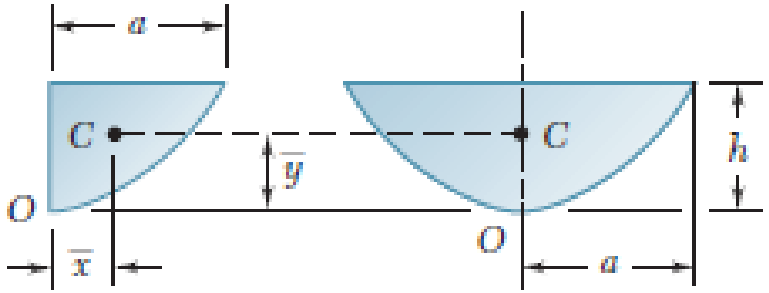
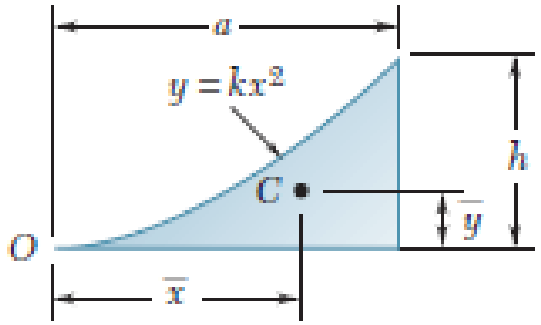
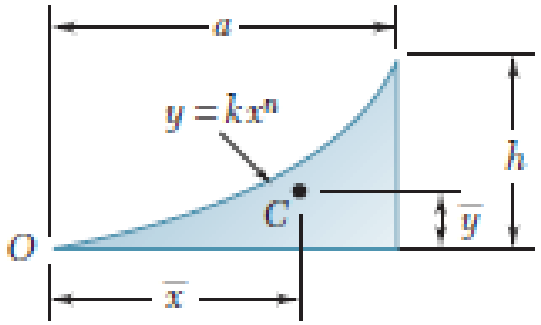


$$\bar{X} = \frac{2a}{3} \qquad \bar{y} = \frac{b}{3}$$

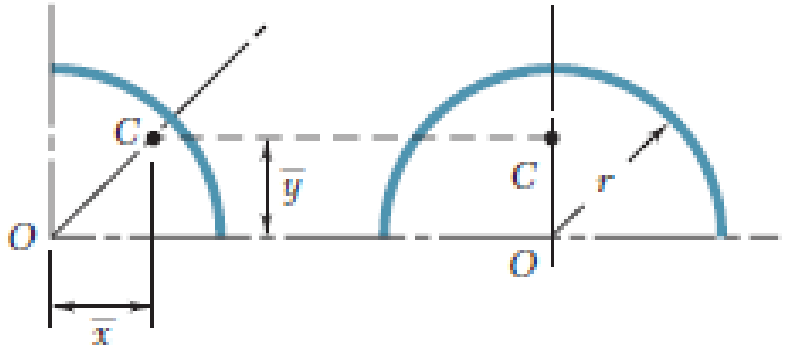
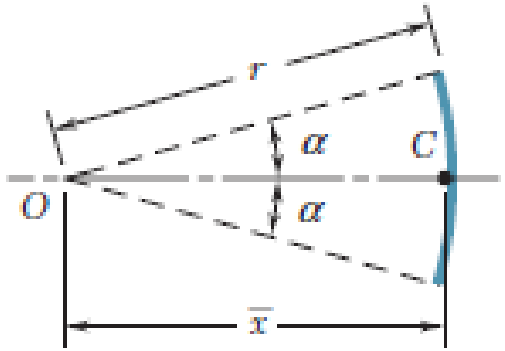
$$A = \frac{b * a}{2}$$



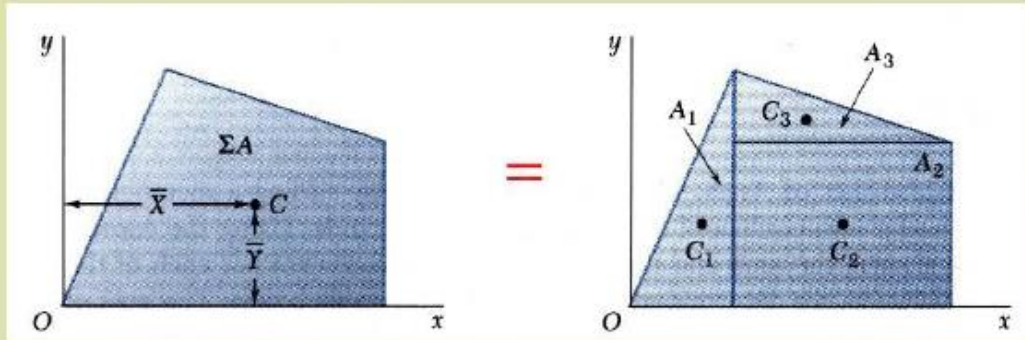
Shape		$\bar{x}$	$\bar{y}$	Area
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2

Shape		$\bar{x}$	$\bar{y}$	Area
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$

Lines :

Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

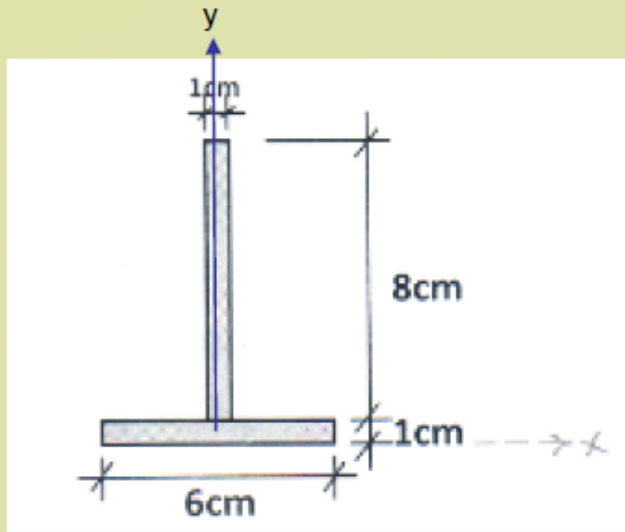
Composite Areas



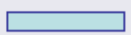
$$\bar{X} = \frac{M_y}{\sum A} \rightarrow \qquad \bar{X} = \frac{\sum \mathrm{A.x}}{\sum A}$$

$$\bar{y} = \frac{M_x}{\sum A} \rightarrow \qquad \bar{y} = \frac{\sum \mathrm{A.y}}{\sum A} \rightarrow$$

Ex/2



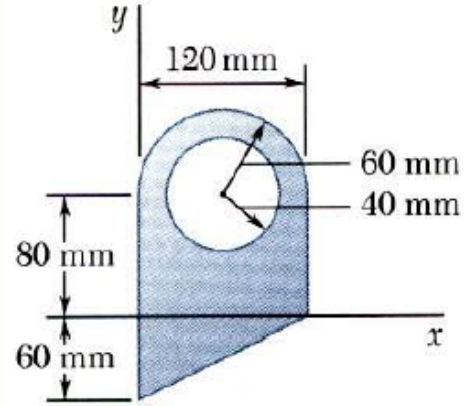
The dimensions of the (T) section of a cast-iron beam are shown in figure. How far is the centroid of the area above the base?

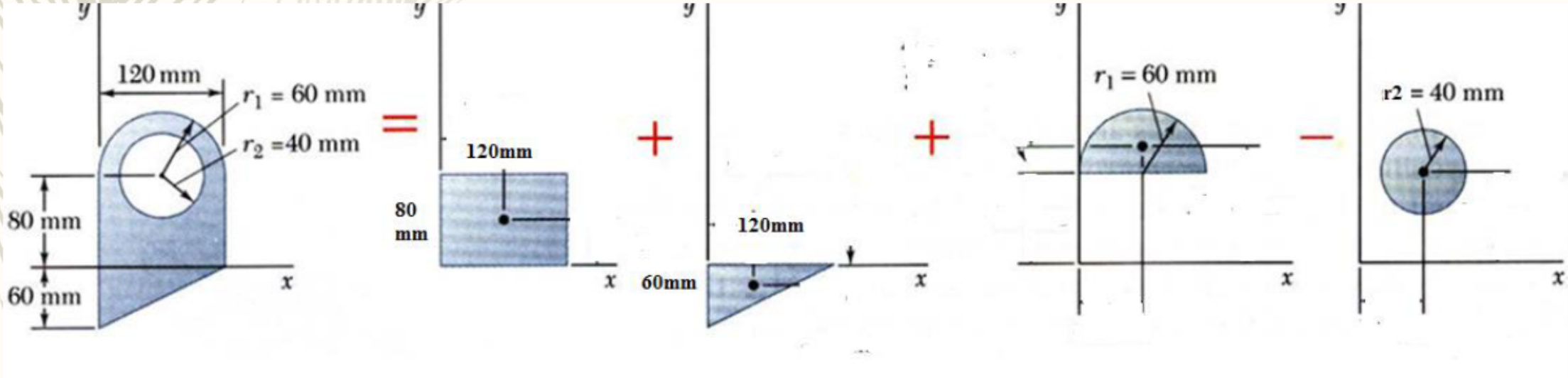
shape	A (cm^2)	Y (cm)	A.y (cm^2.cm)
	8	5	40
	6	0.5	3
	$\sum$A=14		Mx $= \sum A.y$ =43

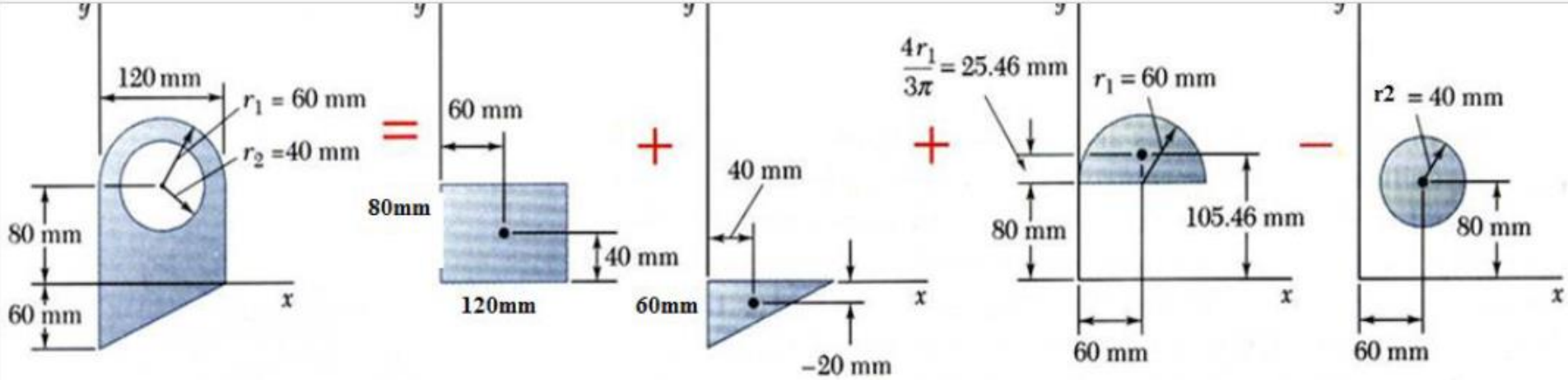
$$\bar{X} = \frac{M_y}{\sum A} \rightarrow \quad \bar{X} = \frac{\sum \text{A.x}}{\sum A} \rightarrow \quad \bar{X} = \frac{0}{14} = 0$$

$$\bar{y} = \frac{M_x}{\sum A} \rightarrow \quad \bar{y} = \frac{\sum \text{A.y}}{\sum A} \rightarrow \quad \bar{y} = \frac{43}{14} = 3.07 \text{ cm}$$

Ex3/For the plane area shown, determine the location of the centroid with respect to the *x* and *y* axes and.







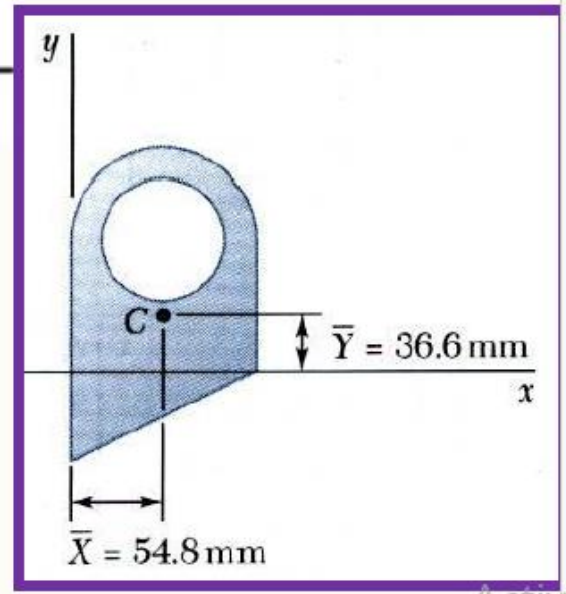
Component	A, mm^2	$\bar{x}$, mm	$\bar{x}A$, mm^3	$\bar{y}$, mm	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	$+576 \times 10^3$	40	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	$+144 \times 10^3$	−20	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	$+339.3 \times 10^3$	105.46	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	-301.6×10^3	80	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$		$\Sigma\bar{x}A = +757.7 \times 10^3$		$\Sigma\bar{y}A = +506.2 \times 10^3$

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{+757.7\times10^3\,\text{mm}^3}{13.828\times10^3\,\text{mm}^2}$$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{+506.2\times10^3\,\text{mm}^3}{13.828\times10^3\,\text{mm}^2}$$

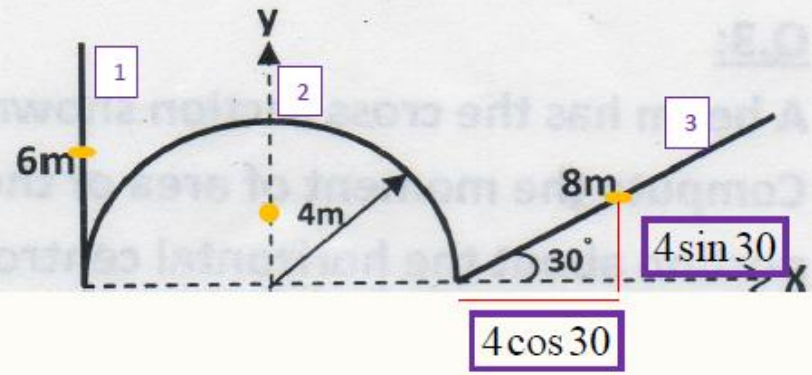
$\bar{X} = 54.8\,\text{mm}$

$\bar{Y} = 36.6\,\text{mm}$



Q.4:

A slender homogeneous wire of uniform cross section is bent into the shape shown in figure. Determine the coordinates of its centroid.



shape	L (m)	x (m)	L.X (m^2)	Y (m)	L.y (m^2)
1	6	-4	-24	3	18
2	π r=π4= 12.56	0	0	2(r)/π=2.548	32
3	8	4cos30+4= 7.464	59.7	4*sin 30=2	16
	∑L=26.56		My $= \sum L.x$ 35.7		Mx $= \sum L.y$ =66

$$\overline{X} = \frac{\sum L.x}{\sum L} = \frac{+35.7\text{m}^2}{26.56\text{m}} \qquad \overline{X} = 1.344\ \text{m}$$

$$\overline{Y} = \frac{\sum L.y}{\sum L} = \frac{+66\text{m}^2}{26.56\,\text{m}} \qquad \overline{Y} = 2.485\ \text{m}$$

H.W2

Q1/A beam has the cross section shown in figure 1, determine the coordinates of its center.

$$\bar{X} = \frac{M_y}{\sum A} \rightarrow \qquad \bar{X} = \frac{\sum A.x}{\sum A} \rightarrow \qquad \bar{X} = \frac{0}{30} = 0$$

$$\bar{y} = \frac{M_x}{\sum A} \rightarrow \qquad \bar{y} = \frac{\sum A.y}{\sum A} \rightarrow \qquad \bar{y} = \frac{171}{30} = 5.7 \text{ cm}$$

shape	A (cm^2)	Y (cm)	A.y (cm^3)
1	6	13.5	81
2	12	7	84
3	12	0.5	6
	$\sum A=30$		Mx $= \sum A.y = 171$

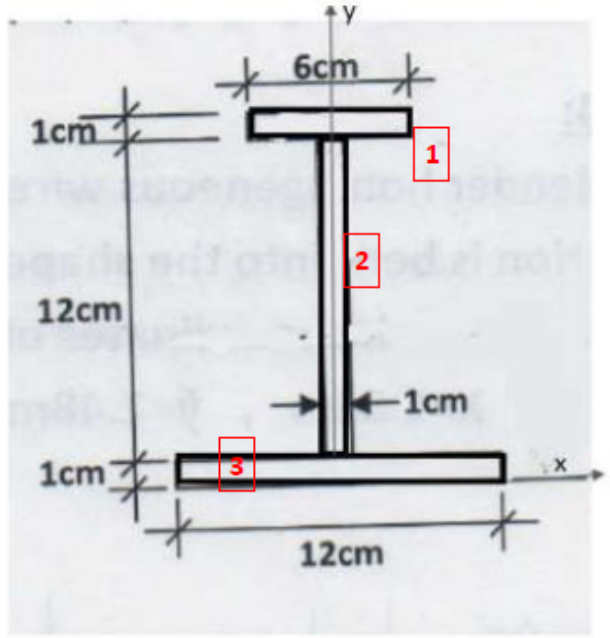


figure 1

Q2/ A beam has the cross section shown in figure 2, determine the coordinates of its center.

$$\bar{y} = \frac{M_x}{\sum A} \rightarrow \qquad \bar{y} = \frac{\sum A.y}{\sum A} \rightarrow \qquad \bar{y} = \frac{0}{30} = 0$$

$$\bar{x} = \frac{M_y}{\sum A} \rightarrow \qquad \bar{x} = \frac{\sum A.x}{\sum A} \rightarrow \qquad \bar{x} = \frac{171}{30} = 5.7 \text{ cm}$$

shape	A (cm^2)	x (cm)	A.x (cm^3)
1	6	13.5	81
2	12	7	84
3	12	0.5	6
	$\sum A=30$		My $= \sum A.x = 171$

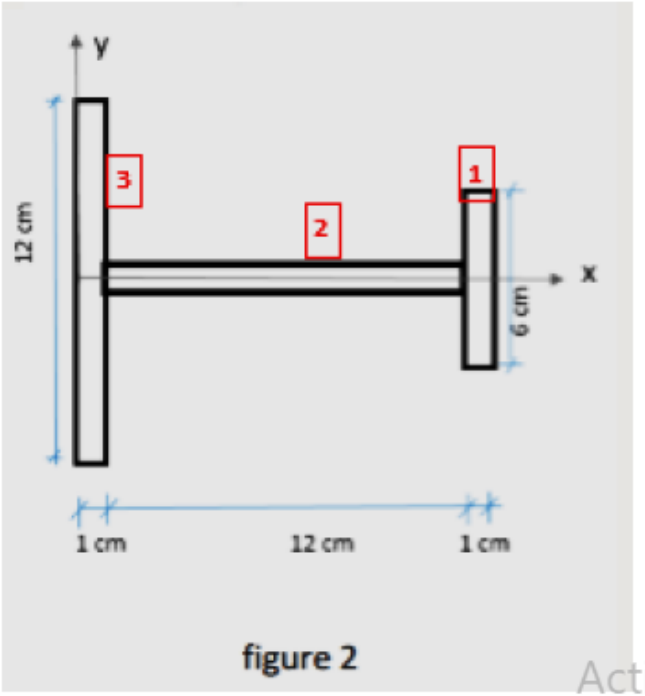
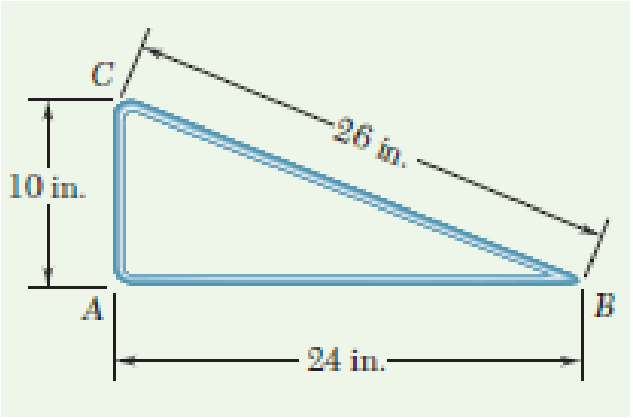


figure 2

The figure shown is made from a piece of thin, homogeneous wire. Determine the location of its center of gravity.

Segment	L, in.	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}L$, in^2	$\bar{y}L$, in^2
AB	24	12	0	288	0
BC	26	12	5	312	130
CA	10	0	5	0	50
	$\Sigma L = 60$			$\Sigma \bar{x}L = 600$	$\Sigma \bar{y}L = 180$



ANALYSIS: Substituting the values obtained from the table into the equations defining the centroid of a composite line gives

$\bar{X}\Sigma L = \Sigma \bar{x}L$: $\quad \bar{X}(60 \text{ in.}) = 600 \text{ in}^2$ $\quad \bar{X} = 10 \text{ in.}$ ◀

$\bar{Y}\Sigma L = \Sigma \bar{y}L$: $\quad \bar{Y}(60 \text{ in.}) = 180 \text{ in}^2$ $\quad \bar{Y} = 3 \text{ in.}$ ◀

5.29 The frame for a sign is fabricated from thin, flat steel bar stock of mass per unit length 4.73 kg/m. The frame is supported by a pin at C and by a cable AB. Determine (a) the tension in the cable, (b) the reaction at C.

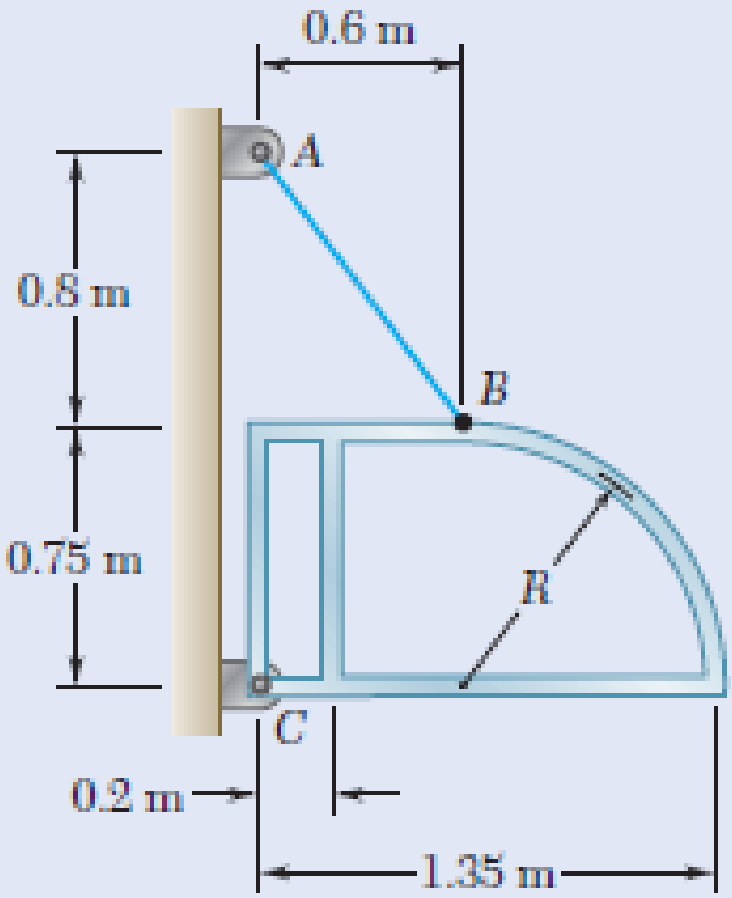
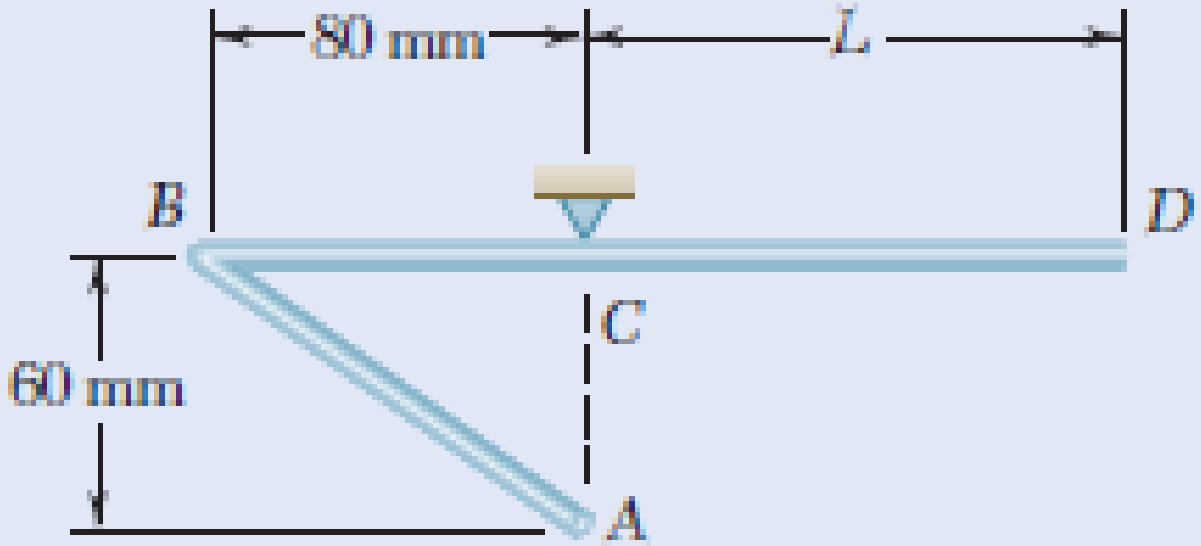


Fig. P5.29

5.30 The homogeneous wire $ABCD$ is bent as shown and is attached to a hinge at C. Determine the length L for which portion BCD of the wire is horizontal.



Locate the centroid of the plane area shown.

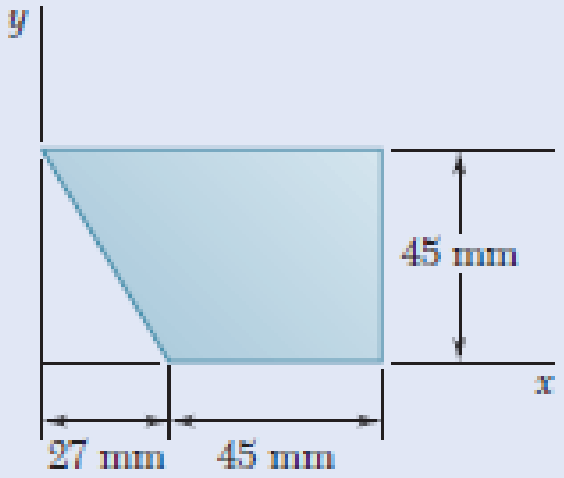


Fig. P5.1

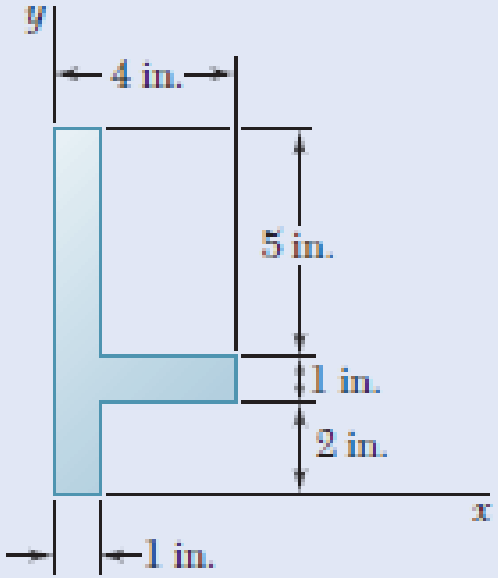


Fig. P5.2

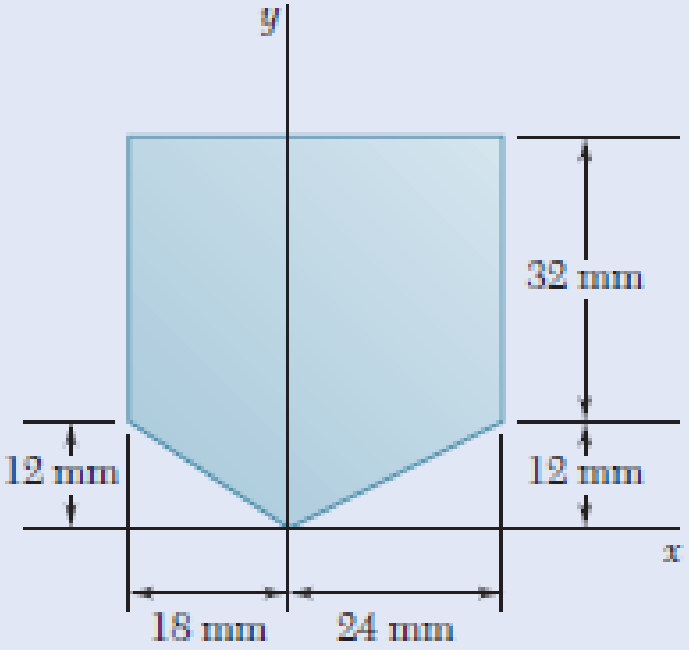


Fig. P5.3

Locate the centroid of the plane area shown.

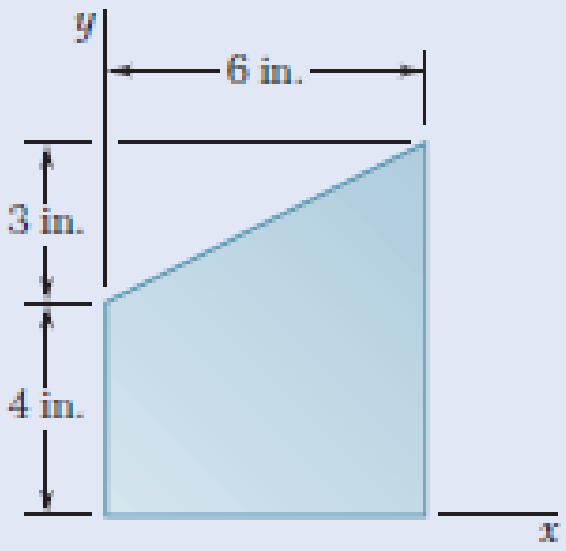


Fig. P5.5

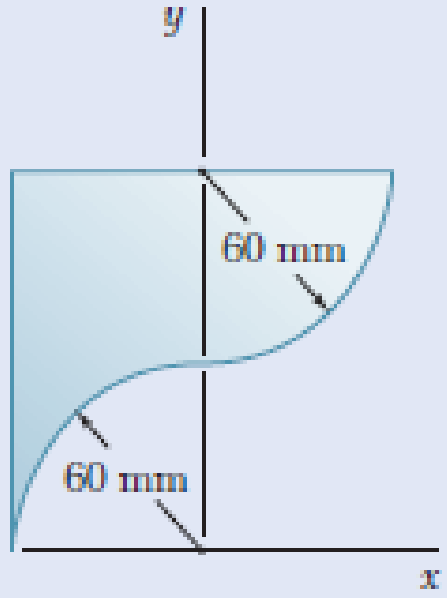


Fig. P5.6

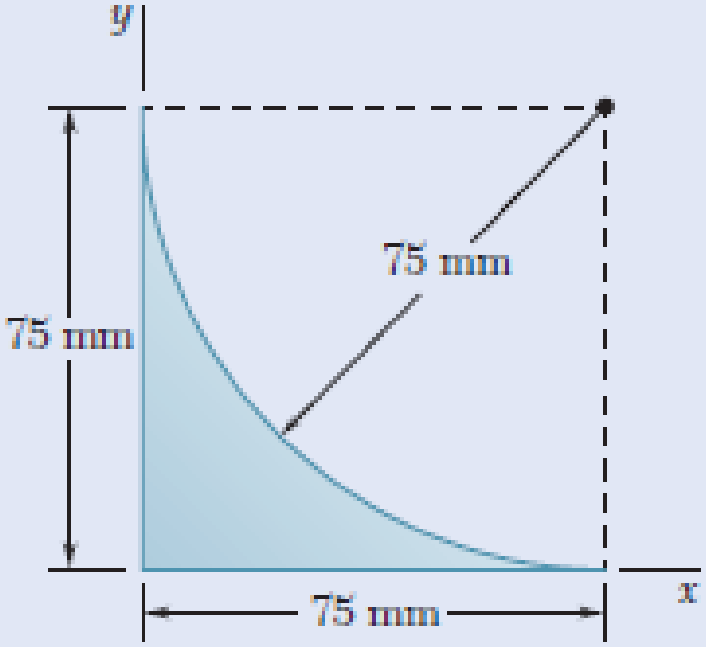
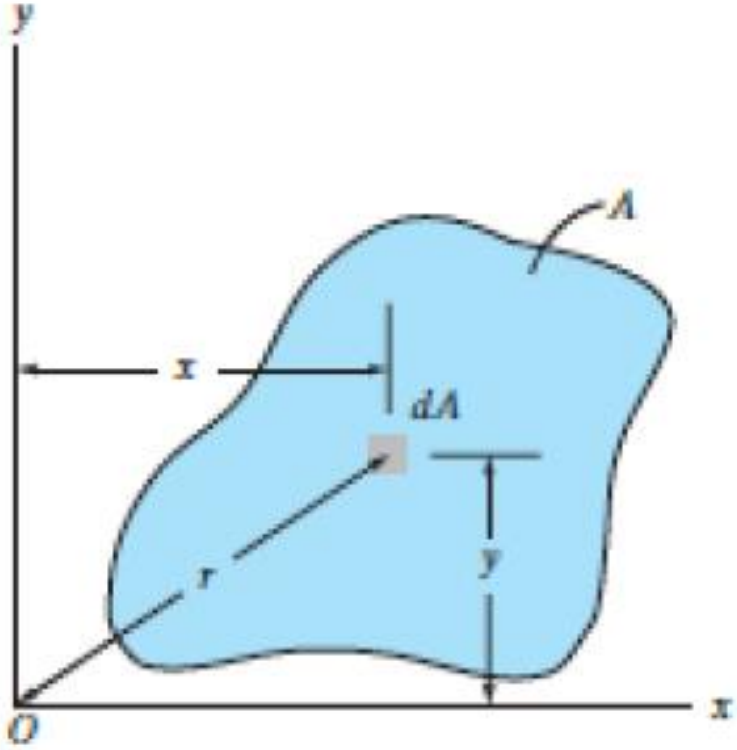


Fig. P5.9

Moment of inertia : The intrinsic strength of a material is its resistance strength, through which every body, as far as it can, seeks to maintain its current state, whether that state is rest or uniform forward motion in a straight line. The moment of inertia is geometric property of an area that is used to determine the strength of a structural member.it is sometimes referred to as the second moment of the area about an axis.



By definition, the moments of inertia of a differential area about the x and y axis are:-

$dI_x = y^2 dA \quad dI_Y = x^2 dA$

For the entire area A the moments of inertia are determined by integration;

$I_x = \int y^2 A dA \qquad I_y = \int x^2 A dA$

1-Moments of Inertia of an area

The area moment of inertia represents the second moment of the area about an axis. It is frequently used in formulas related to the strength and stability of structural members or mechanical elements. For the entire area A the moments of inertia are determined by integration:

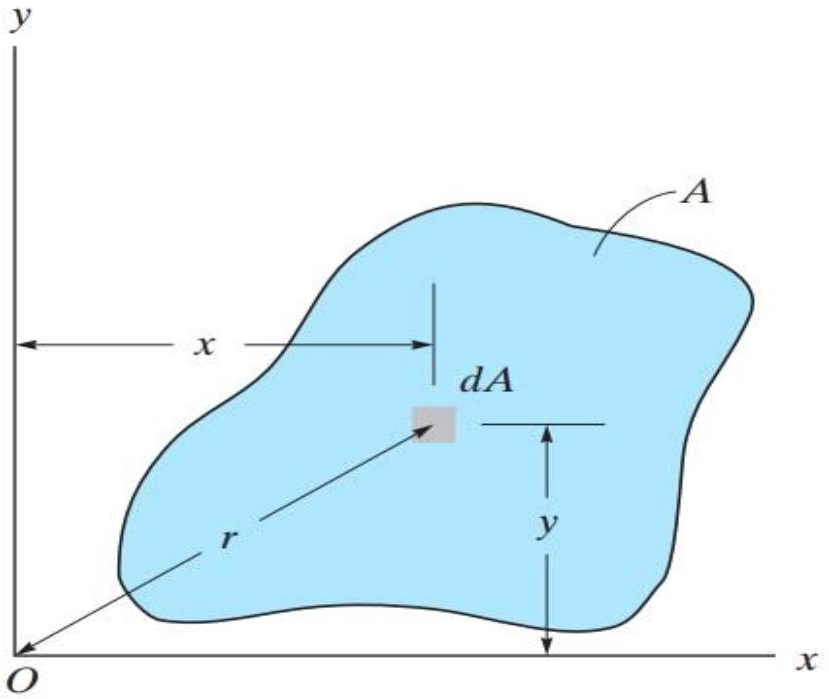
$$I_x = \int_A y^2\, dA$$

$$I_y = \int_A x^2\, dA$$

From the above formulations it is seen that I_x and I_y will always be positive since they involve the product of distance squared and area.

Furthermore, the units for moment of inertia involve product of distance squared and area.

Furthermore, the units for moment of inertia involve **length raised to the fourth power , e.g., m^4 ,mm^4, or ft^4, in^4**



Parallel-Axis Theorem for an Area

The parallel-axis theorem can be used to find the moment of inertia of an area about any axis that is parallel to an axis passing through the centroid and about which the moment of inertia is known.

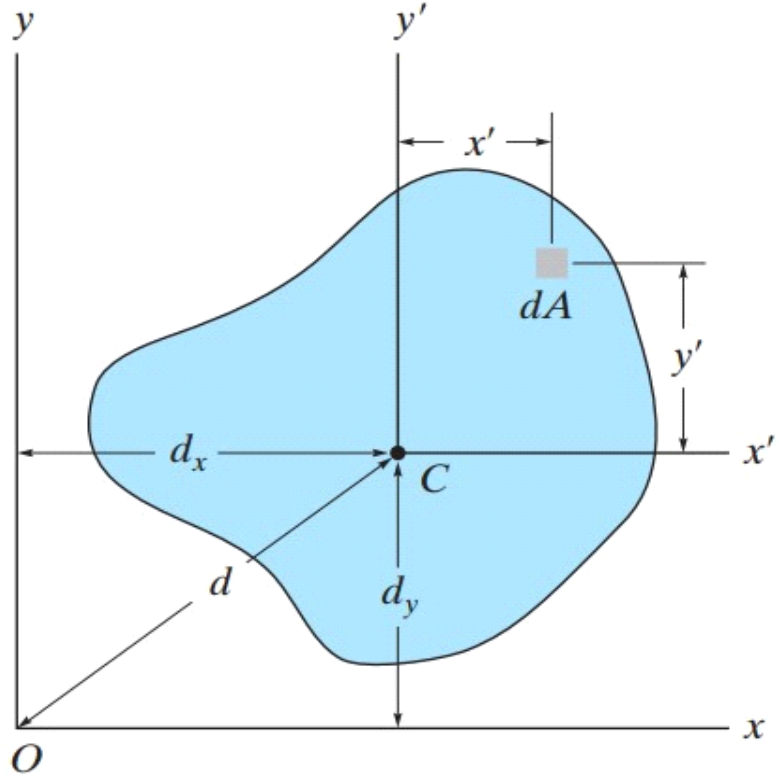
$$I_x = \bar{I}_{x'} + Ad_y^2$$

$$I_y = \bar{I}_{y'} + Ad_x^2$$

$I_{x'}$ and $I_{y'}$: The first integral represents the moment of inertia of the area about the centroidal axis.

A:The total area.

dy and dx :The distance between the parallel x' and x and y' and y respectively.



EXAMPLE: Determine the moment of inertia for the rectangular area shown in Fig. 10–5 with respect to (a) the centroidal x' axis, and (b) the axis x_b passing through the base of the rectangle.

Part (a). The differential element shown in Fig. 10–5 is chosen for integration. Because of its location and orientation, the *entire element* is at a distance y' from the x' axis. Here it is necessary to integrate from $y' = -h/2$ to $y' = h/2$. Since $dA = b\,dy'$, then

$$\bar{I}_{x'} = \int_A y'^2\,dA = \int_{-h/2}^{h/2} y'^2(b\,dy') = b\int_{-h/2}^{h/2} y'^2\,dy'$$

$$\bar{I}_{x'} = \frac{1}{12}bh^3 \qquad \textit{Ans.}$$

Part (b). The moment of inertia about an axis passing through the base of the rectangle can be obtained by using the above result of part (a) and applying the parallel-axis theorem, Eq. 10–3.

$$I_{x_b} = \bar{I}_{x'} + A d_y^2$$

$$= \frac{1}{12}bh^3 + bh\left(\frac{h}{2}\right)^2 = \frac{1}{3}bh^3 \qquad \textit{Ans.}$$

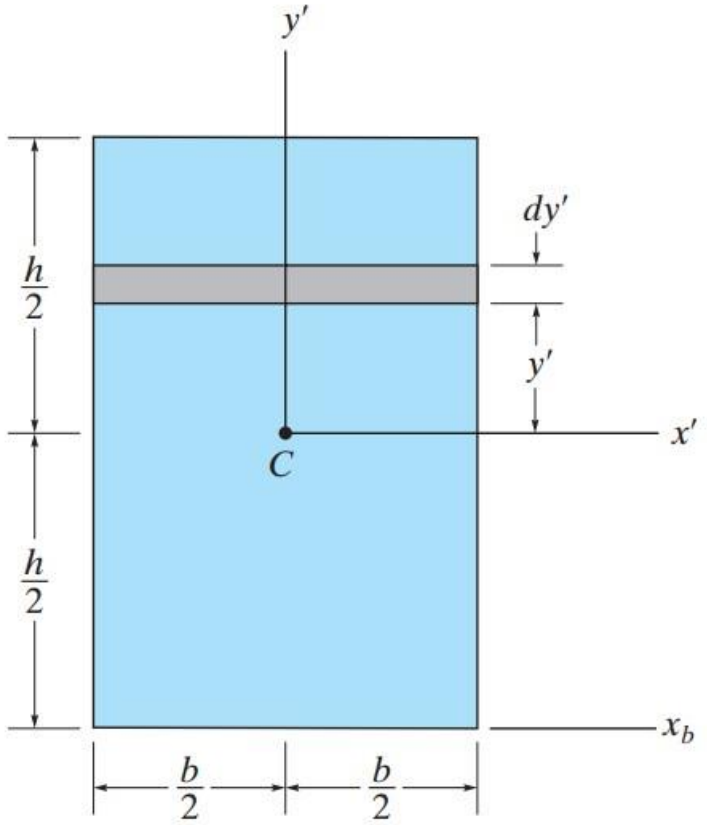


Fig. 10–5

EXAMPLE: Determine the moment of inertia for the shaded area shown in Fig. 10-6a about the x-axis

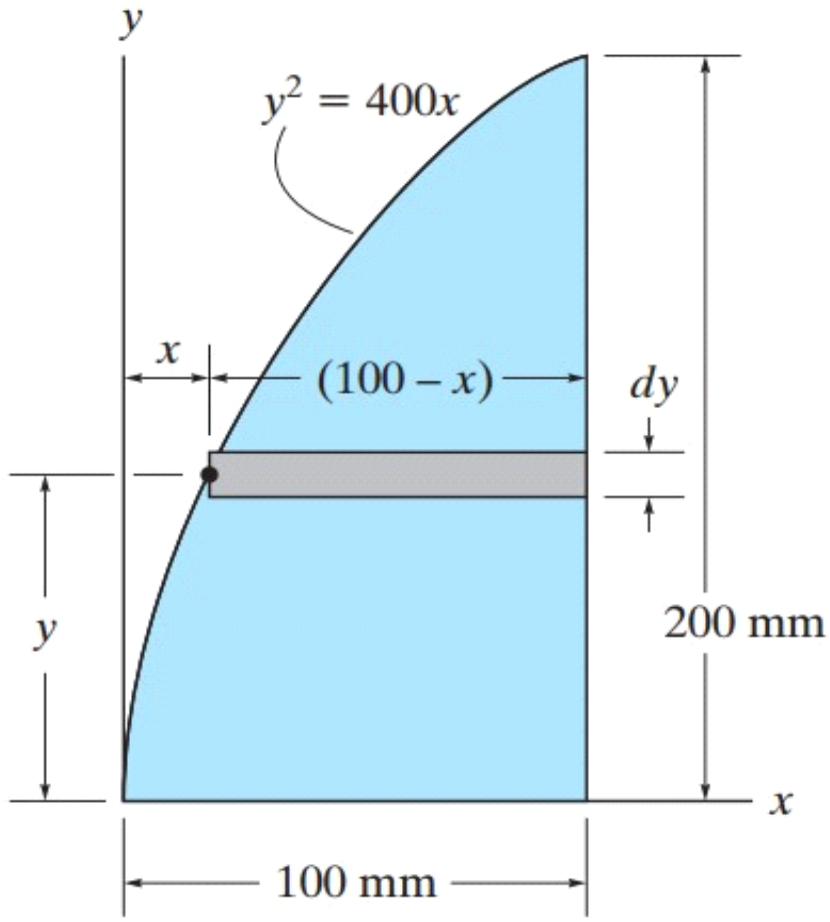
SOLUTION:

A differential element of area that is *parallel* to the x axis, as shown in Fig. 10–6*a*, is chosen for integration. Since this element has a thickness dy and intersects the curve at the *arbitrary point* (x, y), its area is $dA = (100 - x)\,dy$. Furthermore, the element lies at the same distance y from the x axis. Hence, integrating with respect to y, from $y = 0$ to $y = 200$ mm, yields

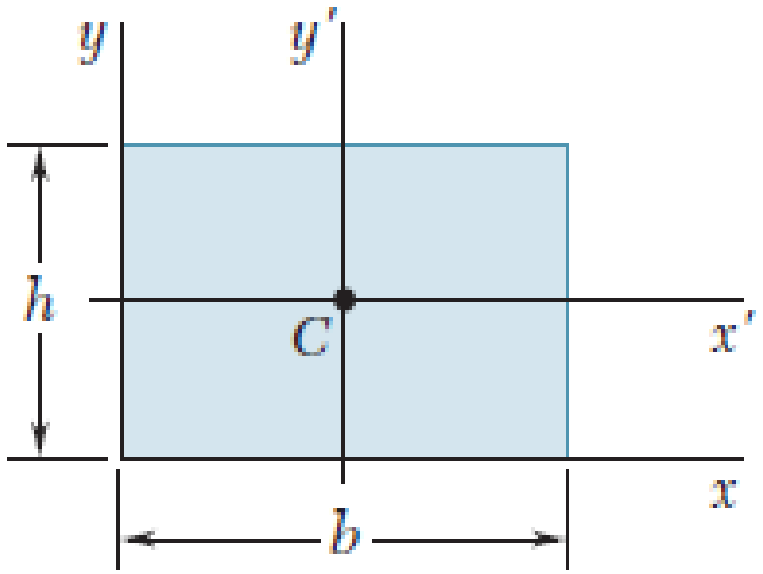
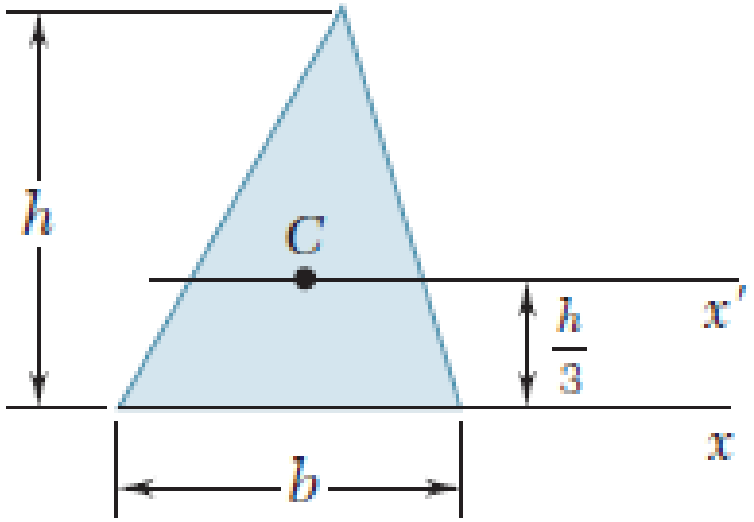
$$I_x = \int_A y^2\,dA = \int_0^{200\text{ mm}} y^2(100 - x)\,dy$$

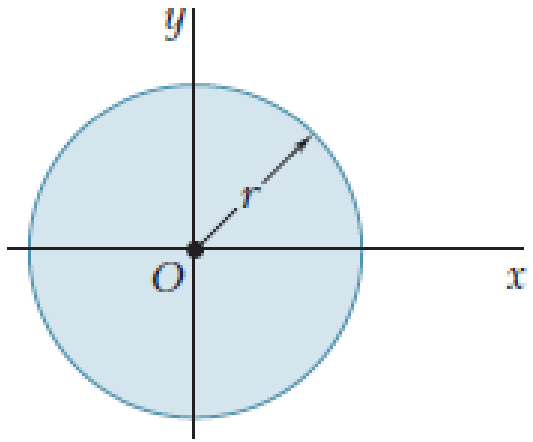
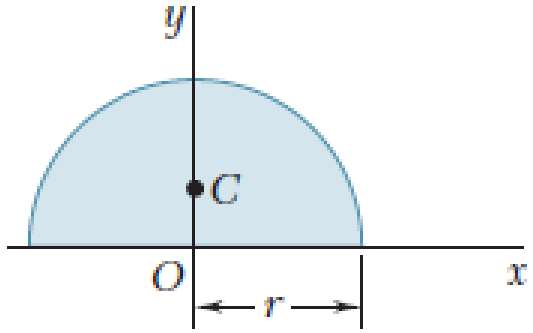
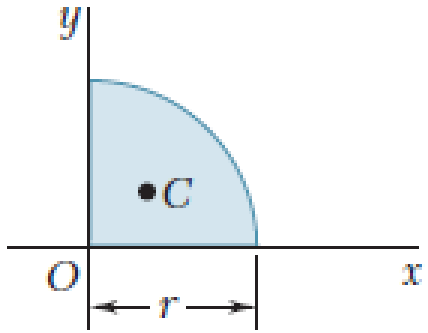
$$= \int_0^{200\text{ mm}} y^2\left(100 - \frac{y^2}{400}\right)dy = \int_0^{200\text{ mm}} \left(100y^2 - \frac{y^4}{400}\right)dy$$

$$= 107(10^6)\text{ mm}^4 \qquad \textit{Ans.}$$

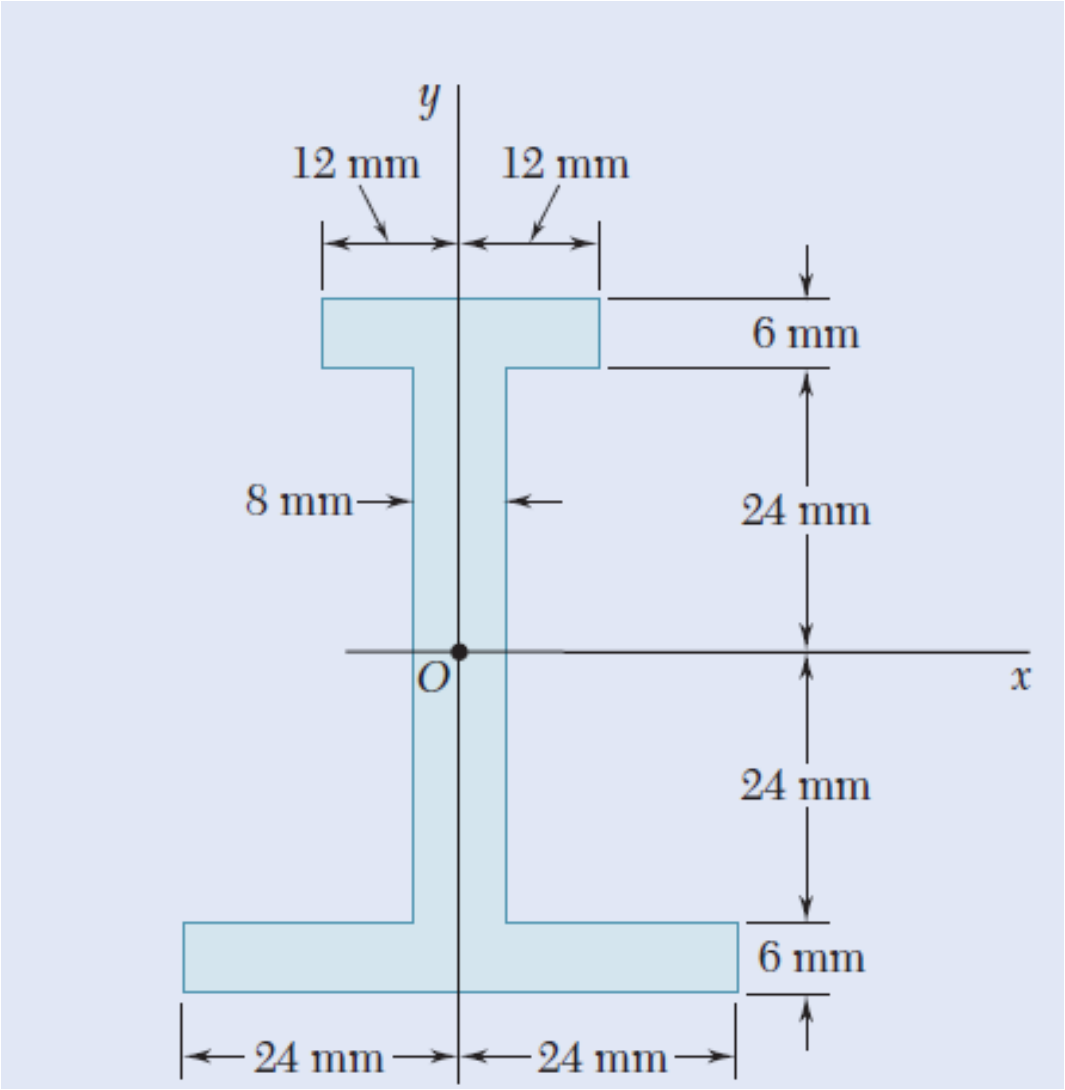


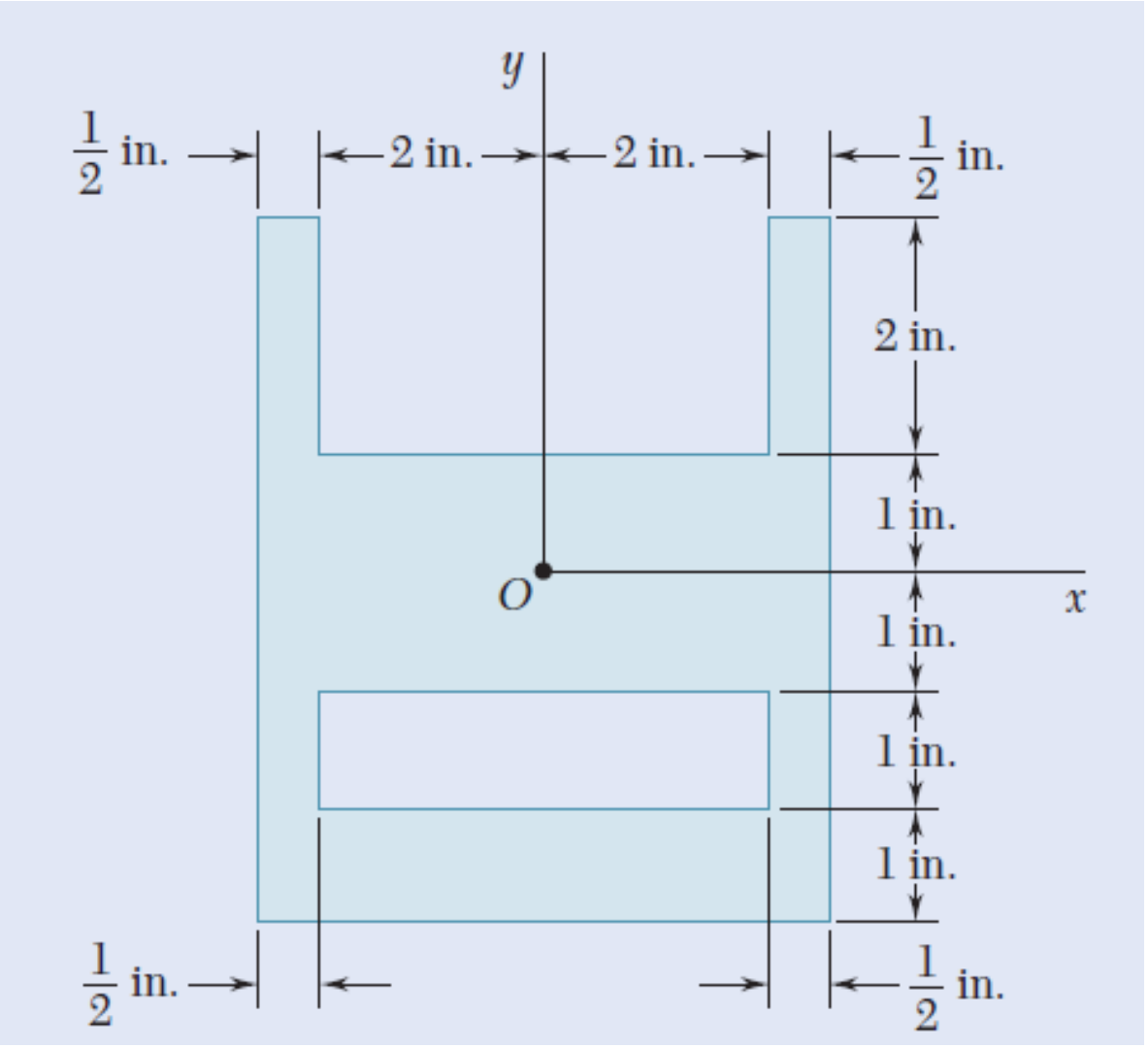
(a)

Rectangle		$\overline{I}_{x'} = \frac{1}{12}bh^3$ $\overline{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$
Triangle		$\overline{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$

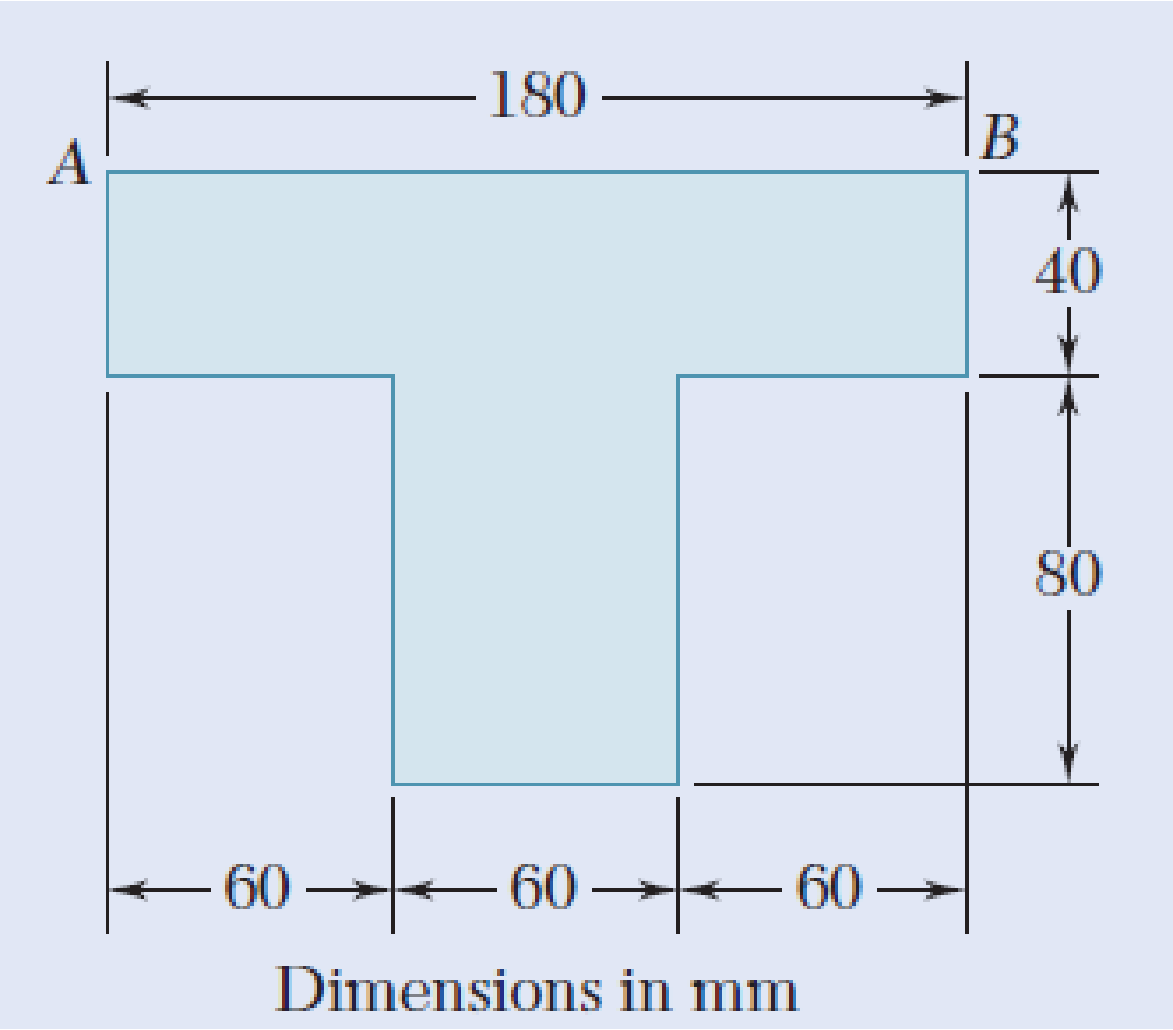
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$

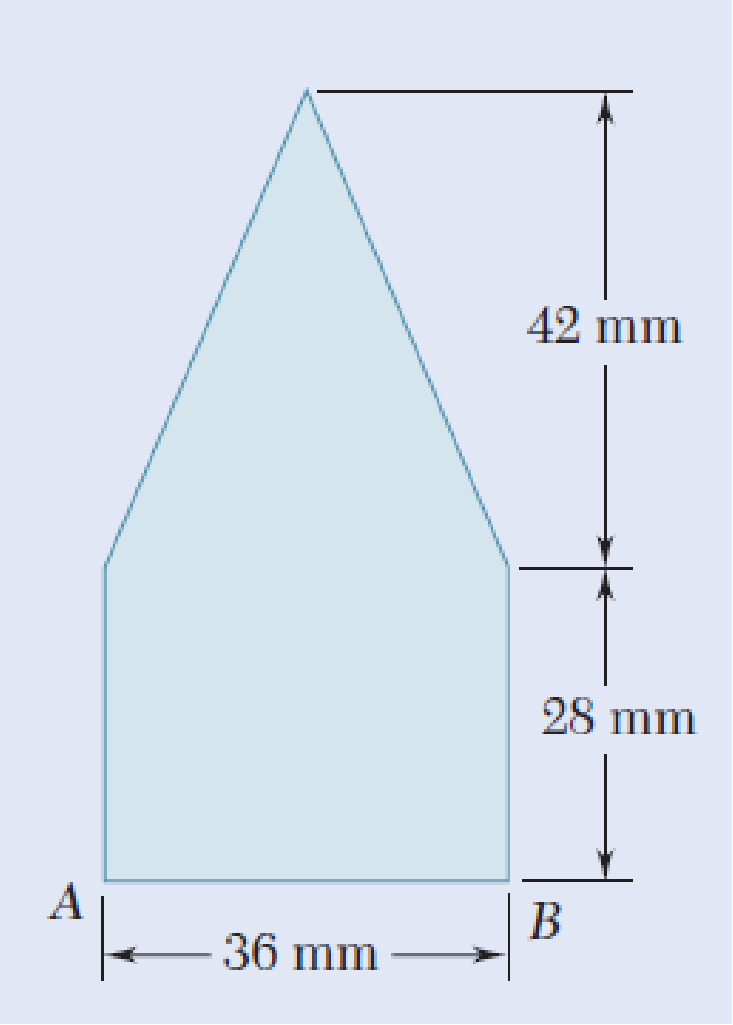
Determine the moment of inertia of the shaded area with respect to the *x* axis.





Determine the moments of inertia *I x* and *I y* of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.





Engineering Mechanics / Dynamics

Statics

Deals with bodies at rest

Dynamics

Deals with bodies in motion

Dynamics: Is that branch of mechanics which deals with the motion of bodies under the action of forces.

Dynamics

Kinematics

which is the study of motion without reference to the forces which cause motion

Kinetics

which relates the action of forces on bodies to their resulting motions.

A dynamic study could be done on two levels

Particle

An object whose size and shape can be ignored when studying its motion.

Rigid Body

collection of particles that remain at fixed distance from each other at all times and under all conditions of loading.

FORCE, MASS, AND ACCELERATION

Newton's Second Law

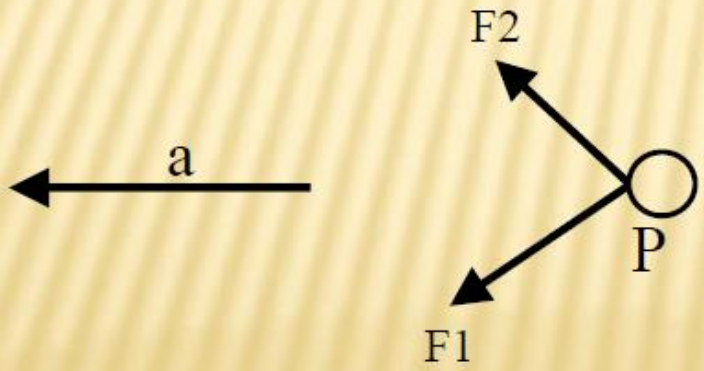
If the resultant force acting on a particle is not zero, the particle wil have an acceleration proportional to the magnitude of resultant and in the direction of the resultant.

$$\sum F = ma$$

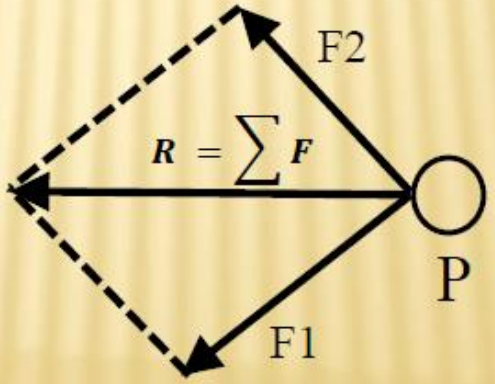
$$\sum F = 0$$

$$R = \sum F$$

F = resultant force acting on the particle
m = mass of the particle
a = acceleration of the particle



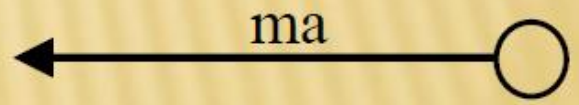
A



Free body diagram

B

=



Kinetic diagram

C

Systems of Units

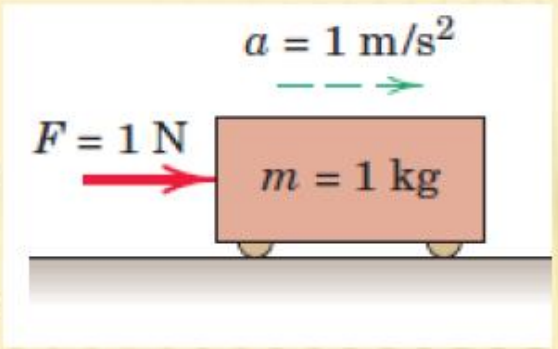
Of the units for the four primary dimensions (force, mass, length, and time), three may be chosen arbitrarily. The fourth must be compatible with Newton's 2nd Law.

International System of Units (SI Units): base units are the units of **length (m)**, **mass (kg)**, and **time (second)**.

The unit of force is derived

$$F = ma$$

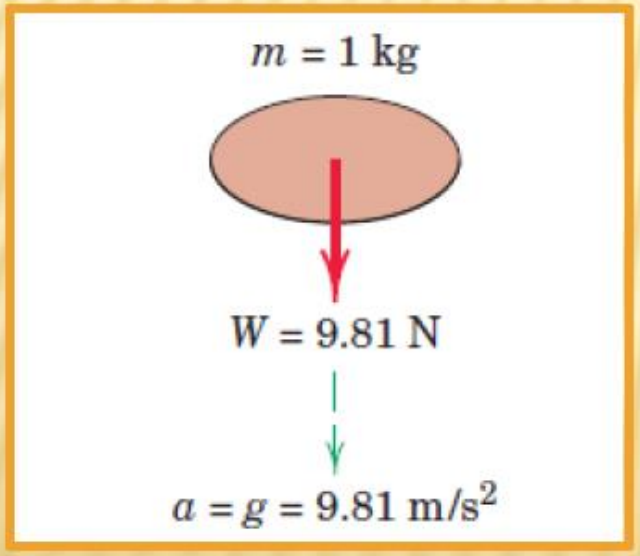
$$N = kg\frac{m}{s^2}$$



A: Newton's Second Law

1 Newton
is the force required to give a mass of 1 kg an acceleration of 1 m/s²

$$1\text{N} = (1\text{kg})\left(1\frac{\text{m}}{\text{s}^2}\right) = 1\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$



B: Gravitational Free Fall

$$W = mg$$

Kinetics of Particles

Equation of motion for a System of Particles

$$\sum F = ma$$

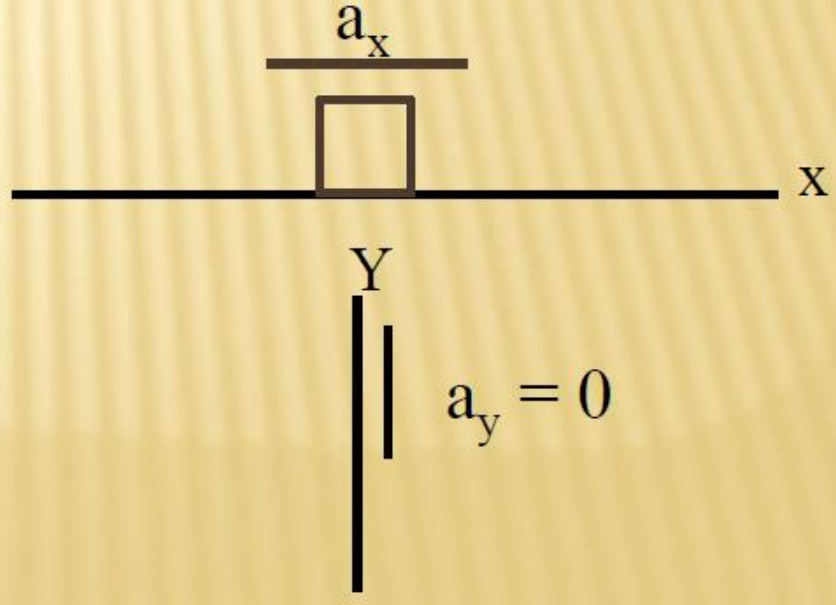
F = resultant force acting on the particle
m = mass of the particle
a = acceleration of the particle

Rectilinear Motion

- If the x-axis is the direction of the rectilinear motion:

$$\sum F_x = m.ax$$

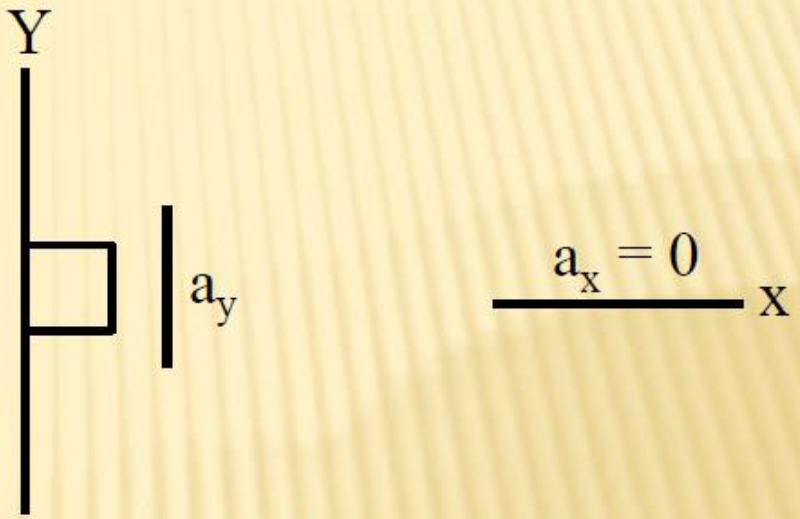
$$\sum F_y = 0$$



- If the Y-axis is the direction of the rectilinear motion:

$$\sum F_x = 0$$

$$\sum F_y = m \,.\, ay$$



- If we are not free to choose a coordinate direction along the motion, the nonzero acceleration component will be shown up in all equations:

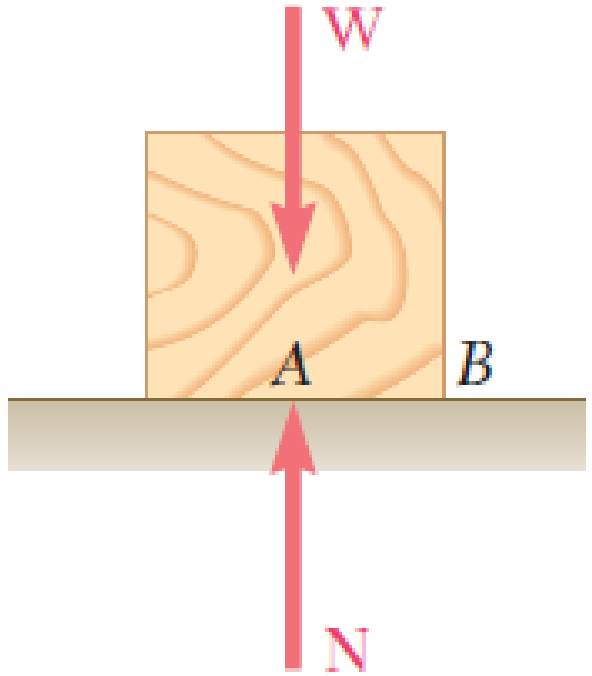
$$\sum F_x = m \,.\, ax$$

$$\sum F_y = m \,.\, ay$$

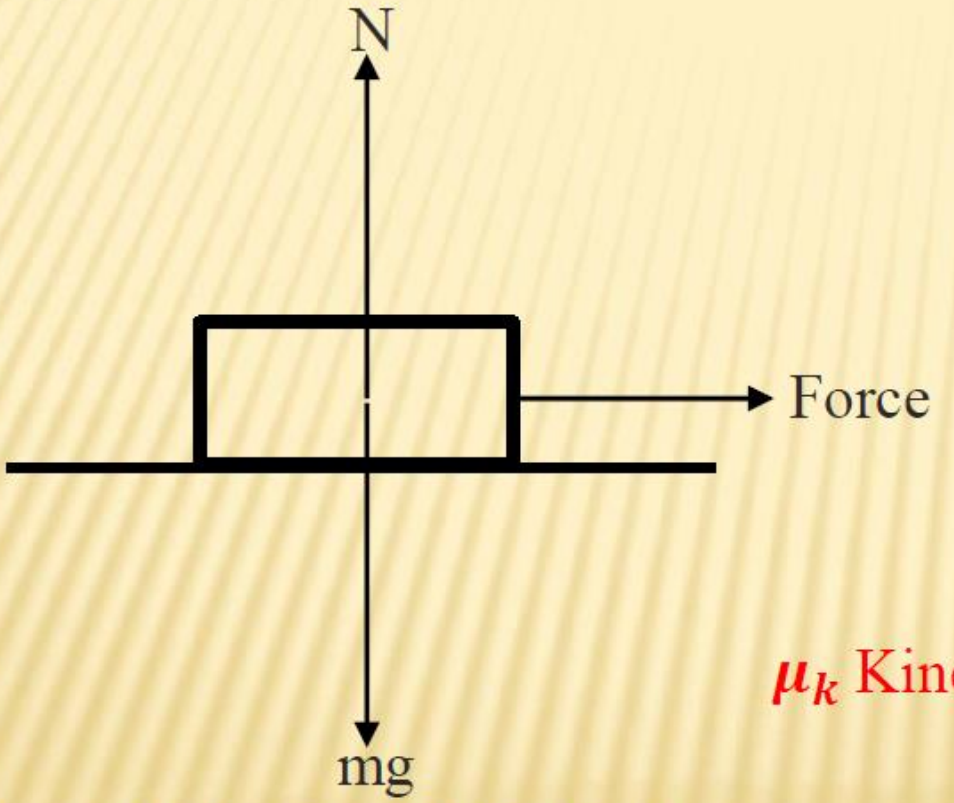
Procedure of analysis

- Select the inertial coordinate system. Most often, rectangular or x, y coordinates are chosen to analyze problems for which the particle has rectilinear motion.
- Once the coordinates are established, draw the particle's Freebody diagram. Drawing this diagram is very important since it a provides a graphical representation that accounts for all forces ($\sum F$) which act on the particle, and thereby makes it possible to resolve these forces into their x, y components.
- The direction and sense of the particle's acceleration a should also be established. If the sense is unknown, for mathematical convenience assume that the sense of each acceleration component acts in the same direction as its positive inertial coordinate axis.
- The acceleration may be represented as the ma vector on the kinetic diagram
- Identify the unknowns in the problem.
- If the forces can be resolved directly from the free-body diagram, apply the equations of motion in their scalar component form.

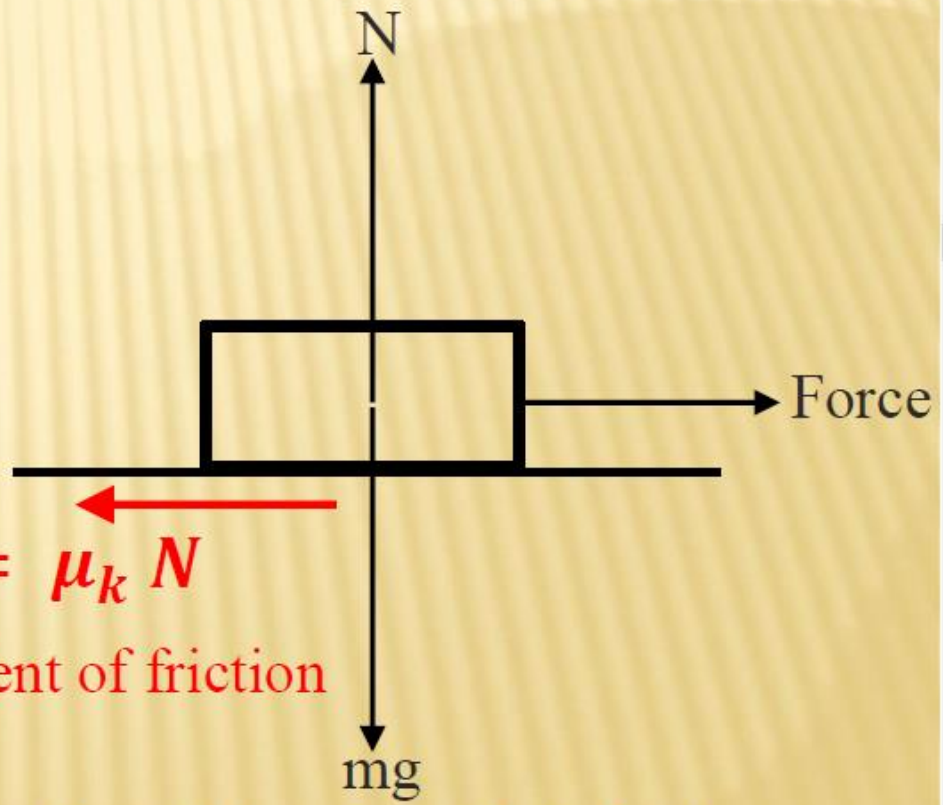
If surfaces in contact are rough, tangential forces can develop that prevent the motion of one surface with respect to the other. When two surfaces are in contact, tangential forces, called **friction forces**, Since the weight has no horizontal component, the reaction of the surface also has no horizontal component; the reaction is therefore *normal* to the surface and is represented by **N**



- **Friction**. If a moving particle contacts a rough surface, it may be necessary to use the frictional equation, which relates the frictional and normal forces F_f and N acting at the surface of contact by using the coefficient of kinetic friction, i.e., $F_f = \mu_k N$. Remember that F_f always acts on the free-body diagram such that it opposes the motion of the particle relative to the surface it contacts.



Smooth Surface (Frictionless)

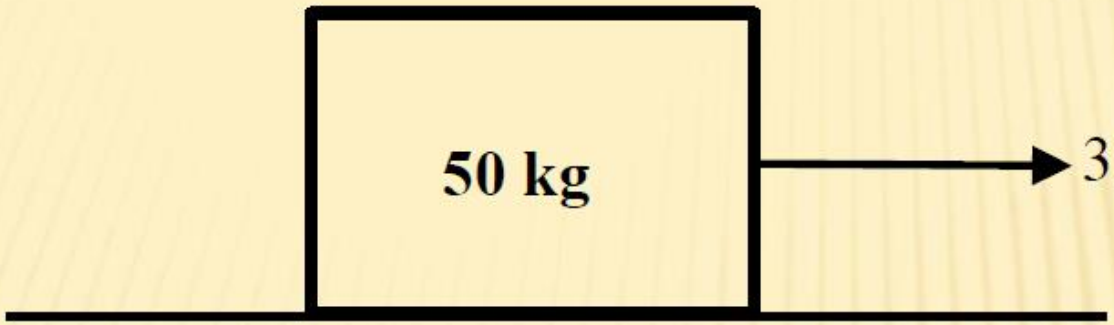


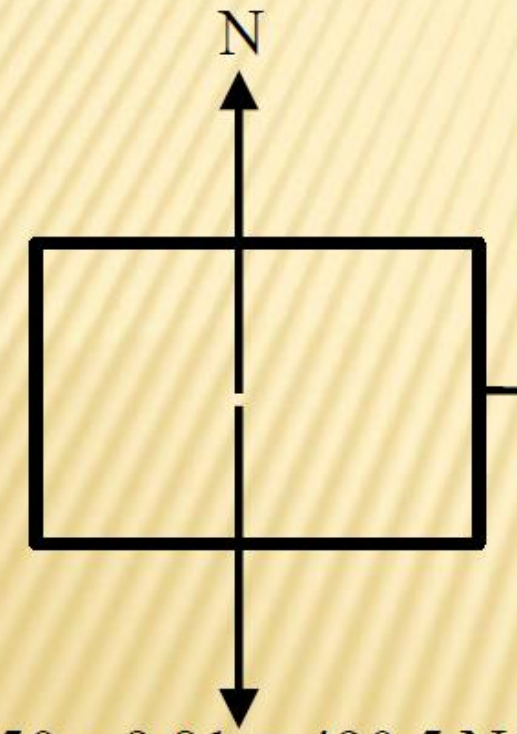
$F_f = \mu_k N$

μ_k Kinetic coefficient of friction

Rough Surface (Friction)

Example: The 50 kg crate shown in Figure rests on a horizontal smooth surface. If the crate is subjected to a 300 N towing force as shown, determine the acceleration of the crate.





FBD



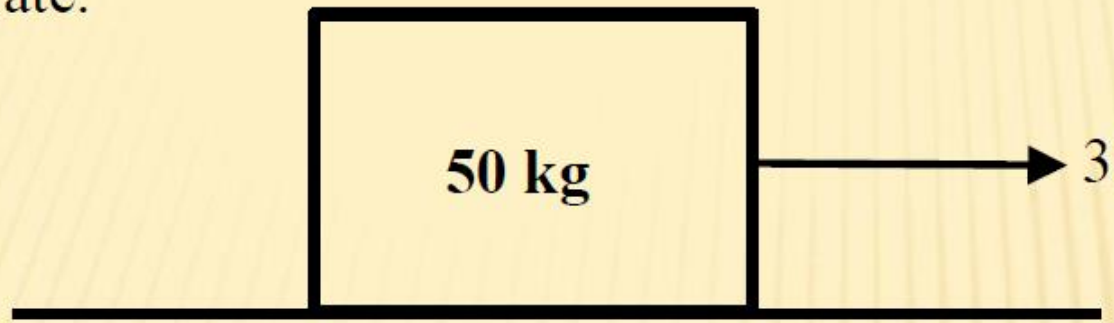
Kinetics Diagram

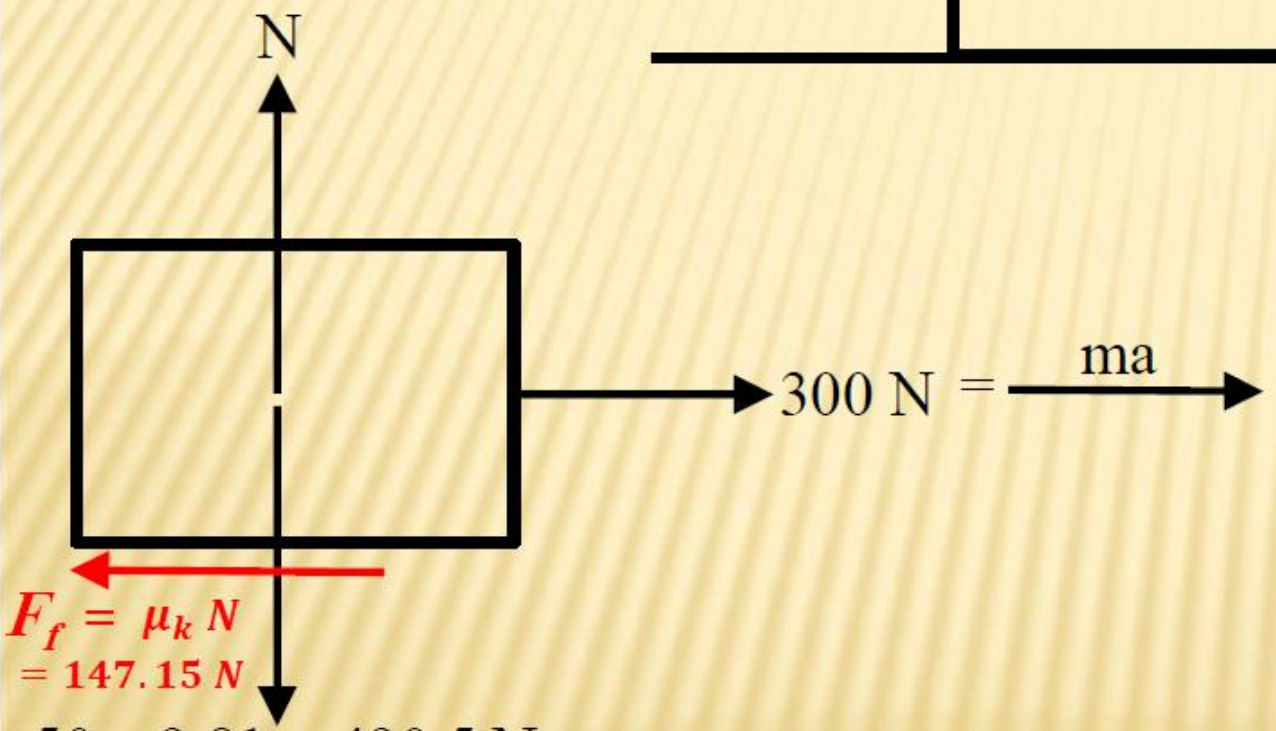
$$\sum F_x = m \,.\, ax$$

$$300 = 50.\, a_x$$

$$a_x = 6\ \frac{m}{s^2}$$

Example: The 50 kg crate shown in Figure rests on a horizontal surface which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 300 N towing force as shown, determine the acceleration of the crate.



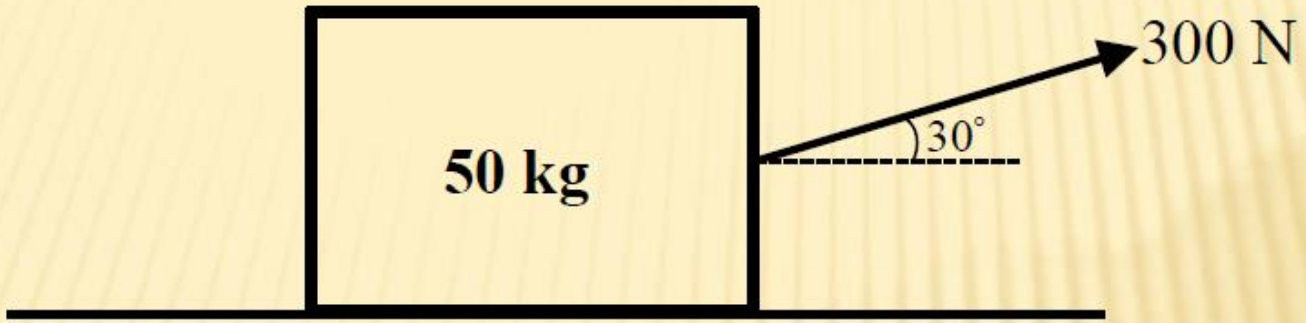


FBD

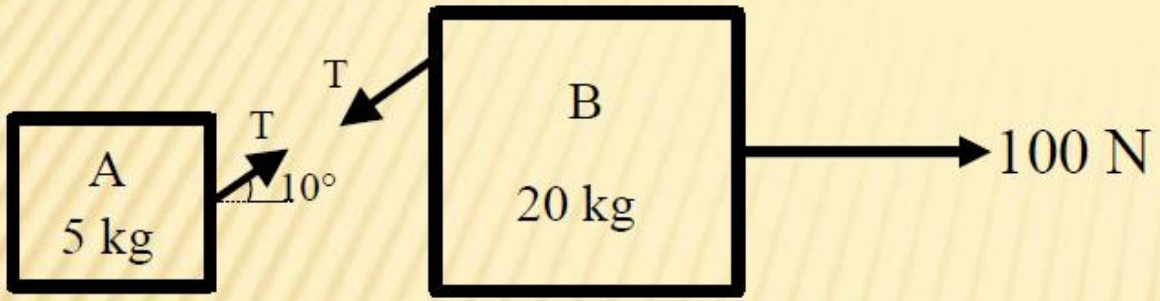
Kinetics Diagram

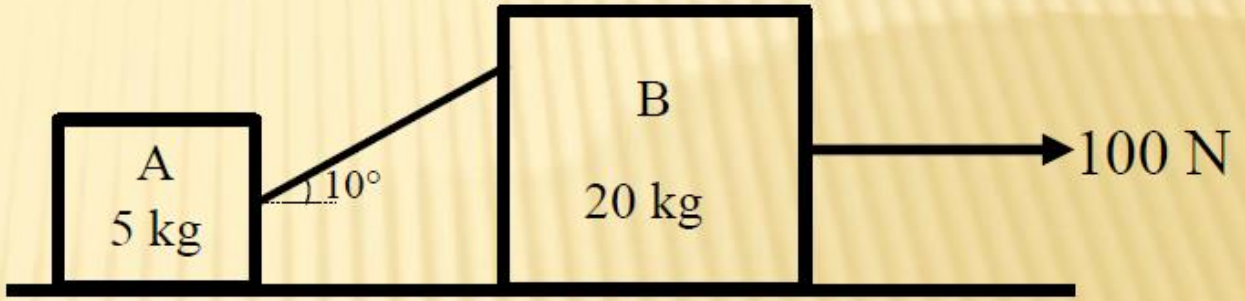
$\sum F_y = m\,.\,ay$	$\sum F_x = m\,.\,ax$
$a_y = 0$	$300 - F_f = 50\,.\,a_x$
$\sum F_y = 0$	$F_f = 0.3\ x\ 490.5 = 147.15\ N$
$N - 490.5 = 0$	$300 - 147.15 = 50\,.\,a_x$
$N = 490.5N$	$a_x = 3.057\ \frac{m}{s^2}$

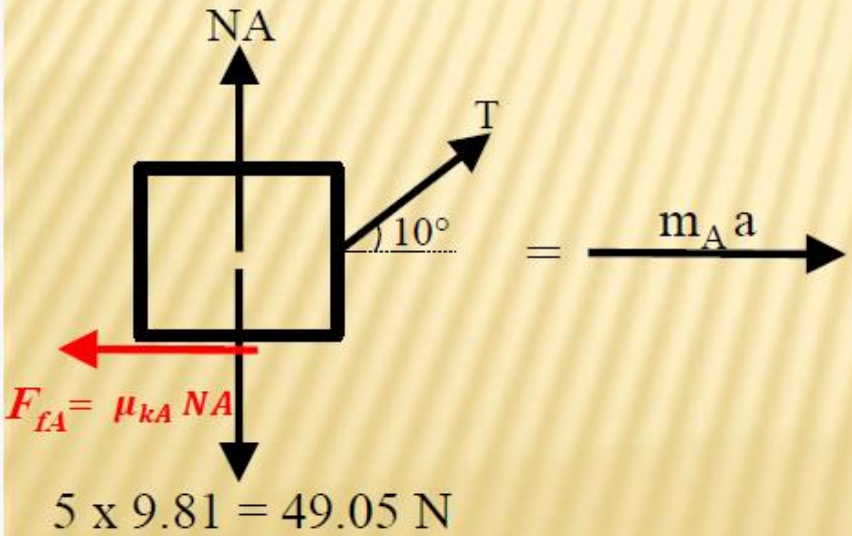
H.W.: The 50 kg crate shown in Figure rests on a horizontal surface which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 300 N towing force as shown, determine the acceleration of the crate.

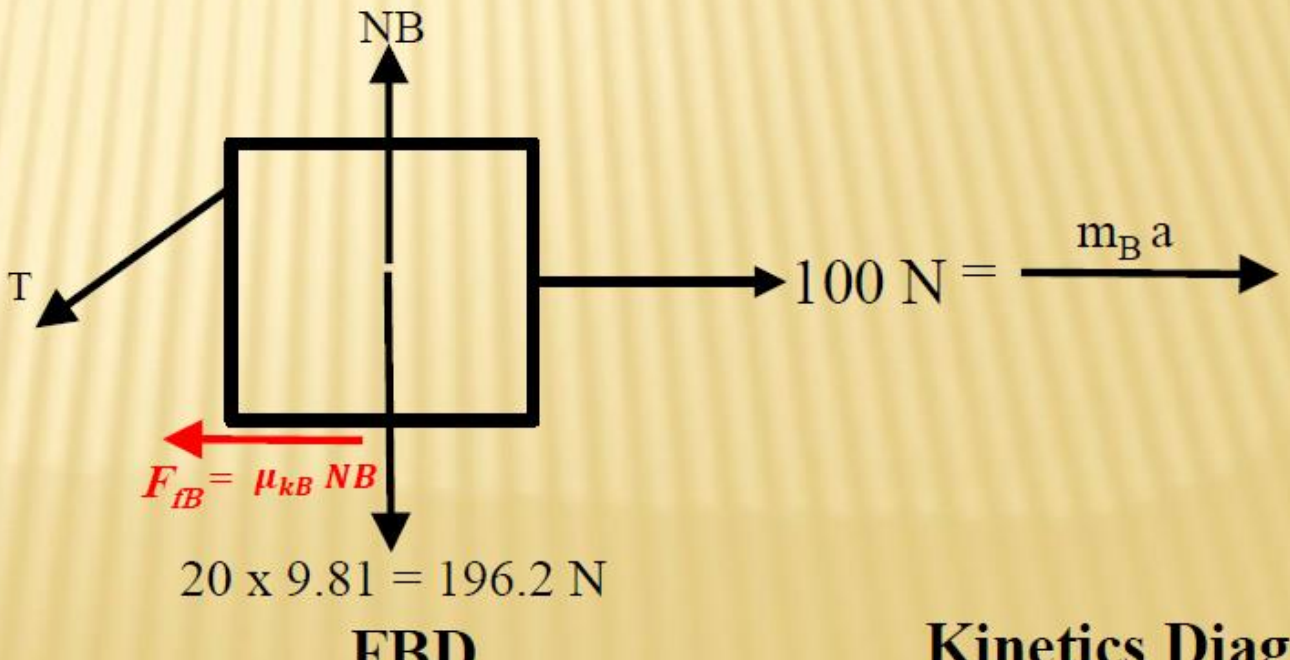


Example: Two blocks A and B of masses 5 kg and 20 kg are connected by an inclined cord . A horizontal force 100 N is applied to block B as shown in figure. The coefficient of friction between block A and the surface is 0.5 and between block B and the surface is 0.25. Calculate the tension in the cord, and the acceleration of the system.



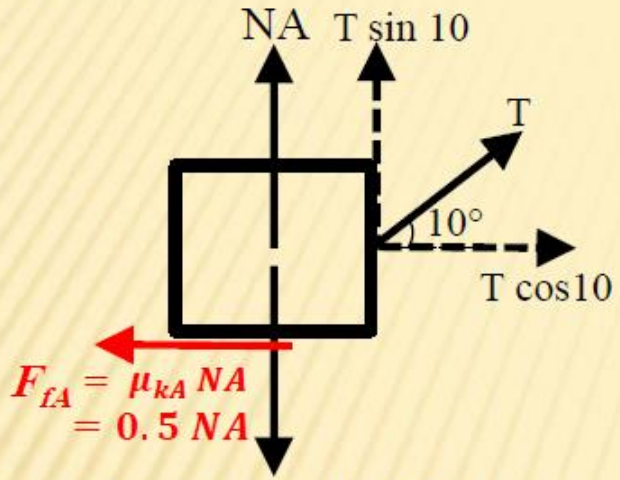






FBD **Kinetics Diagram** **FBD** **Kinetics Diagram**

Block A



5 x 9.81 = 49.05 N

$$\sum F_y = 0$$

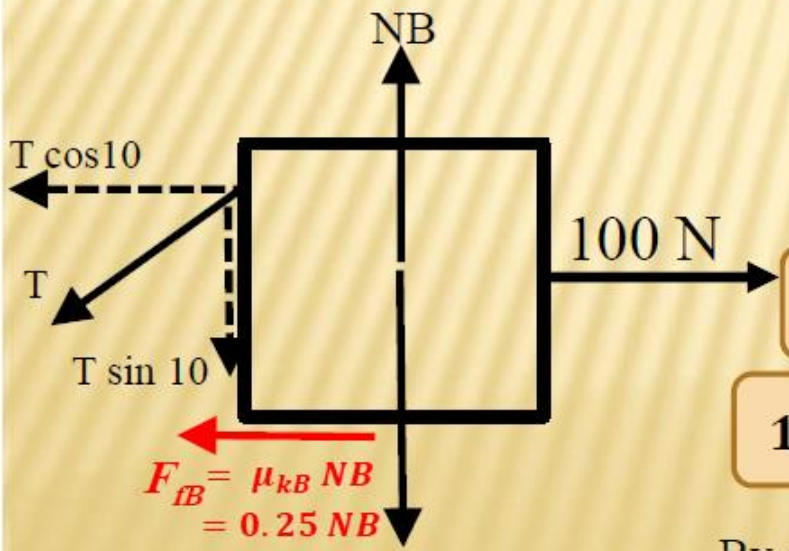
$$N_A + T\ sin\ 10 - 49.05 = 0 \Rightarrow N_A = 49.05 - T\ sin\ 10$$

$$\sum F_x = mA\ .\ ax$$

$$T\ cos\ 10 - F_{fA} = 5\ .\ a_x$$

$$T\ cos\ 10 - 0.5NA = 5\ .\ a_x \Rightarrow 1.072\ T - 24.525 = 5\ .\ a_x \quad (1)$$

Block B



20 x 9.81 = 196.2 N

$$\sum F_y = 0$$

$$N_B - T\ sin\ 10 - 196.2 = 0 \Rightarrow N_B = 196.2 + T\ sin\ 10$$

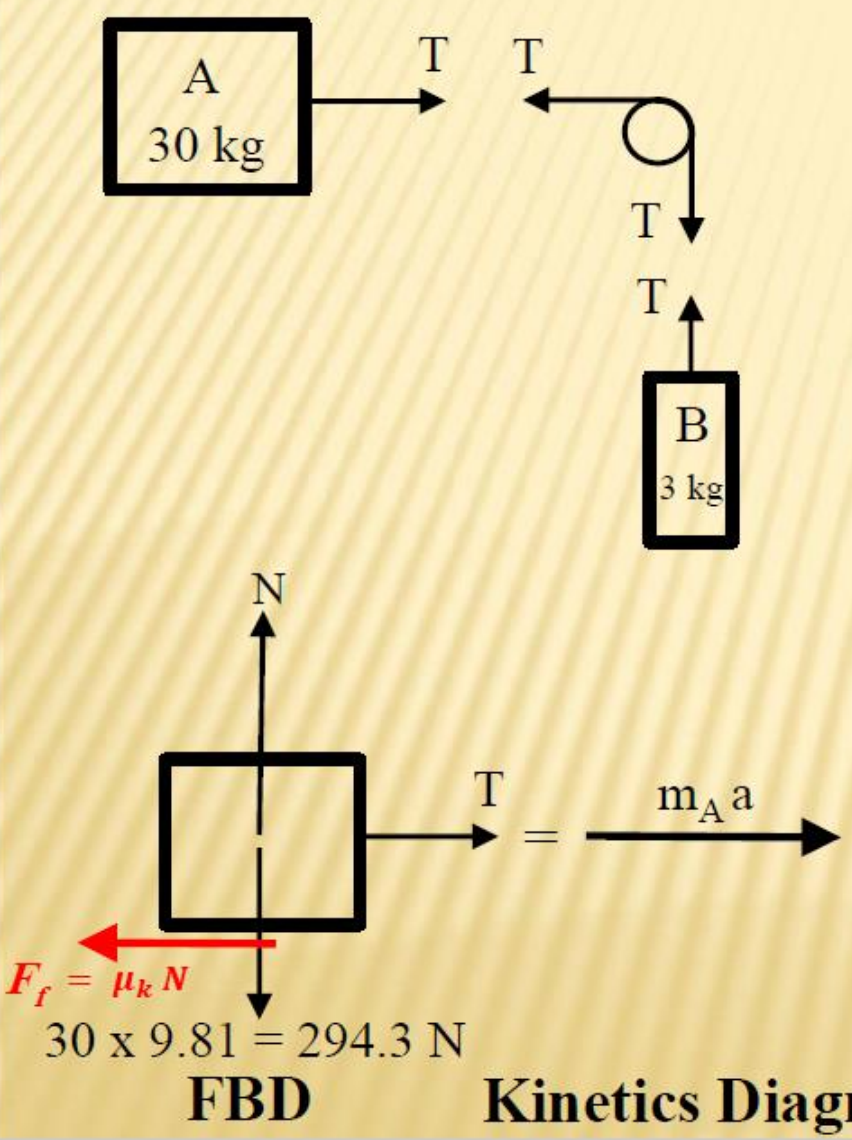
$$\sum F_x = mB\ .\ ax$$

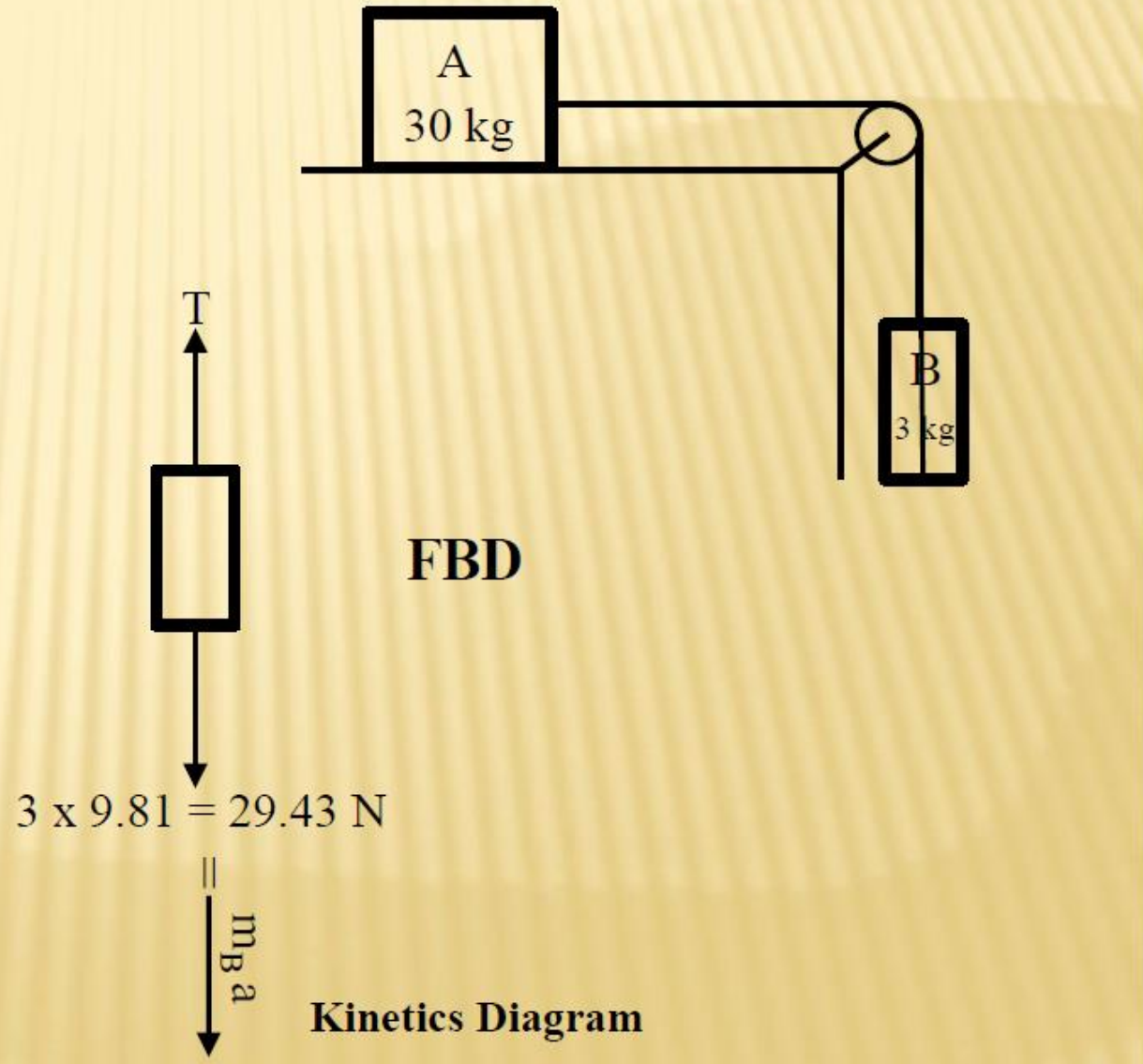
$$100 - T\ cos\ 10 - F_{fB} = 20\ .\ a_x$$

$$100 - T\ cos\ 10 - 0.25\ NB = 20\ .\ a_x \Rightarrow -1.028\ T + 50.95 = 20\ .\ a_x \quad (2)$$

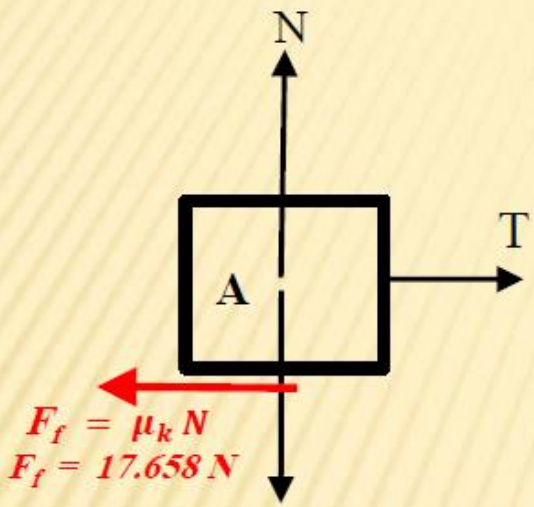
By solving (1) and (2) We get $\Rightarrow$ $T = 28.038\ N$ $\quad ax = 1.106\ \frac{m}{s^2}$

Example: A block A of mass 30 kg is resting on a horizontal table. The block is attached to a cord whose other end is carrying a body B of mass 3 kg. If the coefficient of friction between block A and the table is 0.06. Calculate the acceleration of the system.





Block A



30 x 9.81 = 294.3 N

$$\sum F_y = 0$$

$$N - 294.3 = 0 \rightarrow N = 294.3\ N$$

$$\sum F_x = mA\,.\,a$$

$$T - F_f = 30\,.\,a$$

$$T - 17.658 = 30\,.\,a \quad \cdots\cdots\cdots (1)$$

Block B



3 x 9.81 = 29.43 N

$$\sum F_Y = mB\,.\,a$$

$$29.43 - T = 3\,.\,a \quad \cdots\cdots\cdots (2)$$

By solving (1) and (2) We get ⇒ $a = 0.357\ \frac{m}{s^2}$

Inclined Plane

Example: A 50 kg mass is placed on an inclined plane that makes an angle 30 degree with the horizontal. The coefficient of friction between the mass and inclined is 0.2. Calculate the acceleration.

$$\sum F_y = 0$$

$$N - 424.785 = 0 \rightarrow N = 424.785N$$

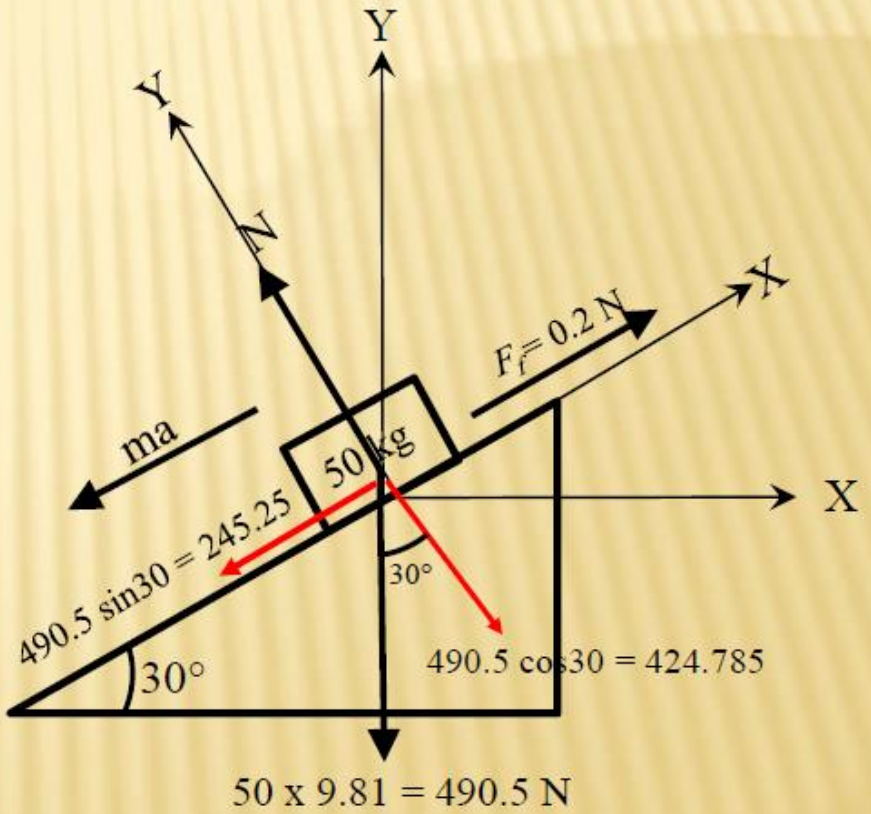
$$F_f = \mu_k \; N = 0.2 \; x \; 424.785 = 84.957N$$

$$\sum F_x = m \, . \, a$$

$$245.25 - F_f = 50 \, . \, a$$

$$245.25 - 84.957 = 50 \, . \, a$$

$$a = 3.206 \frac{m}{s^2}$$

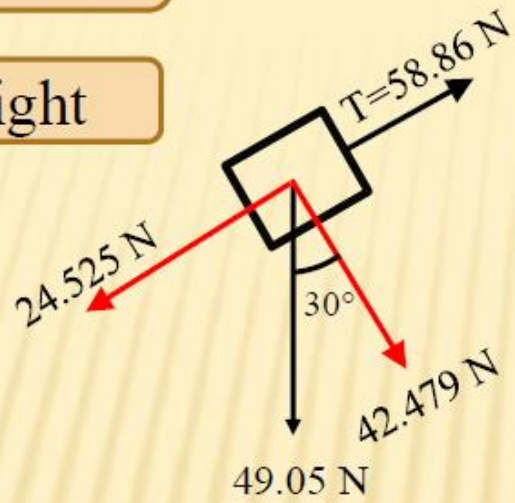


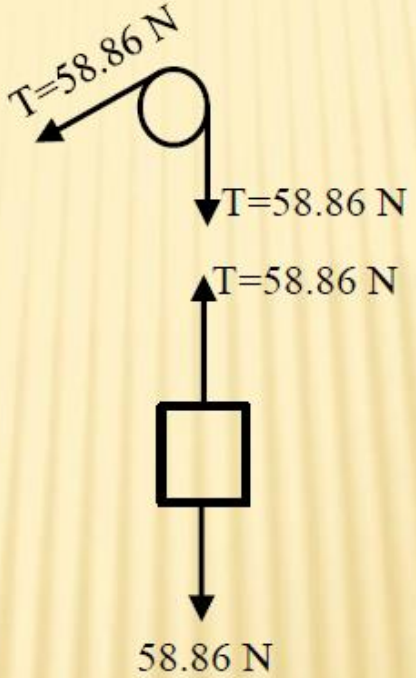
Example: A block A of mass 5 kg is placed on a 30 degree inclined plane, and is attached to a rope whose other end is carrying a body B of mass 6 kg. If the coefficient of friction between block A and the inclined is 0.2. Calculate the acceleration of the system and the tension in the rope

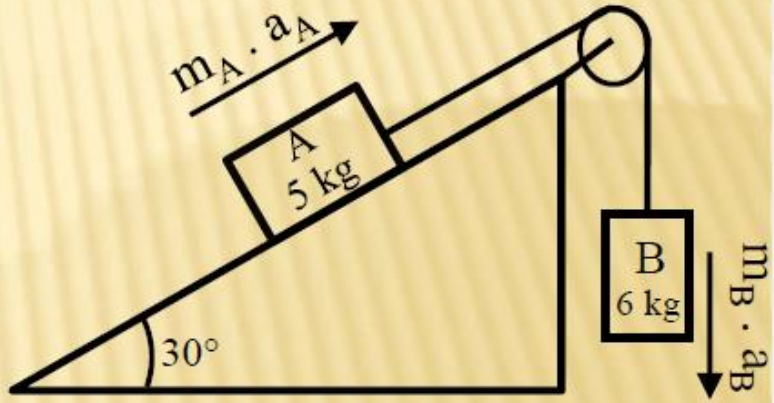
.

Step 1: Direction of motion Static

T(58.86) > 24.525 Right







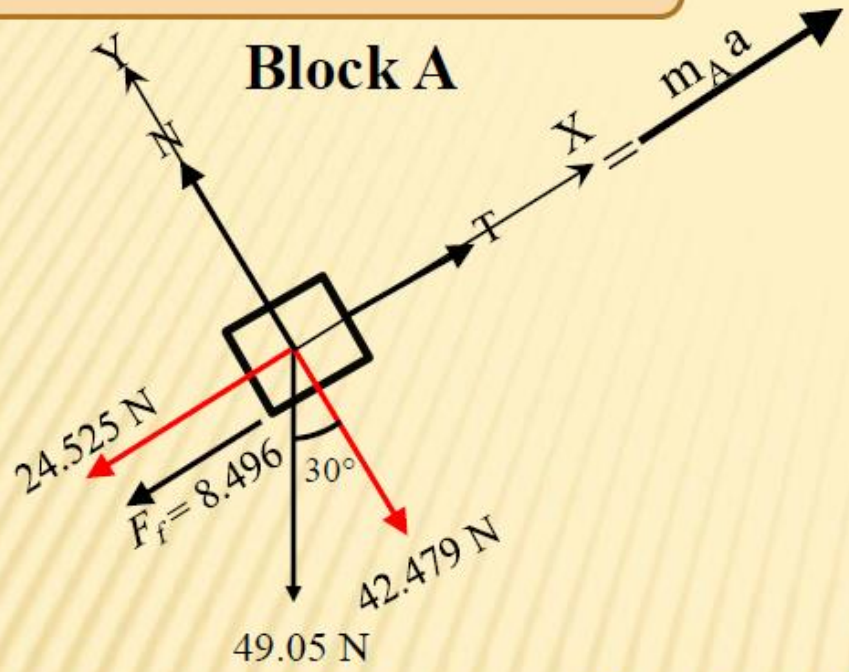
Step 2: Acceleration relation between Blocks A & B

$$\sum T.a = 0$$

$$a_A.T - a_B.T = 0 \Rightarrow a_A = a_B = a$$

Step 3: Motion analysis

Block A



FBD **Kinetics Diagram**

$$\sum F_y = 0$$

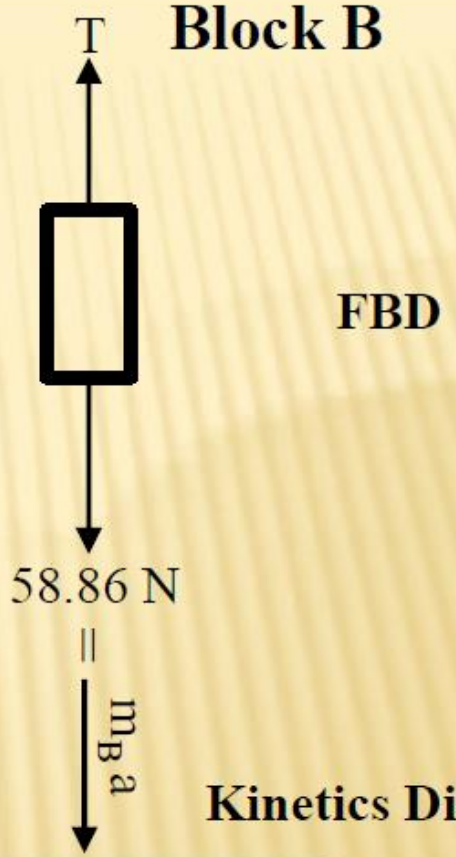
$$N - 42.479 = 0 \rightarrow N = 42.479N$$

$$\sum F_x = mA \,.\, a$$

$$T - 24.525 - F_f = 5 \,.\, a$$

$$T - 33.021 = 5 \,.\, a \quad \text{(1)}$$

Block B



Kinetics Diagram

$$\sum F_Y = mB \,.\, a$$

$$58.86 - T = 6 \,.\, a \quad \text{(2)}$$

By solving (1) and (2) We get

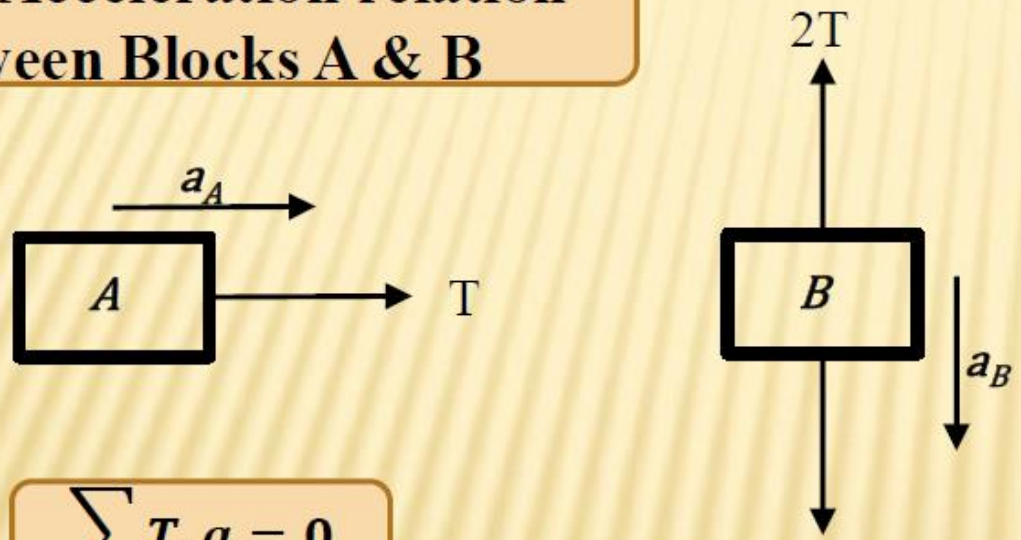
$$a = 2.349 \frac{m}{s^2}$$

$$T = 44.766\ N$$

Example: The two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in each cord.

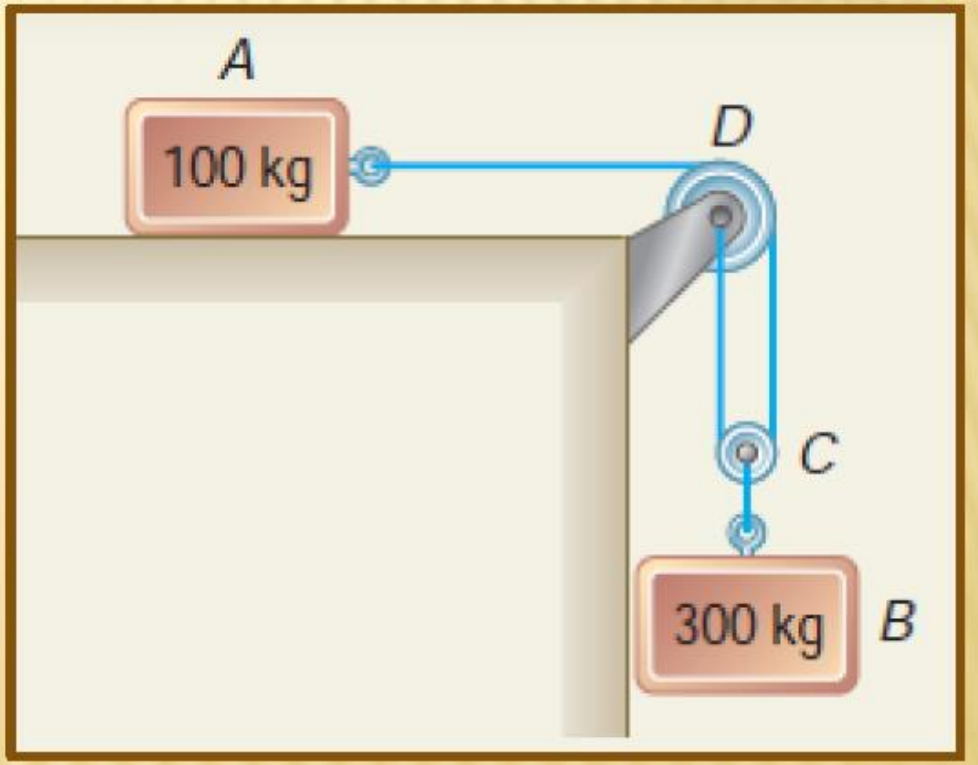
Step 1: Direction of motion

Step 2: Acceleration relation between Blocks A & B



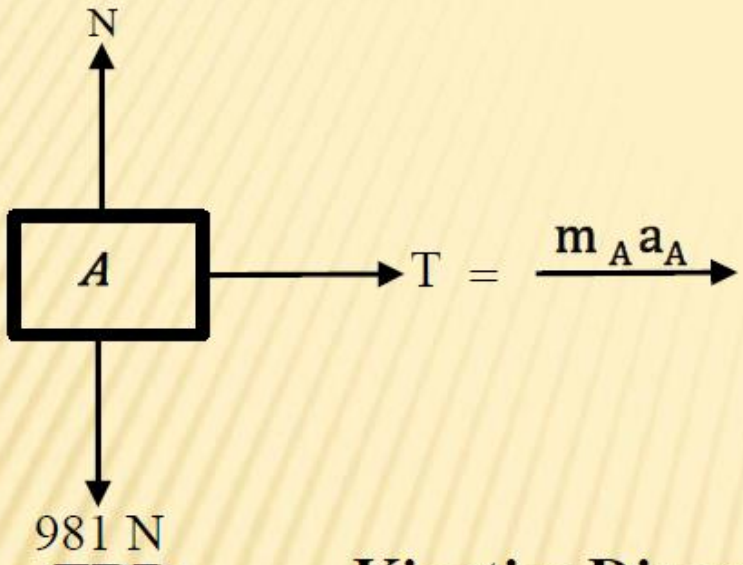
$$\sum T.a = 0$$

$$a_A.T - a_B.2T = 0 \Rightarrow a_A = 2a_B$$



Step 3: Motion analysis

Block A



FBD **Kinetics Diagram**

$$\sum F_y = 0$$

$$N - 981 = 0 \rightarrow N = 981\ N$$

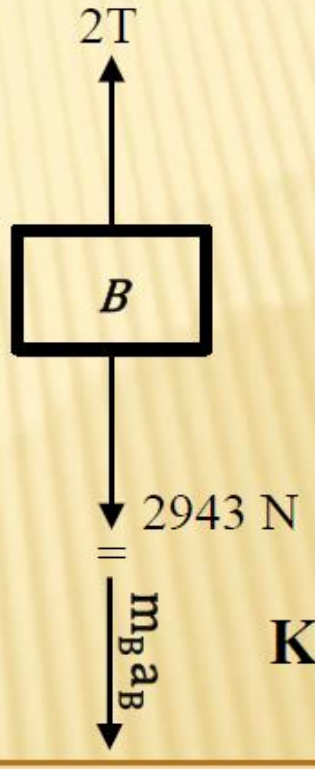
$$\sum F_x = mA\,.\,aA$$

$$T = 100\,.\,aA$$

$$a_A = 2a_B$$

$$T = 200\,.\,aB \quad \text{.....} \quad (1)$$

BlockB



FBD **Kinetics Diagram**

$$\sum F_Y = mB\,.\,aB$$

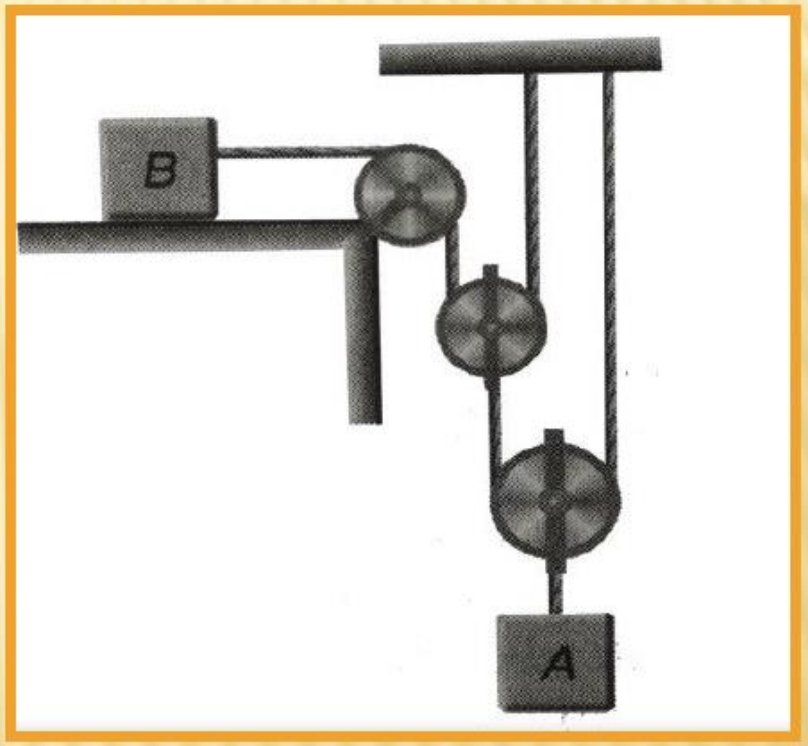
$$2943 - 2T = 300\,.\,aB \quad \text{.....} \quad (2)$$

By solving (1) and (2) We get

$$a_B = 4.204\,\frac{m}{s^2} \qquad a_A = 8.409\,\frac{m}{s^2}$$

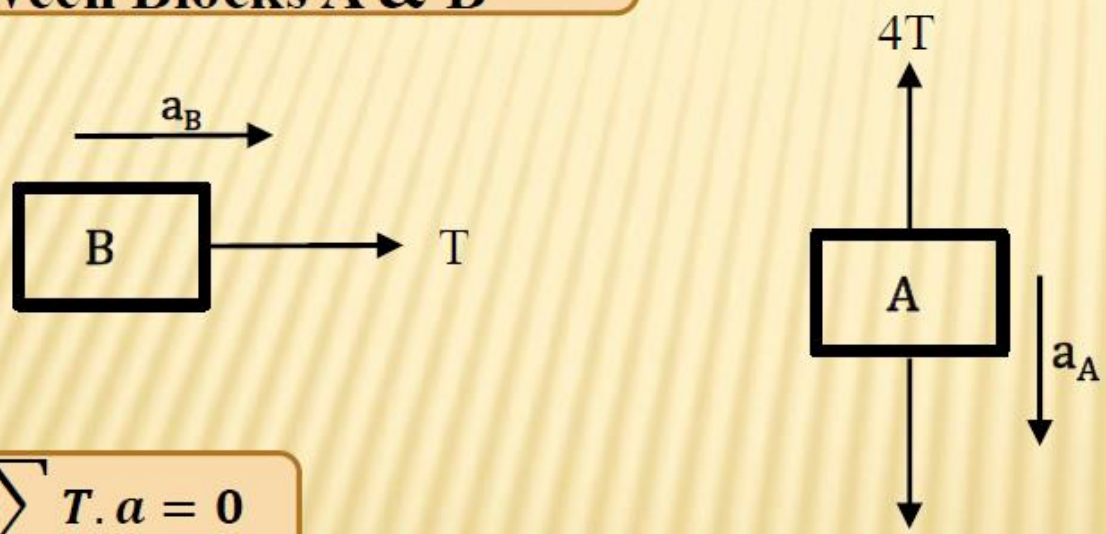
$$T = 840.857\ N$$

Example: Determine the acceleration of the 5 kg block A. Neglect the mass of the pulleys and cords. The block B has a mass of 10 kg. The coefficient of kinetics friction between block B and the surface is 0.1.



Step 1: Direction of motion

Step 2: Acceleration relation between Blocks A & B

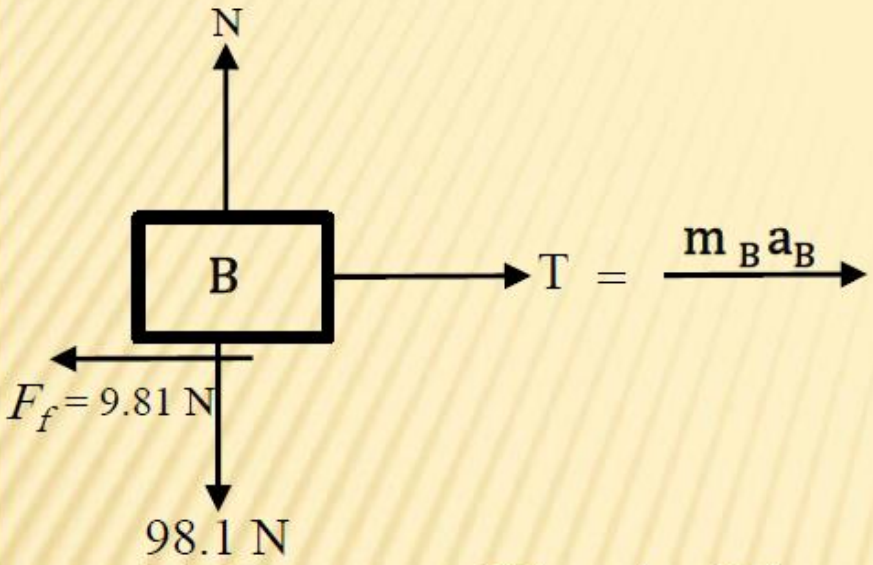


$$\sum T.a = 0$$

$$a_B.T - a_A.4T = 0 \Rightarrow a_B = 4a_A \quad \text{.....} (1)$$

Step 3: Motion analysis

Block B



FBD Kinetics Diagram

$$\sum F_y = 0$$

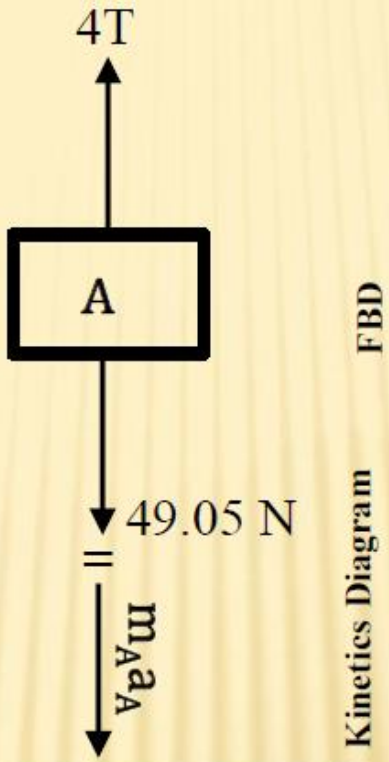
$$N - 98.1 = 0 \rightarrow N = 98.1\ N$$

$$\sum F_x = {}_mB\,.\,aB$$

$$T - 9.81 = 10\,.\,aB$$

$$a_B = \frac{T - 9.81}{10} \quad \text{.....} \quad 2$$

Block A



$$\sum F_Y = mA\,.\,aA$$

$$49.05 - 4T = 5\,.\,aA$$

$$a_A = \frac{49.05 - 4T}{5} \quad \text{.....} \quad 3$$

Substitute 2 and 3 in 1

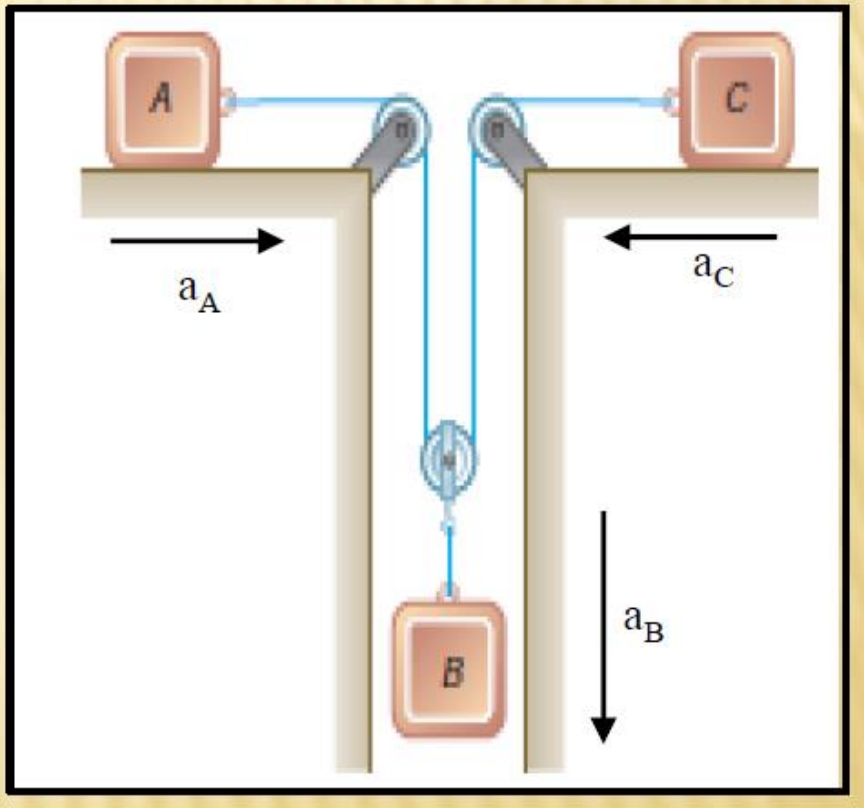
$$a_B = 4a_A$$

$$\frac{T - 9.81}{10} = 4\left(\frac{49.05 - 4T}{5}\right)$$

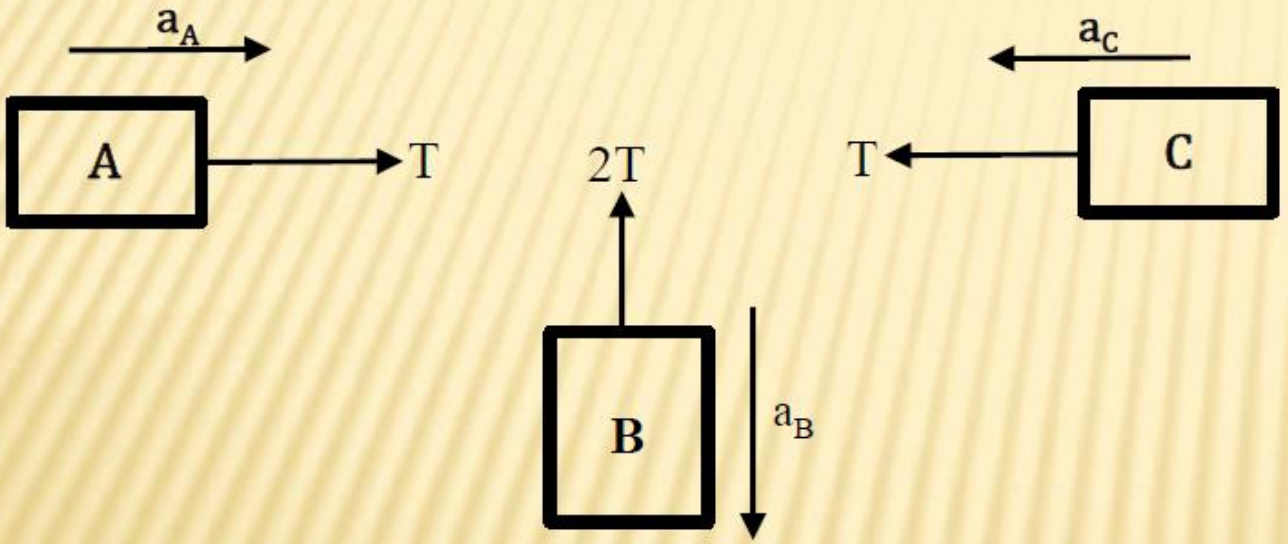
$$T = 12.188\ N$$

$$a_A = 0.0595\ \frac{m}{s^2}$$

Example: A system of blocks A, B, and C are connected as shown in figure. The coefficients of friction between blocks A and C with the horizontal surfaces $\mu_k = 0.2$. Knowing that $m_A = 5$ kg, $m_B = 10$ kg, and $m_C = 10$ kg, determine **(a)** the tension in the cord, **(b)** the acceleration of each block.



Step 1: Acceleration relation between Blocks A , B & C

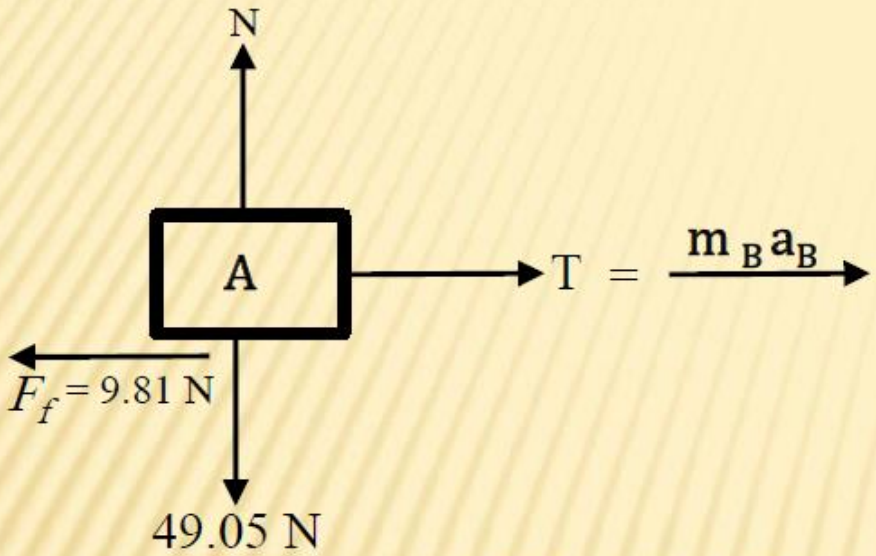


$$\sum T.a = 0$$

$$(a_A.T) - (a_B.2T) + (a_C.T) = 0 \Rightarrow a_A - 2a_B + a_C = 0 \quad \cdots\cdots (1)$$

Step 2: Motion analysis

Block A



FBD

Kinetics Diagram

$$\sum F_y = 0$$

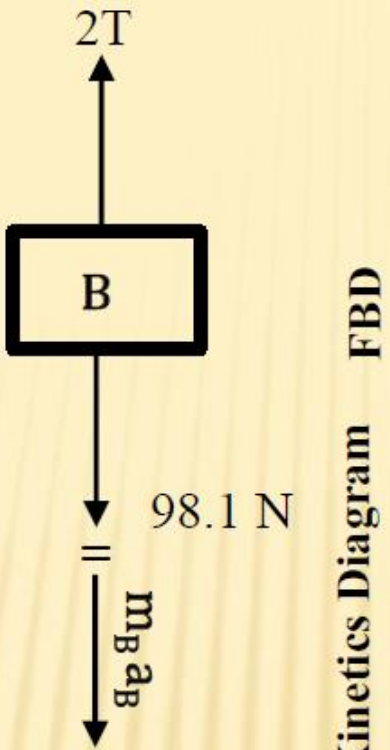
$$N - 49.05 = 0 \rightarrow N = 49.05N$$

$$\sum F_x = mA \,.\, aA$$

$$T - 9.81 = 5 \,.\, aA$$

$$a_A = \frac{T - 9.81}{5} \quad \cdots\cdots (2)$$

Block B



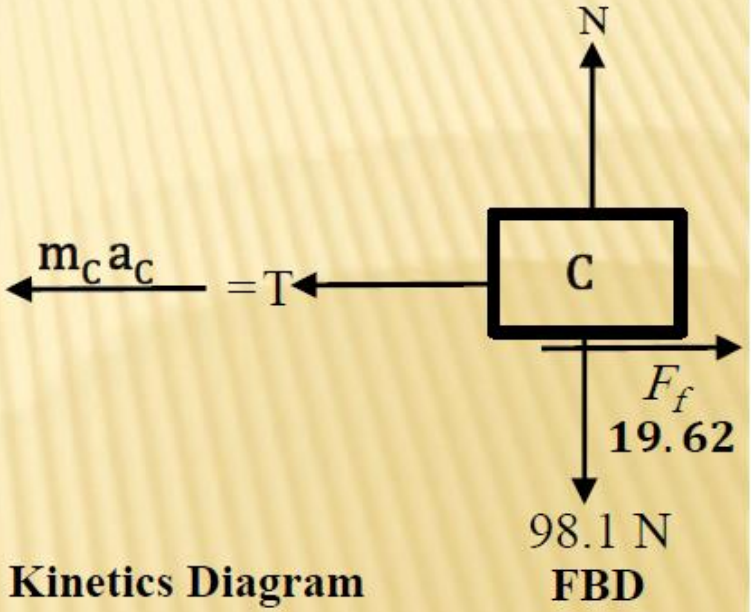
Kinetics Diagram FBD

$$\sum F_Y = mB \,.\, aB$$

$$98.1 - 2T = 10 \,.\, aB$$

$$a_B = \frac{98.1 - 2T}{10} \quad \cdots\cdots (3)$$

Block C



Kinetics Diagram FBD

$$\sum F_y = 0$$

$$N - 98.1 = 0 \rightarrow N = 98.1\ N$$

$$\sum F_x = mC \,.\, aC$$

$$T - 19.62 = 10 \,.\, aC$$

$$a_C = \frac{T - 19.62}{10} \quad \cdots\cdots (4)$$

1

$$a_A - 2a_B + a_C = 0$$

$$a_A = \frac{T - 9.81}{5}$$ (2)

$$a_B = \frac{98.1 - 2T}{10}$$ (3)

$$a_C = \frac{T - 19.62}{10}$$ (4)

Substitute in (1)

$$\frac{T-9.81}{5} - 2\left(\frac{98.1-2T}{10}\right) + \frac{T-19.62}{10} = 0$$

$$T = 33.634\ N$$

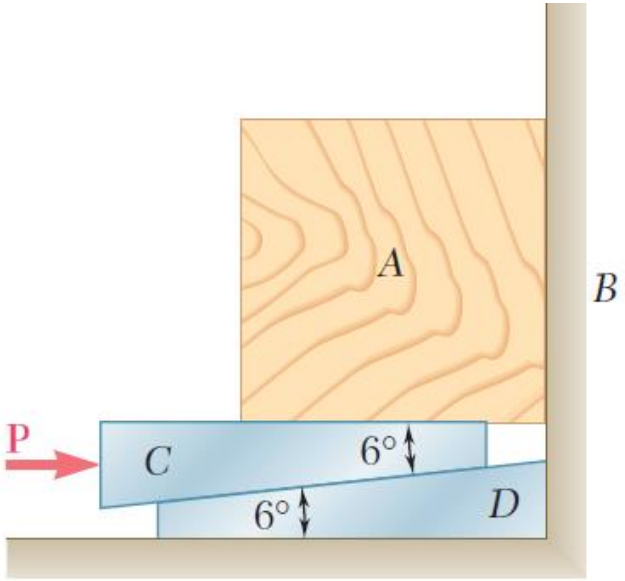
$$a_A = 4.765\ \frac{m}{s^2}$$

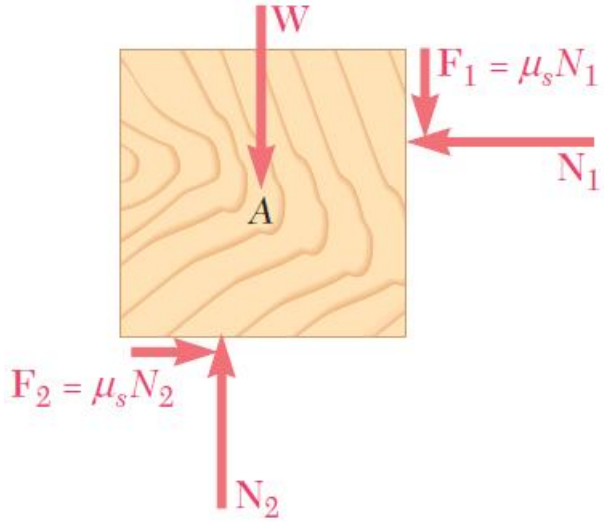
$$a_B = 3.083\ \frac{m}{s^2}$$

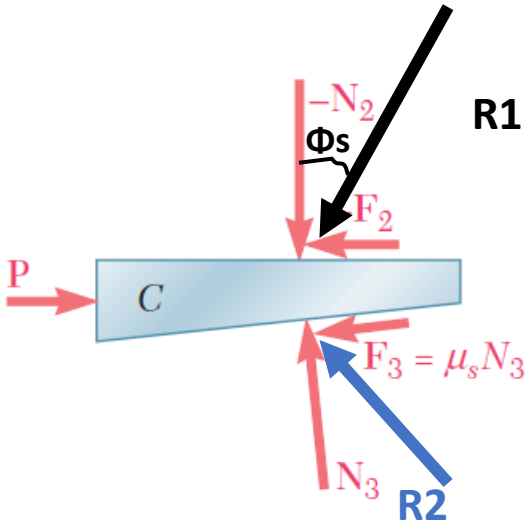
$$a_C = 1.401\ \frac{m}{s^2}$$

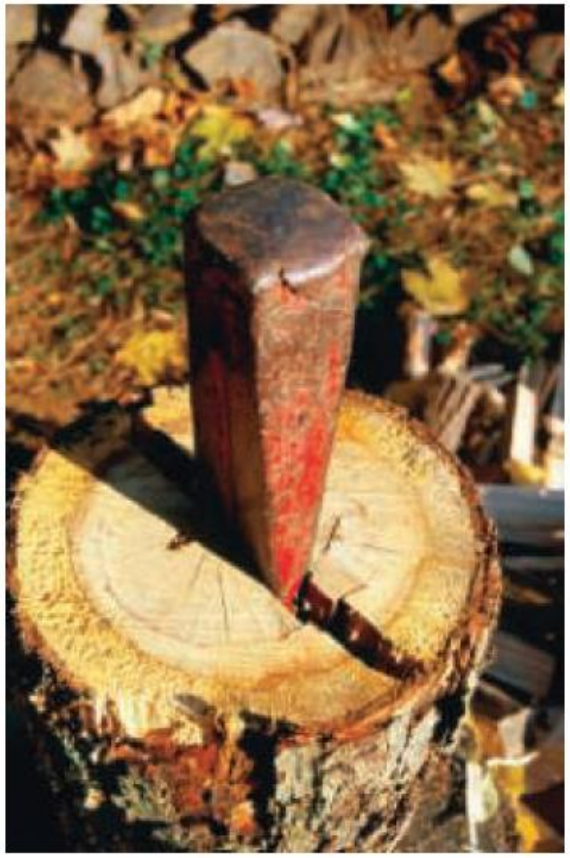
Wedges

Wedges are simple machines used to raise large stone blocks and other heavy loads. These loads are raised by applying to the wedge a force usually considerably smaller than the weight of the load. In addition, because of the friction between the surfaces in contact, a properly shaped wedge remains in place after being forced under the load

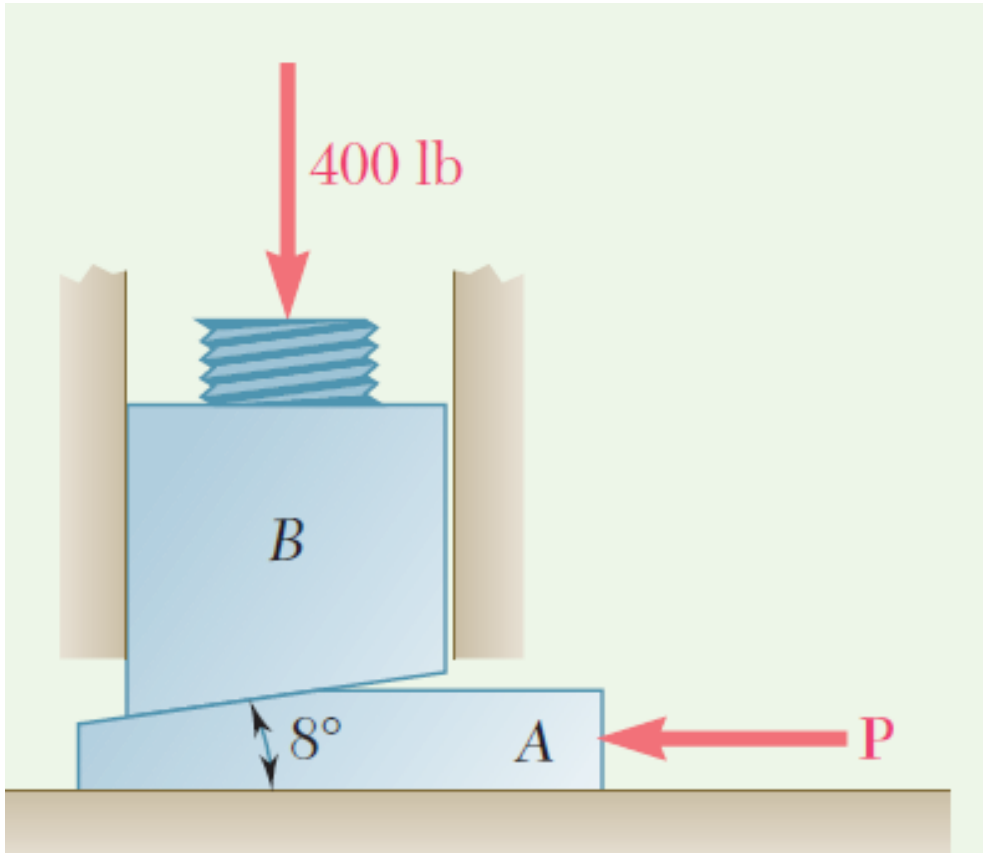




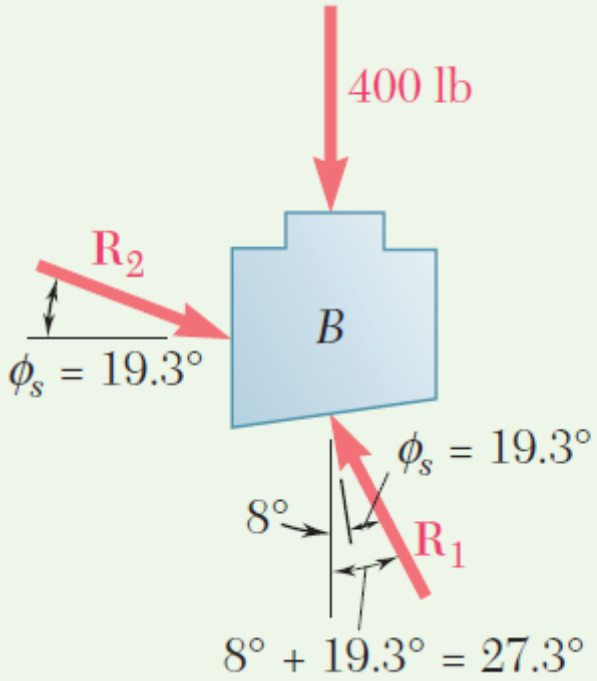


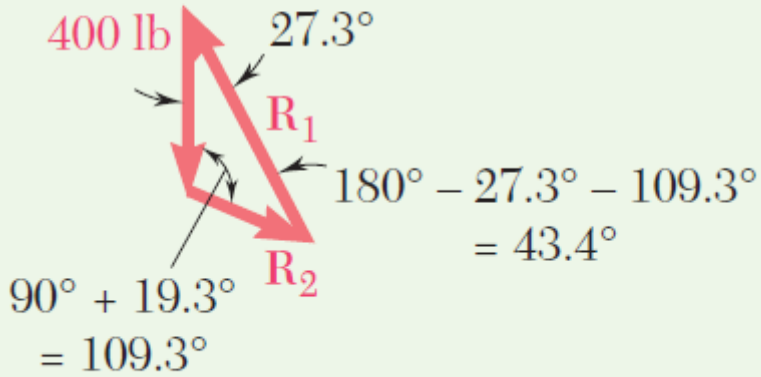


The position of the machine block B is adjusted by moving the wedge A. Knowing that the coefficient of static friction is 0.35 between all surfaces of contact, determine the force **P** required to (*a*) raise block B, (*b*) lower block B.



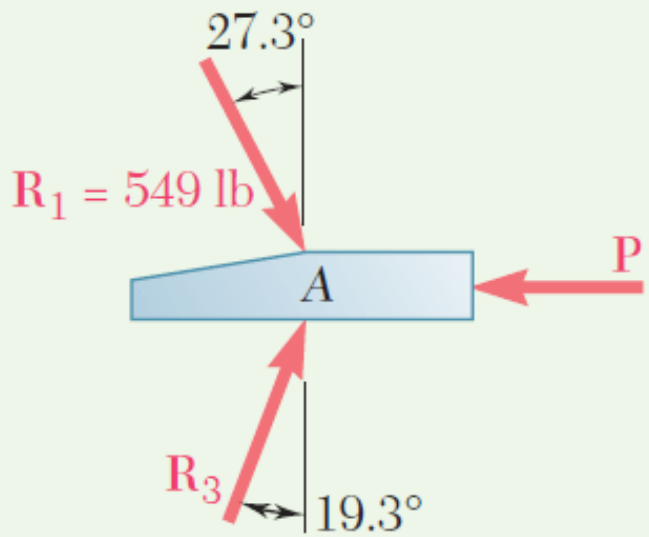
(*a*)

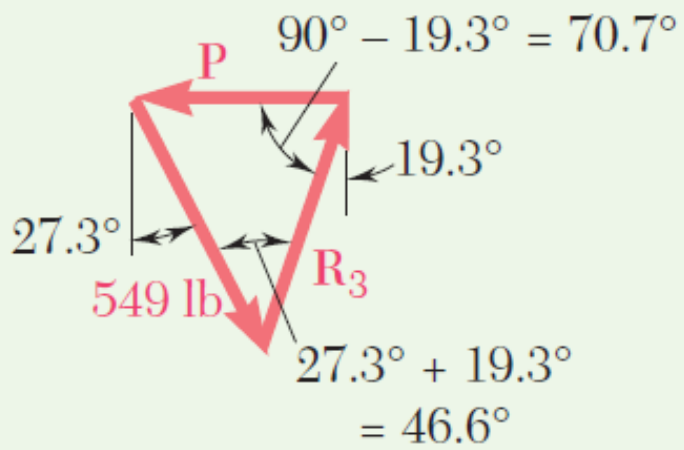




$$\phi_s = \tan^{-1} 0.35 = 19.3°$$

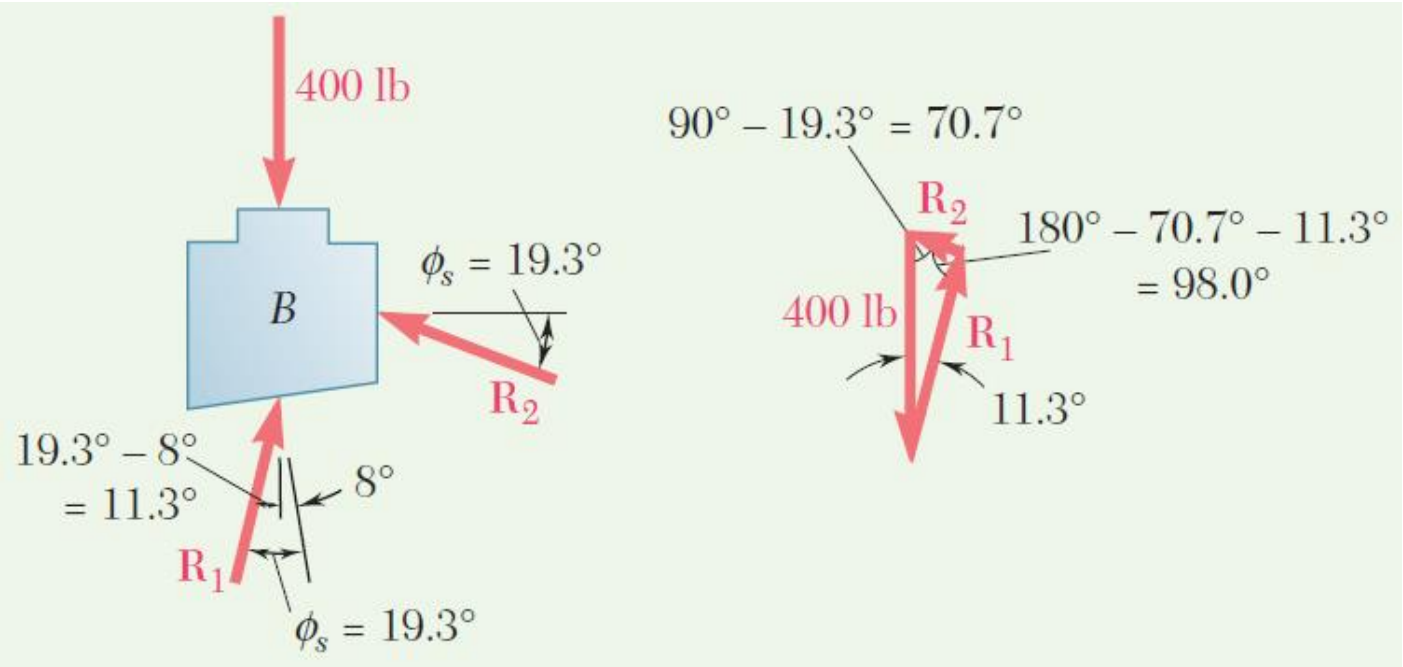
$$\frac{R_1}{\sin 109.3°} = \frac{400 \text{ lb}}{\sin 43.4°} \qquad R_1 = 549 \text{ lb}$$



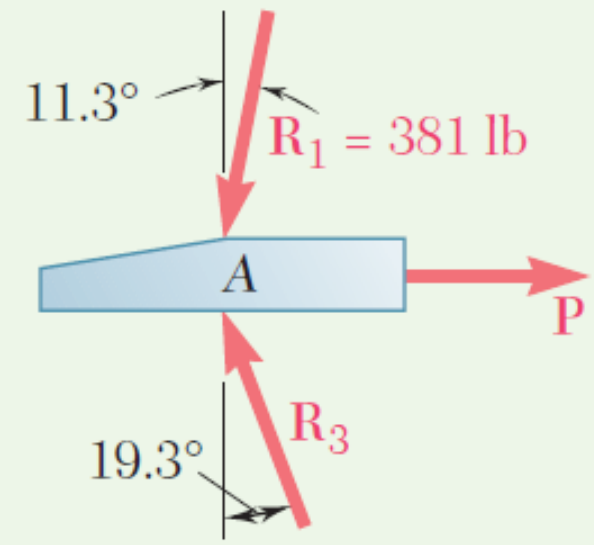


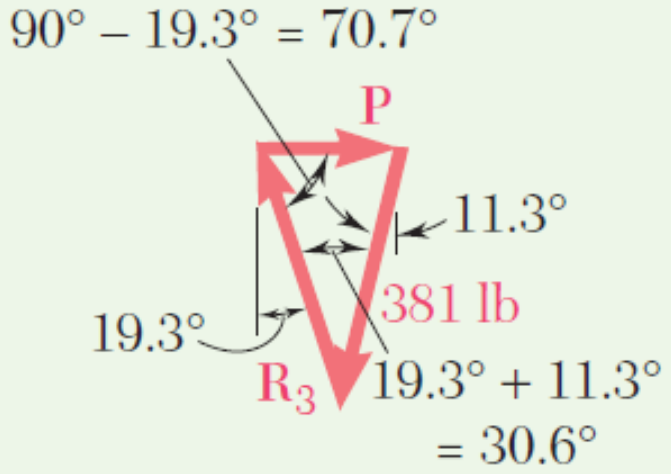
$$\frac{P}{\sin 46.6°} = \frac{549 \text{ lb}}{\sin 70.7°} \qquad \boldsymbol{P} = 423 \text{ lb} \leftarrow$$

(*b*)



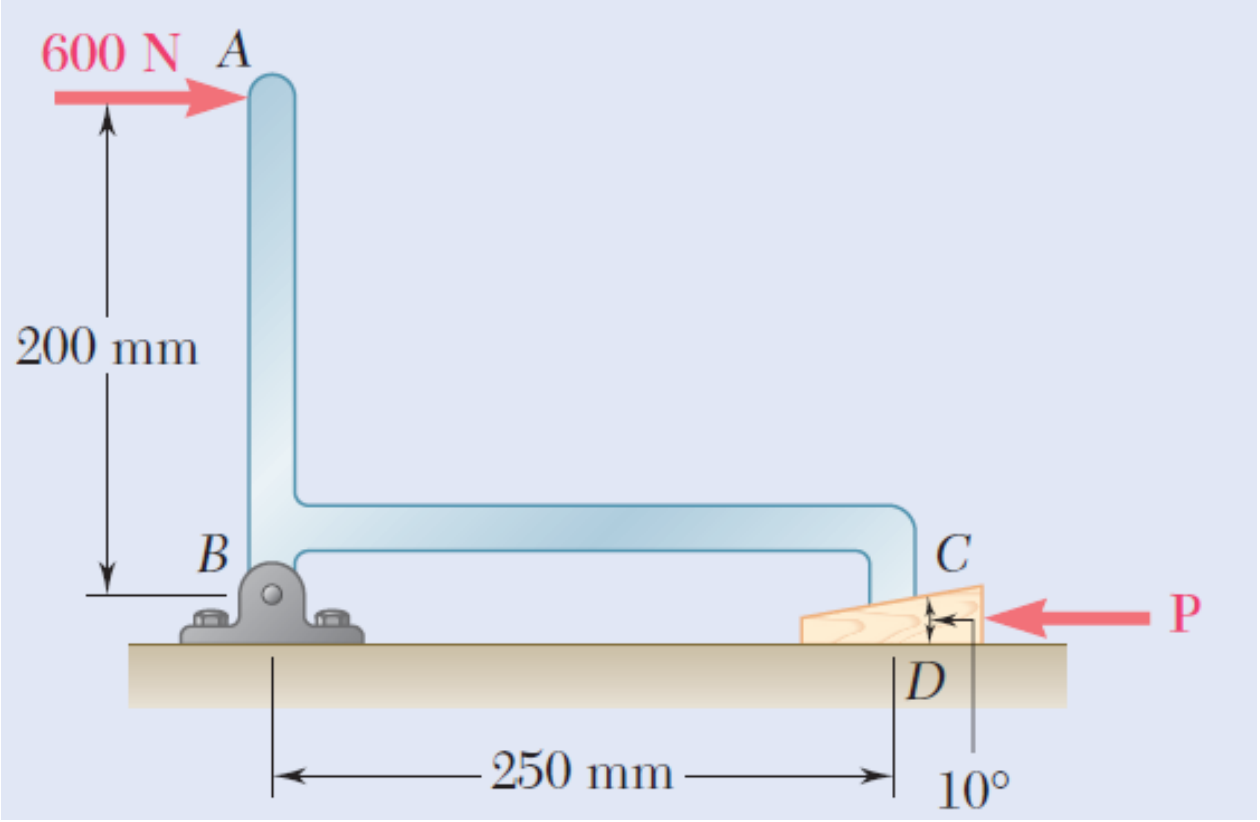
$$\frac{R_1}{\sin 70.7°} = \frac{400 \text{ lb}}{\sin 98.0°} \qquad R_1 = 381 \text{ lb}$$





$$\frac{P}{\sin 30.6°} = \frac{381 \text{ lb}}{\sin 70.7°} \qquad \boldsymbol{P} = 206 \text{ lb} \rightarrow$$

The machine part ABC is supported by a frictionless hinge at B and a 10° wedge at C. Knowing that the coefficient of static friction is 0.20 at both surfaces of the wedge, determine (a) the force **P** required to move the wedge to the left, (b) the components of the corresponding reaction at B.



التاريخ / /

الموضوع :

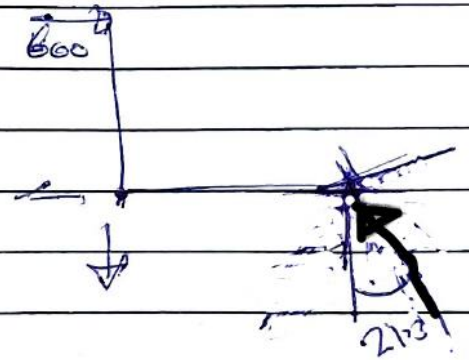
(a)

$\phi_s = \tan^{-1} 0.2 = 11.31^\circ$

$\Sigma M_B = 0$

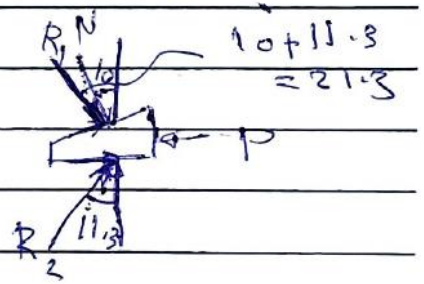
$600 * 200 - R_y * 250 = 0$

$R_y = 480$

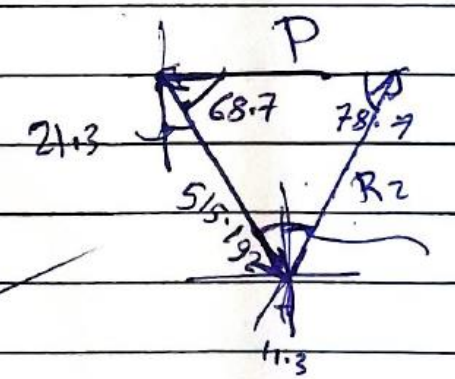


$R_1 \sin 21.3 = 480$

$R_1 = 515.192$



$\frac{P}{\sin 32.6} = \frac{515.192}{\sin 78.7}$



11.3 + 21.3

32.6

$P = 283$

$R_2 = \frac{515.192}{\sin 78.7} * \sin 68.7 =$

$R_2 = 489.488$

$B_x = 412.9$

$B_y = 480$

$\Sigma F_y = 0 : 489.488 \cos 11.3 = B_y$

$+600 + 489.48 * \sin 11.3 - 283 \rightarrow B_x = 0$

Kinetics of Rigid Bodies

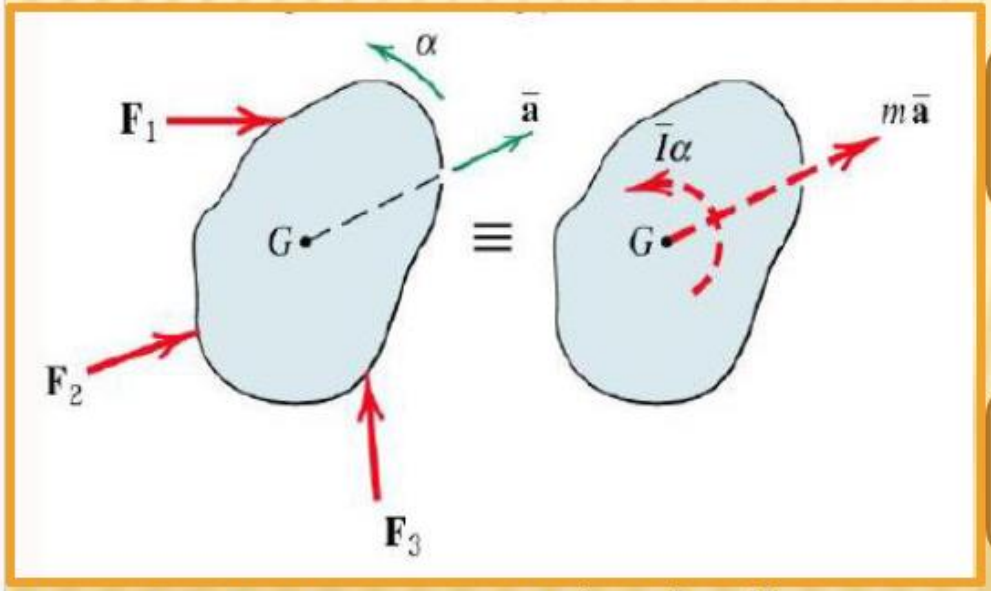
Kinetics: Which relates the action of forces on bodies to their resulting motions.

Rigid Body: Collection of particles that remain at fixed distance from each other at all times and under all conditions of loading

Kinetics of rigid bodies treats the relationships between the external forces acting on a body and the corresponding translational and rotational motions of the body.

FORCE, MASS, AND ACCELERATION

General Equations of Motion



Free body diagram Kinetic diagram

Force Equation

$$\sum \boldsymbol{F} = \boldsymbol{m} \,.\, \boldsymbol{a_G}$$

F = resultant force acting on the rigid body
m = mass of the body
a_G = acceleration of the center of mass, G

Moment Equation (about G)

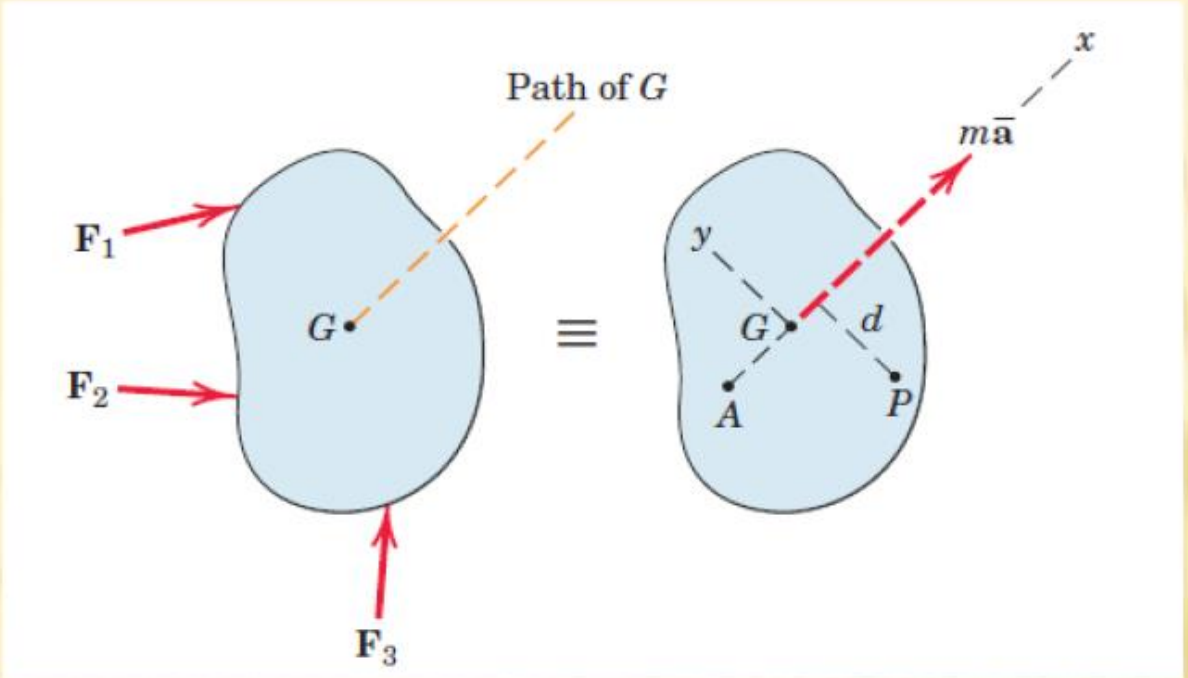
$$\sum \boldsymbol{M_G} = \boldsymbol{I_G} \,.\, \boldsymbol{\alpha}$$

M = Moments about the center, G
I_G = Mass moment of inertia about G
$\boldsymbol{\alpha}$ = the angular acceleration of the body

Activ

Rectilinear Translation

$\alpha = 0$



Force Equation

$$\sum F = m \, . \, a_G$$

Moment Equations Center

$$\sum M_G = I_G \, . \, \cancel{\alpha} \quad \Rightarrow \quad \sum M_G = 0$$

Moment Equations other point

$$\sum M_p = I_G \, . \, \cancel{\alpha} + m \, . \, a \, . \, d \quad \Rightarrow \quad \sum M_p = m \, . \, a \, . \, d$$

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Example: The 1600 kg car has its mass center at G. Calculate the normal forces N_A and N_B between the road and the front and rear Paris of wheels when the acceleration of the car is $2\ \frac{m}{s^2}$ The mass of the wheels are small compared to the mass of the car

$$\sum F_y = 0$$

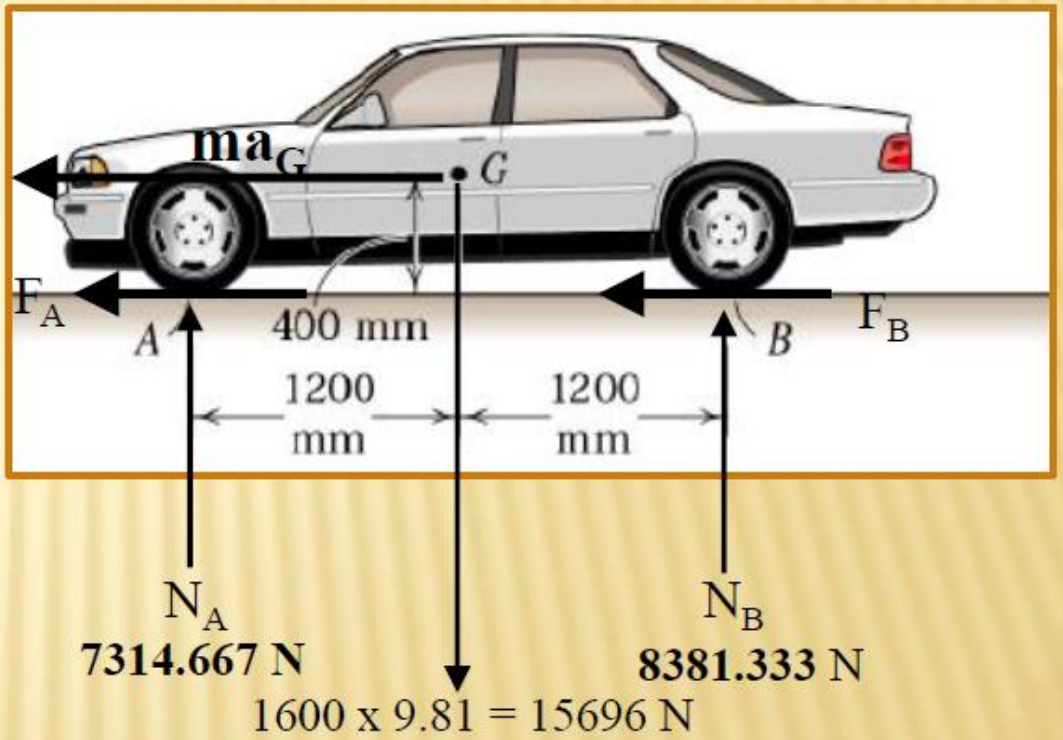
$$N_A + N_B - 15696 = 0 \quad \cdots\cdots\cdots\cdots (1)$$

$$\sum M_A = m.a.d$$

$$15696 \times 1.2 - N_B(2.4) = -1600 \times 2 \times 0.4$$

$$N_B = 8381.333\ N$$ Substitute in (1)

$$N_A = 7314.667\ N$$



Kinetics of Rigid Bodies

Example: The uniform 5 kg bar AB is suspended in vertical position from an accelerating vehicle and restrained by the wire BC. If the acceleration is a = 0.6g, determine the tension T in the wire and the forces at the pin A.

$$a_G = 0.6(9.82) = 5.886\ \frac{m}{s^2}$$

$$\sum M_A = m \,.\, a_G \,.\, d$$

$$0.6\,T\,(0.4) = 5\,x\,5.886\,x\,(0.2)$$

$$T = 24.525\ N$$

$$\sum F_y = 0$$

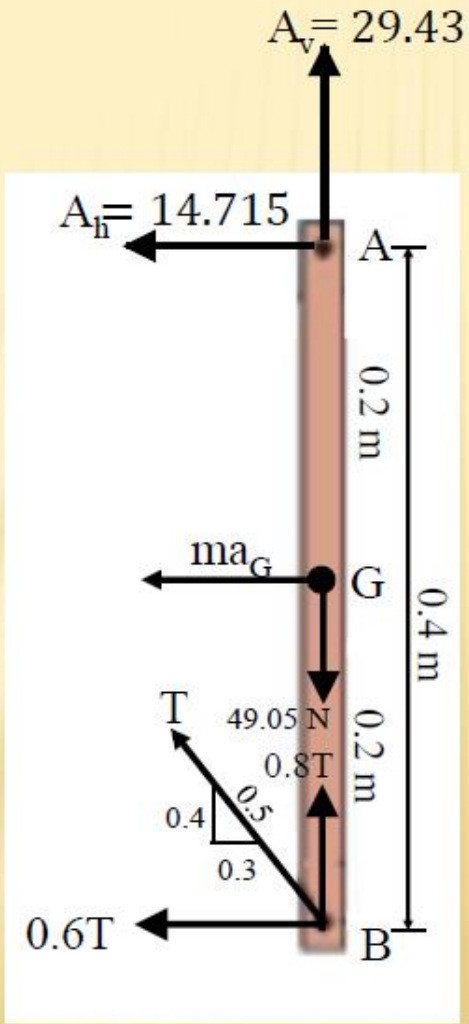
$$0.8\,(24.525) - 49.05 + A_y = 0$$

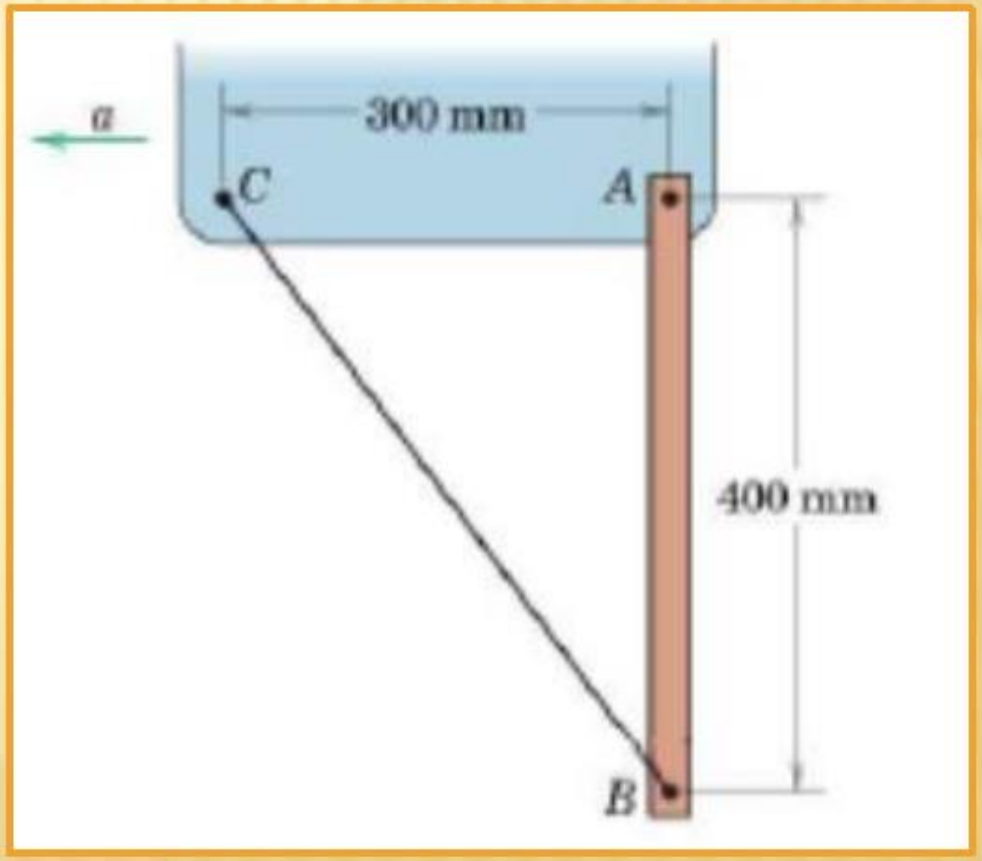
$$A_v = 29.43\ N\ \uparrow$$

$$\sum F_x = m \,.\, a_G$$

$$0.6\,(24.525) + A_h = 5\,x\,5.886$$

$$A_h = 14.715\ \leftarrow$$





Example: The uniform bar of mass 3 kg is pin connected to the collar, which slides along the smooth horizontal rod. If the collar is given a constant acceleration of a= 2 m/s^2, determine the bar's inclination angle θ, and the forces at A. Neglect the collar's mass.

$$\sum F_x = m \, . \, a_G$$

$$A_h = 3 \; x \; 2 \Rightarrow A_h = 6 \; N \rightarrow$$

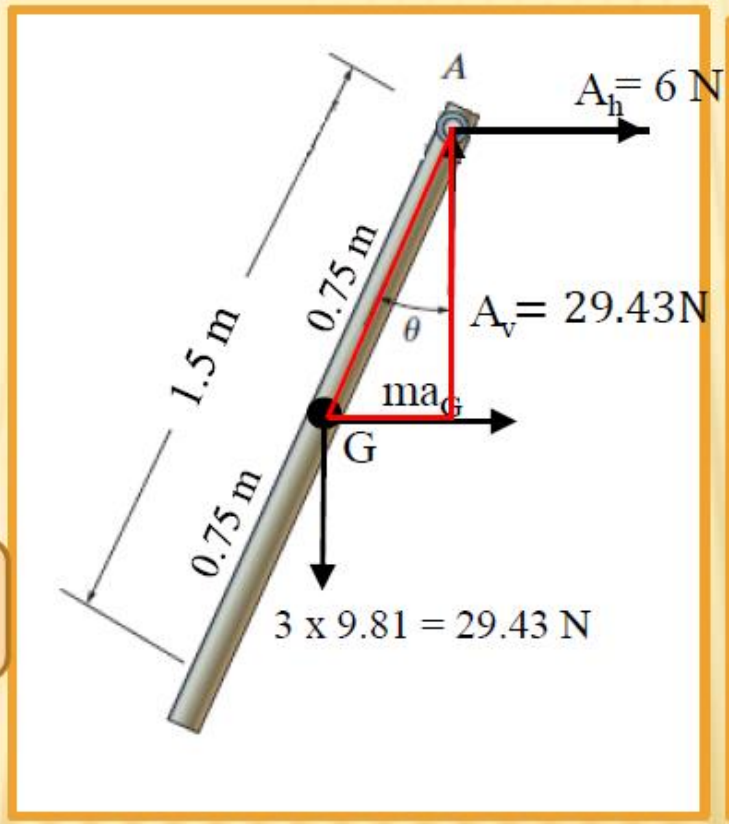
$$\sum F_y = 0$$

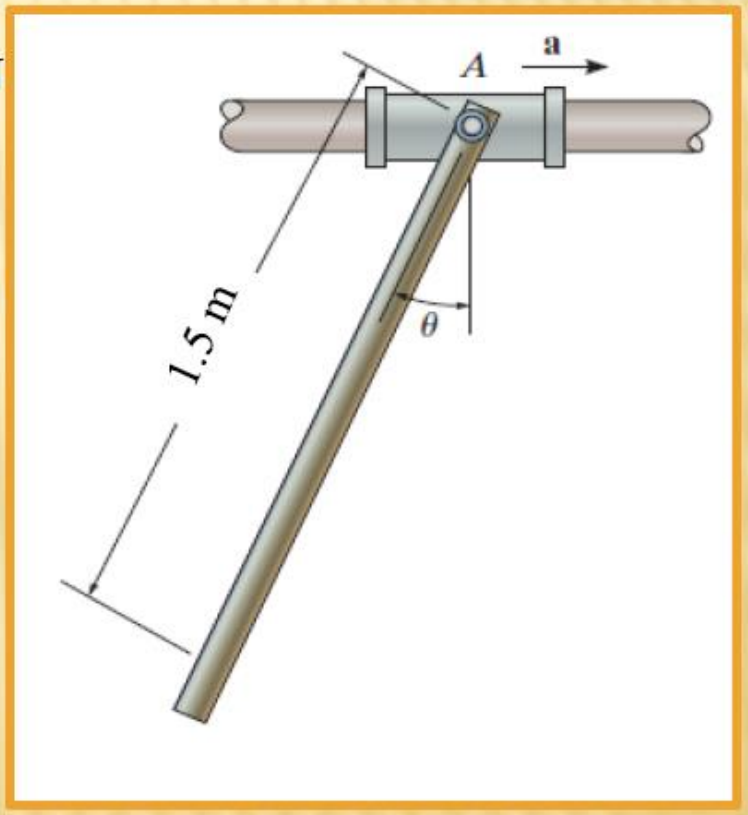
$$A_v - 29.43 = 0 \Rightarrow A_v = 29.43N \uparrow$$

$$\sum M_A = m \, . \, a_G \, . \, d$$

$$-29.43(0.75 \; sin\theta) = -3 \; x \; 2x \; (0.75 \; cos\theta)$$

$$tan \; \theta = 0.20387 \Rightarrow \theta = tan^{-1}(0.20387) = 11.523^{\circ}$$





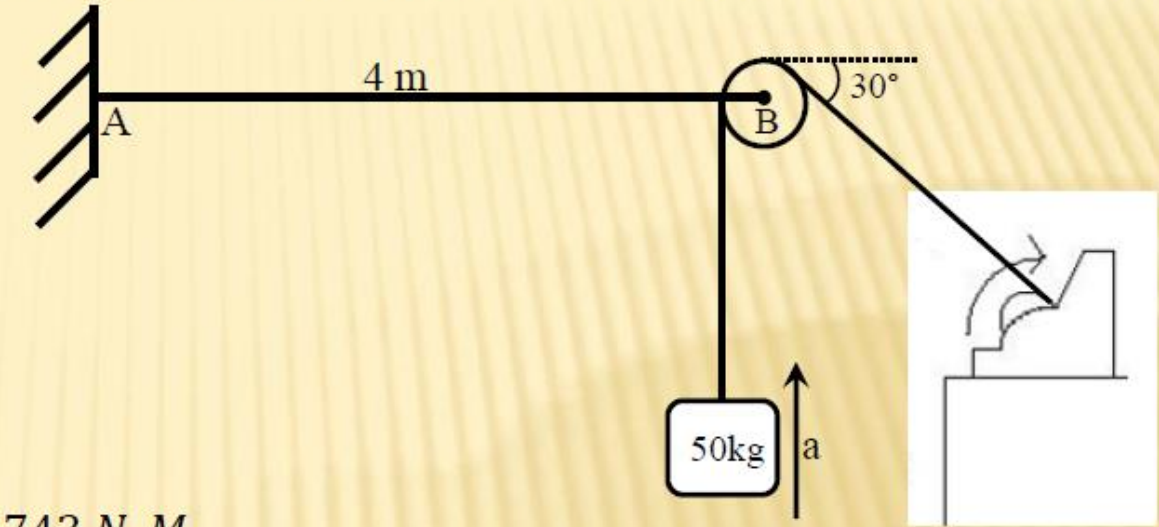
0.75 m θ

0.75 $cos\theta$

0.75 $sin\theta$

Activ

Example: The motor lifts the 50kg crate with an acceleration of 6 m/s². Determine the reactions at the fixed support A.



Beam AB

$$\sum F_x = 0$$

$$Ah = 684.593N \leftarrow$$

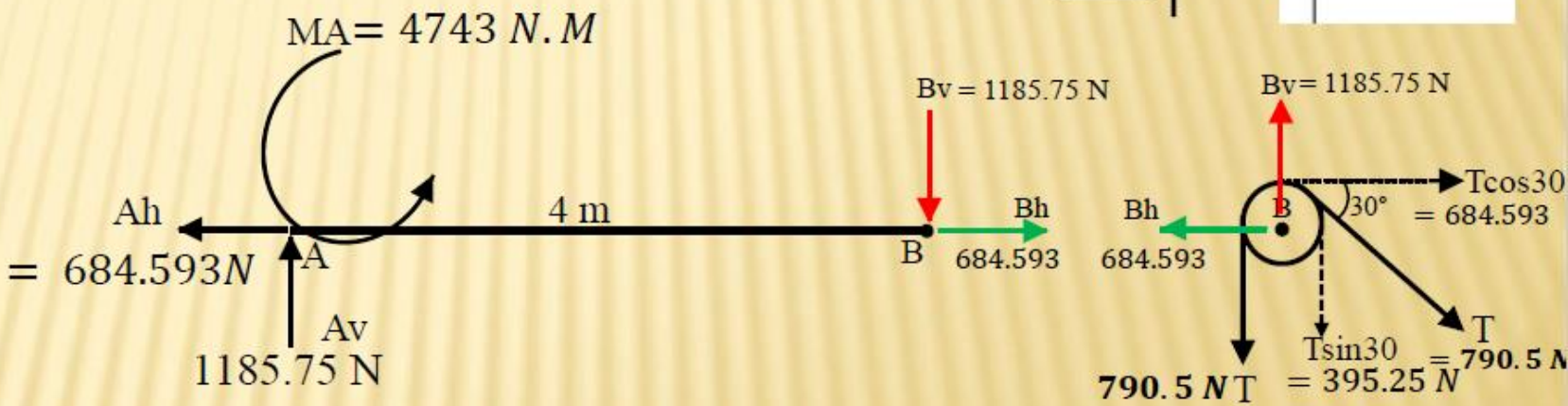
$$\sum F_y = 0$$

$$A_v = 1185.75\ \text{N} \uparrow$$

$$\sum M_A = 0$$

$$-\text{M}A + 1185.75(4) = 0$$

$$MA = 4743\ N.M$$



crate

$$\sum F_y = m.a$$

$$T - 490.5 = 50\ x\ 6 \rightarrow T = 790.5\ N$$

